CURRENT ELECTRICITY

3.1 Introduction: We got familiar with the *static charges* and phenomenon associated with them like *electrostatic forces*, *electric fields*, *electric dipoles*, *potential*, *potential difference*, *conduction*, *induction*, *capacity of a conductor to store the charge* (*capacitance*), *dielectrics* and their effects on *charging capacities of conductors* (*capacitance*) and *potential difference* across them. All these studies come under the *static electricity* branch of the physics. When it comes to the *motion of charge*, it is known as the *dynamics of the charge* and is known as *current* (some type of motion). So, "***the study of electric charges under motion is known as current electricity***".

3.2 Electric Current: If two different bodies are charged to unequal *potentials* and are connected by a conducting wire, the charge begins to flow from one body to another. This flow of charge between two bodies will continue until the *potential* of two bodies become equal.

So, the reader may conclude that the flow of electric charge through a conductor is *electric current*. But more precisely, the *electric current* may be defined as, ***"the charge passing through the cross sectional area of the conductor per unit time (sec) in a direction perpendicular to the cross sectional area of the conductor"***.

If a charge ΔQ passes through a cross sectional area during the time $t \to (t + \Delta t)$, the current I flowing through that conductor at time t may be given as:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \tag{3.1}$$

If the *electric current* is steady, *i.e.* the rate of flow of charge is not changing *w.r.t.* the time, then the *current* flowing through the conductor may be given as:

$$I = \frac{Q}{t} \tag{3.2}$$

or,
$$\text{Electric Current} = \frac{\text{Electric Charge Transferred}}{\text{Time}} \tag{3.3}$$

The examples of flow of steady currents are in the circuit of a torch, electric clocks, electric toys, radio circuits, hearing aids *etc*. The example of flow of a changing current (***transient current***) is Lightening in a thunder storm, which flows for a very short time duration between two clouds or between a cloud and the earth.

SI Unit of Current: *If one coulomb of charge flows through the cross sectional area of a conductor in one second, the current through the conductor is said to be one Ampere* (A).

$$1\ \text{Ampere} = \frac{1\ \text{Coulomb}}{1\ \text{Second}} \qquad \text{or,} \qquad 1\ \text{A} = 1\ \text{C sec}^{-1} \tag{3.4}$$

Let us see the quantity of currents in some daily use applications:

Current in a domestic appliance ≈ 1 A

Current carried by the lightening in a thunder storm ≈ $10^4 - 10^6$ A

Current in electronic circuit's ≈ 1 mA

Current in our nervous system ≈ 1 μA

The reason of Flow of Current: We already have discussed in previous chapters that inside the solids the *mobile charge carrier* is *electron* (*negative charge*) only, the positively charged atom (*ion*) is tightly fixed in the *lattice structure* of the solid. So, ***the reason of flow of current inside a conductor may be***

its free electrons only, which have so small *work function* that they are free to move inside the solid metal / conductor at room temperature and even at subzero temperatures.

Conventional Current and Electronic Current: The flow of current, by convention (*conventional current*), is taken as along the direction of flow of positive charges (*i.e.* from positive terminal to the negative terminal), as shown in the Fig. 3.1 (a). But, we already have discussed that the *mobile charge carriers* are negative charges only inside a solid (*conductor*). So, the current inside a conductor flow due to the flow of *electrons* only, from negative terminal to the positive terminal. So, the direction of flow of current (*electronic current*) is taken as opposite to the direction of flow of electrons inside a conductor.

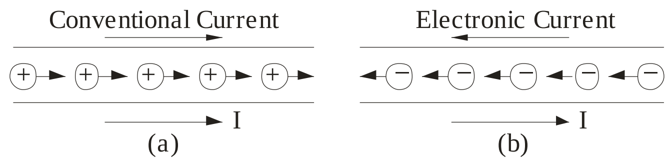

Fig. 3.1

Electric Current is a Scalar Quantity: Although *electric current* has a magnitude and a direction (in which it flows inside a conductor), but it is not a vector for the currents flowing due to batteries or d.c. supplies. ***The d.c. current is a scalar quantity***. This is because of the reason that the rules of basic algebra are applicable to the addition of d.c. currents. Consider the diagram shown in the Fig. 3.2. Two currents I_1 of 5 A (along *AO*) and I_2 of 4 A (along *BO*) are meeting at the point *O*, the current I_3 (along *OC*) may be given as the *algebraic sum of two currents, i.e.* $I_3 = I_1 + I_2 = 5 + 4 = 9$ A.

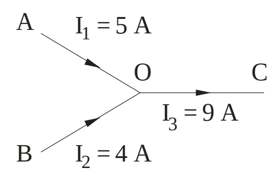

Fig. 3.2

Problem 3.1: *If 10^{20} electrons, each having a charge of 1.6×10^{-19} C, pass from a point A towards another point B in 0.1 sec. What is the value of current in Amperes and its direction?*

Solution: $N = 10^{20}$ electrons, $e = 1.6 \times 10^{-19}$ C, $\Delta t = 0.1$ sec

The current flowing between the points *A* and *B* may be given as:

$$I = \frac{\Delta q}{\Delta t} = \frac{N \times e}{\Delta t} = \frac{10^{20} \times 1.6 \times 10^{-19}}{0.1} = 160 \text{ A}$$

The direction of the flow of current is from the point *B* to the point *A*, *i.e.* in the opposite direction to flow of electrons.

Problem 3.2: *Determine the number of electrons passing through the cross section of a conductor per second for a flow of current of 1 Ampere.* [CBSE 1991-92]

Solution: $I = 1$ A, $\Delta t = 1$ sec, $e = 1.6 \times 10^{-19}$ C

The current flowing through the conductor may be given by the relationship:

$$I = \frac{\Delta q}{\Delta t} = \frac{N \times e}{\Delta t}$$

So, $$N = \frac{I \times \Delta t}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

Problem 3.3: *How many electrons pass through a lamp in one minute, if the current flowing through the lamp is 300 mA.* [Himachal 1994-95, Punjab 2001-02]

Solution: $I = 300$ mA, $t = 1$ min $= 60$ sec, $e = 1.6 \times 10^{-19}$ C

The current flowing through the lamp may be given as:

$$I = \frac{Q}{t} = \frac{N \times e}{t}$$

or, $N = \frac{I \times t}{e} = \frac{300 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = 1.125 \times 10^{20}$ electrons

Problem 3.4: *How many electrons pass through a filament of a 120 V, 60 W electric lamp in one second, if the electric power is product of voltage and the current passing through it.*

Solution: $V = 120$ V, $P = 60$ W, $\Delta t = 1$ sec, $e = 1.6 \times 10^{-19}$ C

Given that, $P = V \times I$

So, $\mathrm{I} = \frac{P}{V} = \frac{60}{120} = 0.5$ A

Now, the current flowing through the filament may be given as:

$$I = \frac{\Delta q}{\Delta t} = \frac{N \times e}{\Delta t}$$

or, $N = \frac{I \times \Delta t}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = 3.125 \times 10^{18}$ electrons

Problem 3.5: *According to Bohr's model, the electrons revolve in the circular path of radius 0.51 Å at a frequency of 6.8 × 10^{15} revolution per second. Determine the equivalent Current.*

Solution: $R = 0.51$ Å, $\nu = 6.8 \times 10^{15}$ rev/sec, $e = 1.6 \times 10^{-19}$ C

The equivalent current may be given as:

$$I = \frac{\Delta q}{\Delta t} = \Delta q \times \nu = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = 1.088 \times 10^{-3} \text{ A} = 1.088 \text{ milli-Amp}$$

Problem 3.6: *An electron moves in an orbit of radius 0.5 Å with a speed of 2.2 × 10^{6} m/sec in a hydrogen atom. Determine the equivalent current.* [Roorkee 1984]

Solution: $R = 0.5$ Å, $v = 2.2 \times 10^{6}$ m/sec, $e = 1.6 \times 10^{-19}$ C

The time taken by the electron in one revolution may be given as:

$$\Delta t = \frac{2\pi r}{v}$$

Now, the equivalent current may be given as:

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{(2\pi r / v)} = \frac{1.6 \times 10^{-19} \times 2.2 \times 10^{6}}{2\pi \times 0.5 \times 10^{-10}} = 1.12 \times 10^{-3} \text{ A} = 1.12 \text{ milli-Amp}$$

Problem 3.7: *The graph of current I through the cross section of a wire is shown in the Fig. 3.3, for a time period of 10 sec. Determine the amount of charge that flows through the wire during this time period.*

Solution: The charge that flows through the circuit during a time period may be given as:

$$q = I \times t$$

But the current is not steady during the time period from $t = 0$ sec to $t = 5$ sec. So, the charge that flows through the circuit for the duration from $t = 0$ sec to $t = 10$ sec may be given by the area under the graph of current I.

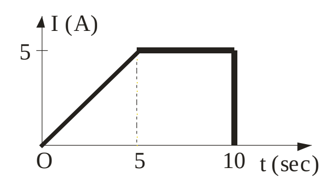

Fig. 3.3

So, $q = \frac{1}{2} \times 5 \times 5 + 5 \times (10 - 5) = 37.5$ C

Problem 3.8: *The amount of charge passing through the cross section of a wire is given by:*

$$q\,(t) = a\,t^2 + b\,t + c$$

i) *Write down the dimensional formula for a, b and c.*

ii) *If the values of a, b and c in SI units are 5, 3 and 1 respectively, determine the value of current at t = 5 sec.*

Solution: Given that, $q\,(t) = a\,t^2 + b\,t + c$, $a = 5$, $b = 3$, $c = 1$

The dimension of $a = \left[\frac{q}{t^2}\right] = \left[\frac{\mathrm{A\,T}}{\mathrm{T}^2}\right] = [\mathrm{A\,T}^{-1}]$

The dimension of $b = \left[\frac{q}{t}\right] = \left[\frac{\mathrm{A\,T}}{\mathrm{T}}\right] = [\mathrm{A}]$

The dimension of $c = q = \mathrm{A\,T} = [\mathrm{A\,T}]$

The value of current at $t = 5$ sec may be given as:

$$I_{(5\,\text{sec})} = \frac{d}{d\,t}\,[q\,(t)]_{t\,=\,5\,\text{sec}} = \frac{d}{d\,t}\,[a\,t^2 + b\,t + c]_{t\,=\,5\,\text{sec}} = [2\,a\,t + b + 0]_{t\,=\,5\,\text{sec}}$$

$$= [2 \times 5 \times 5 + 3 + 0] = 53\ \text{A}$$

Problem 3.9: *One billion electrons pass from a point A towards another point B in 10^{-3} sec. Determine the value of current in Amps and its direction.*

Solution: $N = 10^9$ electrons, $e = 1.6 \times 10^{-19}$ C, $\Delta t = 10^{-3}$ sec

The current flowing between the points *A* and *B* may be given as:

$$I = \frac{\Delta q}{\Delta t} = \frac{N \times e}{\Delta t} = \frac{10^9 \times 1.6 \times 10^{-19}}{10^{-3}} = 1.6 \times 10^{-7}\ \text{A}$$

The direction of the current is from the point *B* towards the point *A*, *i.e.* in the opposite direction of flow of electrons.

Problem 3.10: *If 2.25×10^{20} electrons pass through a wire in one minute, determine the magnitude of the current flowing through the wire.*

Solution: $N = 2.25 \times 10^{20}$ electrons, $e = 1.6 \times 10^{-19}$ C, $t = 1$ min = 60 sec

The current flowing through the wire may be given as:

$$I = \frac{Q}{t} = \frac{N \times e}{t} = \frac{2.25 \times 10^{20} \times 1.6 \times 10^{-19}}{60} = 0.6\ \text{A}$$

Problem 3.11: *A solution of sodium chloride discharges 6.1×10^{16} Na^+ ions and 4.6×10^{16} Cl^- ions in 2 sec. Determine the current passing through the solution.*

Solution: $N_+ = 6.1 \times 10^{16}$ Na^+ ions, $N_- = 4.6 \times 10^{16}$ Cl^- ions, $t = 2$ sec

In an electrolyte both the positive charge as well as the negative charge are mobile carriers, so both ions are responsible for the flow of current. So, the current flowing through the solution may be given as:

$$I = \frac{q_+ + q_-}{t} = \frac{N_+ \times e + N_- \times e}{t} = \frac{(6.1 + 4.6) \times 10^{16} \times 1.6 \times 10^{-19}}{2} = 8.56 \text{ mA}$$

Problem 3.12: *An electric current of 2 μA is passing through a discharge tube. How much charge flows across a cross section of the tube in 5 minutes?*

Solution: $I = 2$ μA, $t = 5$ min. $= 5 \times 60 = 300$ sec

The charge flowing through the cross section of the discharge tube in 5 minutes may be given as:

$$q = I \times t = 2 \times 10^{-6} \times 300 = 6 \times 10^{-4} \text{ C}$$

Problem 3.13: *The only electron in a hydrogen atom makes 6 × 10*^{15} *revolutions per second around the nucleus. Determine the average current at any point on the orbit of electron.*

Solution: $N = 1$, $\nu = 6 \times 10^{15}$ Hz, $e = 1.6 \times 10^{-19}$ C

The equivalent current may be given as:

$$I = \frac{q}{t} = q \times \nu = N\, e \times \nu = 1 \times 1.6 \times 10^{-19} \times 6 \times 10^{15} = 0.96 \times 10^{-3} \text{ A} = 0.96 \text{ mA}$$

Problem 3.14: *An electron moves in a circular orbit of radius 10 cm with a constant speed of 4 × 10*^{6} *m/sec. Determine the electric current at a point on the orbit.*

Solution: $r = 10$ cm, $v = 4 \times 10^6$ m/sec, $e = 1.6 \times 10^{-19}$ C

The time taken by electron in one revolution may be given as:

$$\Delta t = \frac{2\pi r}{v}$$

The equivalent current may be given as:

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{(2\pi r / v)} = \frac{1.6 \times 10^{-19} \times 4 \times 10^6}{2\pi \times 10 \times 10^{-2}} = 1.019 \times 10^{-12} \text{ A}$$

Problem 3.15: *The number of protons drifting across a cross section per second is 1.1 × 10*^{18}*, in a hydrogen discharge tube, while the number of electrons drifting in the opposite direction is 3.1 × 10*^{18} *per second. Determine the current flowing in the discharge tube.*

Solution: $N_+ = 1.1 \times 10^{18}$ protons, $N_- = 3.1 \times 10^{18}$ electrons, $t = 1$ sec

In a gas both the positive charge as well as negative charge are mobile carriers, so both the charges are responsible for the flow of current. So, the current flowing through the discharge tube may be given as:

$$I = \frac{q_+ + q_-}{t} = \frac{N_+ \times e + N_- \times e}{t}$$

$$= \frac{(1.1 + 3.1) \times 10^{18} \times 1.6 \times 10^{-19}}{1} = 0.672 \text{ A}$$

3.3 Maintenance of Steady Current in an Electric Circuit:

Maintenance of Steady Current in an Electric Circuit: The property of flow of the *electric current* is quite similar to the flow of water. We found following similarities between them regarding the flow:

i) Water flows from height towards the depth, similarly the *electric current flows from the higher potential to lower potential*.

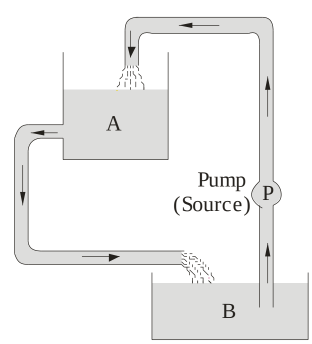

Fig. 3.4

ii) For continuous flow of water in a pipe line between two tanks, as shown in the Fig. 3.4, existence of two pipes is must, one pipe from the tank *A* to the tank *B* and another pipe from the tank *B* to the tank *A* in the form of a closed loop. Also, we can observe that the water is flowing from the tank *A* to the tank *B*, automatically, due to the action of gravity, but for continuous flow of water between two tanks we must transfer the same amount of water per sec from the tank *B* to the tank *A* with the help of a pump (***source***).

So, in case of the continuous flow of the *electric current* in an *electrical circuit* following components are necessary in the circuit:

i) For continuous flow of *electric current* from *source to load*, continuous wiring is required from *source to load to source* in the form of a closed loop.

ii) At least one *source* (*electromotive force*) and one *load* is required, *source* is supplying the *energy* to the *load* by means of the flowing *electric current* through the connecting wires, as shown in the Fig. 3.5.

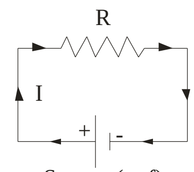

Fig. 3.5

The *source* required to flow the desired *electric current* in an *electrical circuit* has a certain *potential difference* across itself and is known as an *electromotive force* (*emf*), which may be a *battery* or an *electricity generator*.

3.4 **Electromotive Force (EMF):** The schematic diagram of a battery, which maintains a *potential difference* across its two terminals *A* and *B*, is shown in the Fig. 3.6. We know that due to certain chemical reactions a force (of non-electrostatic nature) is exerted on the charges of the electrolyte to accumulate them on positive and negative plates of the battery. Let us assume that, the force on a positive charge is $\vec{F}_n$. As the charges build up in the electrolyte and accumulates on the plates of opposite polarity under the influence of the force $\vec{F}_n$, an electric field $\vec{E}$ is setup inside the battery directed from positive to negative plate of the battery, which exerts a force $\vec{F}_e = q\,\vec{E}$ on them in opposite direction to that of $\vec{F}_n$. Under equilibrium state $\vec{F}_e = \vec{F}_n$ and charges stop accumulating further.

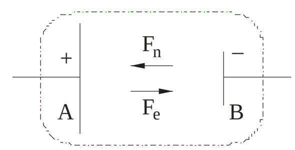

Fig. 3.6

The work done by the chemical (non-electrostatic) force during the accumulation of a positive charge on the plate *A* may be given as:

$$W = F_n\, d \qquad (3.5)$$

Where, $d \rightarrow$ the distance between the plates *A* and *B*.

The work done per unit charge may be given as:

$$E = \frac{W}{q} = \frac{F_n\, d}{q} \qquad (3.6)$$

The quantity *E* is known as the *electromotive force* (*emf*) of the battery / any source.

So, "*the* ***electromotive force*** **(*emf*)** *of a source may be defined as the work done by the source in moving a unit positive charge from lower potential to the higher potential*".

If the battery is open circuited, *i.e.* two terminals are not connected externally:

$$F_n = F_e = q\,\vec{E}$$

or, $$F_n\, d = F_e\, d = q \times \left|\vec{E}\right| d = q\,V \qquad (3.7)$$

Where, $V = \left|\vec{E}\right| d$ (the potential difference between the two terminals)

Now, Electromotive force, $E = \frac{F_n\, d}{q} = \frac{qV}{q} = V$ (3.8)

So, we may say that, "*the electromotive force* (*emf*) *of a source is equal to the potential difference between the terminals of a source when it is open circuited, i.e. not supplying the current to any load*".

SI unit of emf is Volt: *If an electrochemical cell supplies the energy of 1 joule for the flow of 1 coulomb of charge through the whole circuit* (*including the battery*)*, then its emf is said to be one volt.*

Comparison between EMF and Potential Difference (p.d.):

The comparison between *electromotive force* (*emf*) and the *potential difference* may be given as:

Sr. No	Electromotive Force	Potential Difference
1	It is the work done by a source in taking a unit charge once round the complete circuit.	It is the amount of work done in moving a unit charge from one point of circuit to another point of circuit at a different potential.
2	It is the potential difference between the terminals of the source when it is open circuited.	It is the difference of potential between any two points of the circuit at different potentials.
3	It exists even when the circuit is open and not working.	It exists during the working condition of the circuit, *i.e.* when the circuit is closed.
4	It arises due to chemical (non- electrostatic) forces.	It arises due to the electrostatic forces setup by the charges accumulated on two plates of the battery due to chemical forces.
5	It is the cause for developing the potential difference between various points of a circuit on its application.	It is the effect of application of emf.
6	It exists across the terminals of a source only.	Each and every component has its own potential difference across its terminals in a closed circuit caused due to the applied emf across the circuit.
7	It is independent of the external resistance in the circuit.	It depends on the external resistance and current flowing through the external resistance.

3.5 Ohms Law (Applied to Resistance): The great Scientist *George Simon Ohm* observed on the basis of various experiments that there is a definite relationship between the *emf* applied across the circuit and the *electric current* in the circuit and the *potential difference* across the elements of the circuit due to the flow of *current* through them. He gave the law for this relationship, which is universally known as "*Ohm's Law*".

The Ohm's Law states that, "*the current flowing through an element is directly proportional to the potential difference applied across its ends, at a constant temperature and other physical conditions*".

So, Current $\propto$ Potential Difference

or, $I \propto V$ (3.9)

or, $I = \frac{V}{R}$ (3.10)

Where, $\frac{1}{R}$ is proportionality constant, and R is *resistance offered by the element* (*conductor*) *to the flow of current*. The value of R is independent of V and I, but depends on the shape and nature of the conductor, its length and area of cross section and physical conditions like temperature *etc*.

The expression for the *Ohm's law* may also be given as:

$$\frac{V}{I} = R \qquad \text{(a constant for an element / conductor)} \tag{3.11}$$

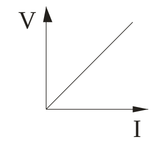

Fig. 3.7

So, the graph between the *potential difference* across a conductor and the *current* flowing through it is a *linear relationship* and is drawn in the Fig. 3.7.

Resistance: *The resistance of a conductor is the property of that material by which it offers a resistance to the flow of current*. This *resistance* is due to the collisions of *electrons* with several atomic and subatomic particles in the *lattice structure* of the material of the conductor during its movement through the metal / conductor.

SI unit of Resistance is Ohm (Ω): Any material having a pure resistance is known as the *resistor* or simply the *resistance*. "*The resistance* (*R*) *of a conductor is said to be one Ohm* (Ω), *if a potential difference of V = 1 Volt across a conductor causes the flow of current of I = 1 Amp in the conductor*".

$$1\ \text{Ohm}\ (\Omega) = \frac{1\ \text{Volt}}{1\ \text{Amp}}$$

or, $$1\ \Omega = 1\ \text{V A}^{-1} \tag{3.12}$$

Symbols for Resistances and some Meters: Various standard symbols for the representation of *resistances* and some *electrical meters* for electrical circuits are shown in the Fig. 3.8.

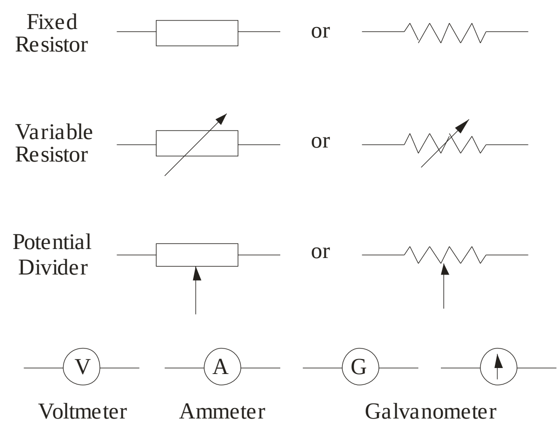

Fig. 3.8

Measurement of Resistance: A suitable circuit diagram for *resistance* measurement is shown in the Fig. 3.9. A battery (d.c. supply source) of emf *E* is supplying a d.c. current (*I*) to the resistance (*R*) to be measured. If the *current* supplied by the source (*I*) is measured with the help of an ammeter in series with the resistor (*R*), and the *potential difference* (*V*) across the resistor due to flow of the current is measured; the resistance of the resistor (*R*) may be calculated as:

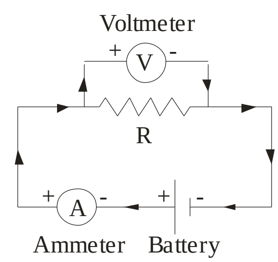

Fig. 3.9

$$R = \frac{V}{I}\ \text{Ohm}\ (\Omega) \tag{3.12}$$

3.6 Factors affecting the Resistance (Length, Cross Sectional Area of Conductor and Resistivity): The resistance of a conductor depends on the following factors at a constant temperature:

Length of the Conductor: The resistance (*R*) of the conductor is directly proportional to the length of the conductor,

i.e, $$R \propto l \tag{3.13}$$

Area of Cross Section of Conductor: The resistance (*R*) of the conductor is inversely proportional to the area of cross section of the conductor,

i.e, $$R \propto \frac{1}{A} \tag{3.14}$$

Resistivity (Inherited property from parent metal): The resistance also depends on the nature of the material used for the conductor. This characteristic property of the material, inherited from the parent metal, is known as *resistivity* or *specific resistance* of the conductor.

Combining equation (3.13) and (3.14):

$$R \propto \frac{l}{A} \qquad \text{or,} \qquad R = \frac{\rho l}{A} \tag{3.15}$$

Where, ρ is proportionality constant and is known as the *resistivity* or *specific resistance* of the material used for the conductor.

So, the resistivity or specific resistance of a conductor may be given as:

$$\rho = \frac{RA}{l} \tag{3.16}$$

So, "*the resistivity or specific resistance may be defined as the resistance offered by a conductor of unit length having a unit cross sectional area*". The resistance is dependent on the shape and size of the conductor but ***resistivity is the basic inherited property of a material and is independent of shape and size of the material***.

SI Unit of Resistivity is Ohm-meter (Ω-m): The unit of resistivity / specific resistance may be given as:

$$\rho = \frac{RA}{l} = \frac{\text{Ohm}\times\text{Meter}^2}{\text{Meter}} = \text{Ohm-meter } (\Omega\text{-m}) \tag{3.17}$$

3.7 Current Density, Conductance and Conductivity:

Current Density: "*The current density at any point inside a conductor may be defined as the amount of current* (*charge*) *flowing normal to per unit area of cross section of the conductor at that point*". The current density is a vector quantity, as it is normal to the cross sectional area (*area is a vector*). It may be represented by $\vec{J}$.

If a current *I* is flowing uniformly and normal to an area of cross section *A* of the conductor, as shown in the Fig. 3.10 (a), then the magnitude of the current density at any point of this cross section may be given as:

$$J = \frac{I}{A} \tag{3.18}$$

If the area *A* is not perpendicular to the direction of flow of the current *I*, but normal to this area *A* makes an angle θ with the direction of current, as shown in the Fig. 3.10 (b), then the component of area *A* along the direction of flow of current may be given as:

$$A_n = A \cos\theta$$

So, now the current density may be given as:

$$J = \frac{I}{A\cos\theta} \tag{3.19}$$

or,

$$I = JA\cos\theta = \vec{J}\cdot\vec{A} \tag{3.20}$$

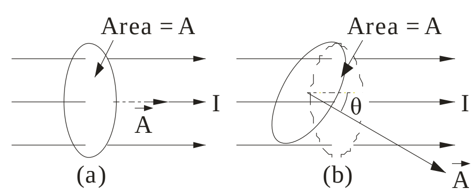

Fig. 3.10

This equation indicates that the electric current is an scalar quantity being dot product of two vector quantities ($\vec{J}$ and $\vec{A}$).

The SI unit of current density is ***Ampere per square meter*** (A m^{-2}) and its dimension is [L^{-2} A].

Conductance: *The conductance of a conductor may be defined as the ease with which the current / charge may flow through a conductor*. Mathematically, the *conductance is equal to the reciprocal of the resistance of the conductor* and may be denoted by *G*.

$$G = \frac{1}{R} \tag{3.21}$$

The SI unit of the conductance is **Ohm^{-1}** or **Siemens (S)**.

Conductivity: *The reciprocal of the resistivity / specific resistance is known as conductivity* and is denoted by σ.

So, Conductivity, $\sigma = \dfrac{1}{\rho}$ (3.22)

The SI Unit of conductivity / specific conductance may be given as:

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{\text{Meter}}{\text{Ohm}\times\text{Meter}^2} = \text{Siemens-meter}^{-1}\ (\text{S-m}^{-1}) \tag{3.23}$$

Vector Form of Ohm's Law: If $|\vec{E}|$ is the magnitude of the *electric field* applied across a conductor of length *l*, then the *potential difference* across the conductor may be given as:

$$V = |\vec{E}|\ l$$

According to Ohm's Law:

$$V = I\,R = I \times \frac{\rho l}{A}$$

or, $$V = |\vec{E}|\ l = \frac{I}{A} \times \rho\ l$$

or, $$|\vec{E}| = J\,\rho \tag{3.24}$$

The direction of electric field and the direction of current density vectors are same.

So, $$\vec{E} = \rho\ \vec{J} \tag{3.25.1}$$

or, $$\vec{J} = \sigma\ \vec{E} \tag{3.25.2}$$

The equations (3.25) are known as ***Vector Form of Ohm's Law***.

3.8 Classifications of Materials in terms of Resistivity: The solids may be divided into three categories depending on their *resistivity*, as the *resistivity* of different materials varies over a wide range.

Conductors: *The materials which offer very low resistance to the flow of current across them, and can conduct electric current fairly well are known as conductors*. All the metals are good conductors of *electric current*. They have low *resistivity* in the range of 10^{-8} Ω-m to 10^{-6} Ω-m. Copper as well as aluminum have the lowest *resistivity* of all the metals, so their wires are used for transmission of *electric current* over long distances without the appreciable loss of energy. On the other hand the nichrome has a *resistivity* of about 60 times that of the copper, so it is used in the heating elements of electric heater and electric iron.

Insulators: *The materials which offer very high resistance to the flow of current across them, and cannot conduct electric current at all are known as insulators*. They have very high *resistivity*, generally more than 10^4 Ω-m. Insulators like mica, Bakelite, glass and hard rubber have very high *resistivity* in the range of 10^{14} Ω-m to 10^{16} Ω-m. They are used for the blocking of current for the safety purposes of human beings.

Semi-Conductors: *The materials having resistivity in between the conductors and insulators, i.e. between 10^{-6} Ω-m to 10^4 Ω-m, are known as semi-conductors*. Germanium (*Ge*) and Silicon (*Si*) are typical semiconductors. The moderately high resistors (in the range of a few kΩ) are generally made up of carbon (graphite) or semiconducting materials.

Common Commercial Resistors: The commercially used *resistors* are of two types in general:

***i*) Wire-Wound Resistors:** These *resistors* are made up of winding the wires of alloys having high resistivity like: manganin, constantan or nichrome on an insulating base. The advantage of using these alloys is that they are insensitive to temperature, so their resistance remains constant irrespective of their temperature. But their size is inconveniently large due to large length of wire required for making a high resistance.

***ii*) Carbon Resistors:** They are made from the mixture of carbon black, clay and resin binder which are pressed together and then molded into cylindrical rods by heating. These cylindrical rods are enclosed in ceramic or plastic jacket. The carbon resistors are widely used in electronic circuits of radio receivers, amplifiers, antennas *etc*. They have following advantages:

a) They can be made with resistance values ranging from a few ohms to several million ohms.

b) They are quite cheap and compact in size.

c) They are quite light and good enough for use in small circuits.

The only dis-advantage associated with them is that their current rating is very small, so they cannot be used in the circuits where flow of large current is involved in the circuits.

3.9 Carbon Color Coding for Carbon Resistors: These resistances are so small that it is impossible to write their values on them, so a *color coding scheme* is used to identify the value of carbon resistances and the percentage accuracy of the resistors. This coding is a standard all around the world. The color coding scheme is given in the table below:

Number	Color	Initials for memory	Multiplier	Color	Tolerance
0	Black	*B*	10^0	Gold	5%
1	Brown	*B*	10^1	Silver	10%
2	Red	*R*	10^2	No fourth Band	20%
3	Orange	*O*	10^3		
4	Yellow	*Y*	10^4		
5	Green	*G*	10^5		
6	Blue	*B*	10^6		
7	Violet	*V*	10^7		
8	Grey	*G*	10^8		
9	White	*W*	10^9		

B	***B***	***R***	***O***	***Y***	of	***G**reat*	***B**ritain*	had	***V**ery*	***G**ood*	***W**ife*
0	1	2	3	4		5	6		7	8	9

There are two systems according to which the *color coding* may be marked on a carbon resistor.

First System: A set of colored coaxial rings / bands are printed on the resistors which gives the following information:

i) The first ring indicates the *first significant figure*.

ii) The second ring indicates the *second significant figure*.

iii) The third ring indicates the *power of ten to be multiplied* to above two significant figures.

iv) The fourth ring indicates the value of accuracy of the resistor. If fourth ring is absent the value of accuracy is ± 20%.

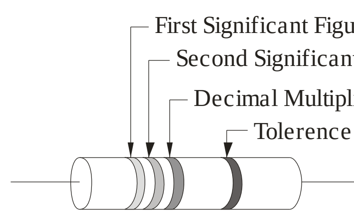

Fig. 3.11

Second System: A second system of colored resistor with a dot and ends with different color is also used, which gives the following information:

i) The color of the body indicates the *first significant figure*.

ii) The color of ends indicates the *second significant figure*.

iii) The color of dot indicates the *power of ten to be multiplied* to above two significant figures.

iii) The color of ring indicates the value of accuracy of the resistor, if the ring is absent the value of accuracy is ± 20%.

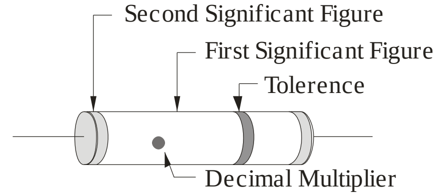

Fig. 3.12

Problem 3.16: *The number of hydrogen ions* (H^+*, i.e. protons*) *drifting across a cross section of a discharge tube is* 1×10^{18} *per second, while the number of electrons* (e^-) *drifting in the opposite direction across another cross section is* 2.7×10^{18} *per second. If the supply voltage is 230 V, determine the effective resistance of the discharge tube.* [NCERT]

Solution: $N_{H+} = 1 \times 10^{18}$ protons / second, $N_{e-} = 2.7 \times 10^{18}$ electrons / second, $V = 230$ V,

$e = 1.6 \times 10^{-19}$ C

The current flowing through the discharge tube may be given as:

$$I = \frac{q_{H^+} + q_{e^-}}{t} = \frac{N_{H^+} \times e + N_{e^-} \times e}{t} = \frac{e \times (N_{H^+} + N_{e^-})}{t}$$

$$= \frac{1.6 \times 10^{-19} \times (1 + 2.7) \times 10^{18}}{1} = 0.592 \text{ A}$$

So, the effective resistance of the discharge tube may be given as:

$$R = \frac{V}{I} = \frac{230}{0.592} = 388.514\ \Omega$$

Problem 3.17: *An electron beam has an aperture of 1* mm^2*. A total of* 6×10^{16} *electrons flow through any perpendicular cross section per second. Determine: i*) *the current , ii*) *the current density in the electron beam.*

Solution: $A = 1$ mm^2, $N_{e-} = 6 \times 10^{16}$ electrons / second

The current flowing through the area may be given as:

$$I = \frac{q}{t} = \frac{N_{e^-} \times e}{t} = \frac{6 \times 10^{16} \times 1.6 \times 10^{-19}}{1} = 9.6 \times 10^{-3} \text{ A}$$

And, the current density (current per unit area) may be given as:

$$J = \frac{I}{A} = \frac{9.6 \times 10^{-3}}{1 \times 10^{-6}} = 9.6 \times 10^{3} \text{ A m}^{-2}$$

Problem 3.18: *A copper wire having a radius of 0.1 mm and resistance of 1 KΩ is connected across a power supply of 20 V. i*) *How many electrons transferred per second between the supply and the wire at one end, ii*) *Determine the current density in the wire.*

Solution: $r = 0.1$ mm, $R = 1$ kΩ, $V = 20$ V

The current flowing through the wire may be given as:

$$I = \frac{V}{R} = \frac{20}{1 \times 10^{3}} = 20 \text{ mA}$$

The charge flowing through the wire per second may be given as:

$$q = N\,e = I \times t = 20 \times 10^{-3} \times 1 = 20 \times 10^{-3}\text{ C}$$

or, $$N = \frac{I \times t}{e} = \frac{20 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 1.25 \times 10^{17}\text{ electrons}$$

The current density in the copper wire may be given as:

$$J = \frac{I}{A} = \frac{I}{(\pi r^2)} = \frac{20 \times 10^{-3}}{\pi \times (0.1 \times 10^{-3})^2} = 6.366 \times 10^{5}\text{ A m}^{-2}$$

Problem 3.19: *A current flows through a constricted conductor as shown in the Fig. 3.13. The two diameters are given as D_1 = 2 mm and D_2 unknown. The current density at cross section with diameter D_1 is given as $J = 1.27 \times 10^6$ A m^{-2}. i) Determine the value of current flowing into the constriction, ii) If the current density is doubled when it emerges from the right side of the constriction, determine the diameter D_2.*

Solution: D_1 = 2 mm, $J_{D1} = 1.27 \times 10^6$ A m^{-2}, $J_{D2} = 2 \times J_{D1}$

The current across the cross section having diameter D_1, and into the constriction may be given as:

$$I = J_{D1} \times A_1 = J_{D1} \times \pi \left(\frac{D_1}{2}\right)^2$$

$$= 1.27 \times 10^6 \times \pi \times \left(\frac{2 \times 10^{-3}}{2}\right)^2 = 3.989\text{ A}$$

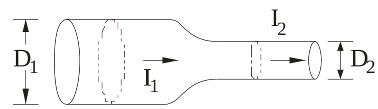

Fig. 3.13

Given that, $J_{D2} = 2 \times J_{D1}$

or, $$\frac{I}{A_2} = 2 \times \frac{I}{A_1} \qquad \text{or,} \qquad A_1 = 2 \times A_2$$

or, $$\pi \left(\frac{D_1}{2}\right)^2 = 2 \times \pi \left(\frac{D_2}{2}\right)^2$$

So, $$D_2 = \frac{D_1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414\text{ mm}$$

Problem 3.20: *A current of 2 mA is passed through a color coded carbon resistor with first, second and third rings of yellow, green and orange colors. Determine the voltage drop across the resistance.* [CBSE 1997-98]

Solution: I = 2 mA, First Ring = Yellow, Second Ring = Green, Third Ring = Orange

The resistance of the carbon resistor may be given as:

Yellow	Green	Orange
4	5	3

$$R = 45 \times 10^3\ \Omega$$

The voltage drop across the resistor may be given as:

$$V = I\,R = 2 \times 10^{-3} \times 45 \times 10^3 = 90\text{ V}$$

Problem 3.21: *An arc lamp operates at 80 V, 10 A. Suggest a method to use it with a 240 V d.c. source. Calculate the value of the electric component required for this purpose.* [CBSE 1993-94]

Solution: V_{Lamp} = 80 V, I_{Lamp} = 10 A, V_{Source} = 240 V

The resistance of the lamp may be given as:

$$R_{\text{Lamp}} = \frac{V_{Lamp}}{I_{Lamp}} = \frac{80}{10} = 8\ \Omega$$

If the lamp of 80 V, 10 A is to be operated on the source of 240 V, a resistor in series with the lamp is required to limit the current to its rated value (10 A). Let the resistance used be R_{ext}.

The current flowing through the series combination of the lamp and the resistor may be given as:

$$I_{\text{Lamp}} = \frac{V_{Source}}{R_{Lamp} + R_{ext}}$$

or, $$R_{\text{Lamp}} + R_{\text{ext}} = \frac{V_{Source}}{I_{Lamp}}$$

or, $$R_{\text{ext}} = \frac{V_{Source}}{I_{Lamp}} - R_{\text{Lamp}} = \frac{240}{10} - 8 = 16\ \Omega$$

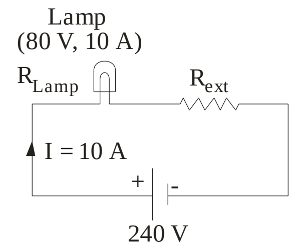

Fig. 3.14

Problem 3.22: *Calculate the resistivity of a material of a wire 10 m long, 0.4 mm in diameter and having a resistance of 2 Ω.* [Punjab 1993-94, Haryana 2001-02]

Solution: l = 10 m, d = 0.4 mm, R = 2 Ω

The resistivity / specific resistance of the material may be given as:

$$\rho = \frac{R\,A}{l} = \frac{R \times (\pi d^2/4)}{l} = \frac{2 \times \pi \times (0.4 \times 10^{-3})^2}{4 \times 10} = 2.513 \times 10^{-8}\ \Omega\text{-m}$$

Problem 3.23: *The external diameter of a 5 meter long hollow copper tube is 10 cm and the thickness of its walls is 5 mm. If specific resistance of the copper is 1.7 × 10^{-8} Ω-m, determine its resistance.*

Solution: l = 5 m, D_{O} = 10 cm, t = 5 mm, ρ = 1.7 × 10^{-5} Ω-m

The internal diameter of the hollow tube may be given as:

$$D_{\text{I}} = (D_{\text{O}} - 2\,t)$$

The area of cross section of the tube may be given as:

$$A = A_{\text{O}} - A_{\text{I}} = \pi \times \frac{(D_O)^2}{4} - \pi \times \frac{(D_O - 2t)^2}{4}$$

$$= \frac{\pi}{4} \times (D_{\text{O}}^2 - D_{\text{O}}^2 - 4\,t^2 + 4\,D_{\text{O}}\,t) = \pi \times (D_{\text{O}}\,t - t^2)$$

$$= \pi \times [0.10 \times 0.005 - (0.005)^2] = 1.49 \times 10^{-3}\ \text{m}^2$$

The resistance of the hollow copper tube may be given as:

$$R = \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 5}{1.49 \times 10^{-3}} = 5.71 \times 10^{-5}\ \Omega$$

Problem 3.24: *Determine the resistivity of a conductor in which a current density of 2.5 A-m^{-2} is found to exist, when an electric field of 15 V-m^{-2} is applied across it.*

Solution: $J = 2.5$ A-m^{-2}, $E = 15$ V-m^2

The current density in a conductor may be given as:

$$J = \sigma E = \frac{E}{\rho}$$

So, $$\rho = \frac{E}{J} = \frac{15}{2.5} = 6 \ \Omega\text{-m}$$

Problem 3.25: *Calculate the electrical conductivity of the material of a conductor of length 3 m area of cross section 0.02 mm^2 having a resistance of 2 Ω.* [CBSE 1995-96]

Solution: $l = 3$ m, $A = 0.02$ mm^2, $R = 2\ \Omega$

The conductivity of the material may be given as:

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{2 \times 0.02 \times 10^{-6}} = 7.5 \times 10^7 \text{ S-m}^{-1}$$

Problem 3.26: *A wire of resistance 4 Ω is used to wind a coil of radius 7 cm. The wire has a diameter of 1.4 mm and the specific resistance of its material is 2 × 10^{-7} Ω-m. Determine the number of turns in the coil.*

Solution: $R = 4\ \Omega$, $r = 7$ cm, $d = 1.4$ mm, $\rho = 2 \times 10^{-7}\ \Omega$-m

The resistance of the wire may be given as:

$$R = \frac{\rho l}{A} = \frac{\rho l}{(\pi d^2/4)} = \frac{4\rho l}{\pi d^2}$$

or, $$l = \frac{\pi d^2 \times R}{4\rho}$$

If there are N turns in the coil, the length of the wire may also be given as:

$$l = 2\pi r \times N$$

So, $$N \times 2\pi r = \frac{\pi d^2 \times R}{4\rho}$$

or, $$N = \frac{\pi d^2 \times R}{4\rho \times 2\pi r} = \frac{\pi \times (1.4 \times 10^{-3})^2 \times 4}{4 \times 2 \times 10^{-7} \times 2\pi \times 7 \times 10^{-2}} = 70 \text{ turns}$$

Problem 3.27: *A wire of resistance 10 Ω is stretched to thrice its original length. Determine it's new: i) Resistivity, ii) Resistance.* [CBSE 1993-94, 1997-98]

Solution: $R_1 = 10\ \Omega$, $l_2 = 3 \times l_1$

The volume of the material of the wire remains constant and may be given as:

$$Volume = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{3l_1}{l_1} = 3$$

The resistivity of the material does not depend on the length and area of the material.

So, $\rho_2 = \rho_1 = \rho$

and, $$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{3l_1}{l_1} \times 3$$

or, $R_2 = R_1 \times 9 = 10 \times 9 = 90\ \Omega$

Problem 3.28: *A wire of resistance 16 Ω is melted and redrawn into a wire of half of its original length. Determine the resistance of the new wire. Also, calculate the percentage change in its resistance.* [CBSE 1993-94]

Solution: $R_1 = 16\ \Omega$, $l_2 = \frac{1}{2} \times l_1$

The volume of the material of the wire remains constant and may be given as:

$$Volume = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{(1/2) \times l_1}{l_1} = \frac{1}{2}$$

The ratio of the resistances of the wire before and after redrawing may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{(1/2) \times l_1}{l_1} \times \frac{1}{2}$$

or, $R_2 = R_1 \times \frac{1}{4} = 16 \times \frac{1}{4} = 4\ \Omega$

The percentage change in the resistance may be given as:

$$\%\ \text{Change} = \frac{R_1 - R_2}{R_1} \times 100\% = \frac{16 - 4}{16} \times 100\% = 75\%$$

Problem 3.29: *If the resistance of a wire is R, determine its new resistance, when it is stretched to n times its original length.*

Solution: $R_1 = R$, $l_2 = n \times l_1$

The volume of the material of the wire remains constant and may be given as:

$$Volume = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{n \times l_1}{l_1} = n$$

The ratio of the resistances of the wire before and after stretching may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{n \times l_1}{l_1} \times n$$

or, $R_2 = R_1 \times n^2 = n^2 R$

Problem 3.30: *A cylindrical wire is stretched to increase its length by 10%. Determine the percentage increase in the resistance.*

Solution: $R_1 = R$, $l_2 = 1.1 \times l_1$

The volume of the material of the wire remains constant and may be given as:

$$Volume = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{1.1\times l_1}{l_1} = 1.1$$

The ratio of the resistances of the wire before and after stretching may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1}\times\frac{A_1}{A_2} = \frac{1.1\times l_1}{l_1} \times 1.1$$

or, $R_2 = R_1 \times 1.1^2 = 1.21\ R$

The percentage increase in the resistance may be given as:

$$\%\ \text{Change} = \frac{R_1 - R_2}{R_1} \times 100\% = \frac{1.21R - R}{R} \times 100\% = 21\%$$

Problem 3.31: *Two wires A and B of equal mass and of the same metal are taken. The diameter of the wire A is half the diameter of wire B. If the resistance of wire A is 24 Ω, determine the resistance of* wire *B.*

Solution: $M_A = M_B$, $D_A = \frac{1}{2}\ D_B$, $R_A = 24\ \Omega$

The mass of the wires as well as the metal of the wires *A* and *B* are same, so the volume of two wires will also be the same.

So, Volume = $A_A \times l_A = A_B \times l_B$

or, $$\pi\ \frac{D_A^2}{4} \times l_A = \pi\ \frac{D_B^2}{4} \times l_B$$

So, $$\frac{l_B}{l_A} = \left(\frac{D_A}{D_B}\right)^2 = \left[\frac{(1/2)\,D_B}{D_B}\right]^2 = \frac{1}{4}$$

The ratio of the resistances of two wires may be given as:

$$\frac{R_B}{R_A} = \frac{(\rho l_B / A_B)}{(\rho l_A / A_A)} = \frac{l_B}{l_A}\times\frac{A_A}{A_B} = \frac{l_B}{l_A}\times\frac{(\pi D_A^2/4)}{(\pi D_B^2/4)} = \frac{l_B}{l_A}\times\left(\frac{D_A}{D_B}\right)^2 = \frac{1}{4}\times\left[\frac{(1/2)\,D_B}{D_B}\right]^2$$

or, $$R_B = R_A \times \frac{1}{16} = 24 \times \frac{1}{16} = 1.5\ \Omega$$

Problem 3.32: *A piece of silver has a resistance of 1 Ω. What will be the resistance of a constantan wire of one third length and one half diameter, if the specific resistance of constantan is 30 times that of silver?*

Solution: $R_{ag} = 1\ \Omega$, $l_{Cn} = \frac{1}{3}\ l_{Ag}$, $D_{Cn} = \frac{1}{2}\ D_{Ag}$, $\rho_{Cn} = 30\ \rho_{Ag}$

The ratio of resistances of two wires may be given as:

$$\frac{R_{Cn}}{R_{Ag}} = \frac{(\rho_{Cn}\, l_{Cn} / A_{Cn})}{(\rho_{Ag}\, l_{Ag} / A_{Ag})} = \frac{\rho_{Cn}\, l_{Cn}}{\rho_{Ag}\, l_{Ag}}\times\frac{A_{Ag}}{A_{Cn}} = \frac{\rho_{Cn}\, l_{Cn}}{\rho_{Ag}\, l_{Ag}}\times\frac{(\pi D_{Ag}^2/4)}{(\pi D_{Cn}^2/4)}$$

$$= \frac{30\rho_{Ag}\times(1/3)\,l_{Ag}}{\rho_{Ag}\times l_{Ag}}\times\left(\frac{D_{Ag}}{(1/2)\,D_{Ag}}\right)^2 = 40$$

So, $R_{Cn} = 40 \times R_{Ag} = 40 \times 1 = 40\ \Omega$

Problem 3.33: *On applying the same potential difference between the ends of wires of iron and copper of the same length, equal current flows in them. Compare their radii. Specific resistance of iron and copper are 1×10^{-7} and 1.6×10^{-8} Ω-m respectively. Can their current densities be made equal by taking appropriate radii?*

Solution: $V_{Fe} = V_{Cu}$, $l_{Fe} = l_{Cu}$, $I_{Fe} = I_{Cu}$, $\rho_{Fe} = 1 \times 10^{-7}\ \Omega$-m, $\rho_{Ag} = 1.6 \times 10^{-8}\ \Omega$-m

If the current flowing in two wires are same on applying the same potential difference, their resistances will also be same,

because, $R_{Fe} = \dfrac{V_{Fe}}{I_{Fe}} = \dfrac{V_{Cu}}{I_{Cu}} = R_{Cu}$

or, $$\frac{\rho_{Fe}\, l_{Fe}}{A_{Fe}} = \frac{\rho_{Cu}\, l_{Cu}}{A_{Cu}}$$

or, $$\frac{\rho_{Fe}}{\pi r_{Fe}^2} = \frac{\rho_{Cu}}{\pi r_{Cu}^2}$$

or, $$\frac{r_{Fe}}{r_{Cu}} = \sqrt{\frac{\rho_{Fe}}{\rho_{Cu}}} = \sqrt{\frac{1\times10^{-7}}{1.6\times10^{-8}}} = 2.5$$

The current densities may not be made equal as the current density in a metal with same voltage (electric field) is the unique property of the metal, given by the formula:

$$J = \sigma E$$

Problem 3.34: *A voltage of 30 V is applied across a color coded carbon resistor with first, second and third rings of blue, black and yellow colors. Determine the current flowing through the resistor.* [Haryana 1998-99, CBSE 1993-94, 2004-05]

Solution: $V = 30$ V, First Ring = Blue, Second Ring = Black, Third Ring = Yellow

The resistance of the carbon resistor may be given as:

Blue	Black	Yellow
6	0	4

$$R = 60 \times 10^4\ \Omega$$

The current flowing through the resistor may be given as:

$$I = \frac{V}{R} = \frac{30}{60\times10^4} = 5 \times 10^{-5}\ \text{A} = 50\ \mu\text{A}$$

Problem 3.35: *A potential difference of 10 V is applied across a conductor of resistance 1 kΩ. Determine the number of electrons flowing through the conductor in 5 minutes.*

Solution: $V = 10$ V, $R = 1$ kΩ, $t = 5$ minutes $= 5 \times 60 = 300$ Sec

The charge flowing through the conductor in 5 minutes may be given as:

$$q = N\, e = I \times t = \frac{V}{R} \times t$$

So, $$N = \frac{V \times t}{R \times e} = \frac{10\times300}{1\times10^3\times1.6\times10^{-19}} = 1.875 \times 10^{19}\ \text{electrons}$$

Problem 3.36: *A copper wire of cross sectional area 0.01 mm^2 has a resistance of 1 kΩ. Determine its length if resistivity of the copper is 1.7 × 10^{-8} Ω-m.*

Solution: A = 0.01 mm^2, R = 1 kΩ, ρ = 1.7 × 10^{-8} Ω-m

Since, $R = \frac{\rho l}{A}$

So, $l = \frac{RA}{\rho} = \frac{1\times10^3\times0.01\times10^{-6}}{1.7\times10^{-8}}$ = 588.235 m

Problem 3.37: *A metal wire of specific resistance 64 × 10^{-8} Ω-m and length 1.98 m has a resistance of 7 Ω. Determine its diameter.*

Solution: ρ = 64 × 10^{-8} Ω-m, l = 1.98 m, R = 7 Ω

Since, $R = \frac{\rho l}{A} = \frac{\rho l}{(\pi d^2/4)}$

So, $d = \sqrt{\frac{4\rho l}{\pi R}} = \sqrt{\frac{4\times64\times10^{-8}\times1.98}{\pi\times7}}$ = 0.48 mm

Problem 3.38: *Determine the resistance of a 2 m long nichrome wire of radius 0.321 mm. The resistivity of nichrome is 15 × 10^{-6} Ω-m. If a potential difference of 10 V is applied across this wire, determine the current flowing through the wire.*

Solution: l = 2 m, r = 0.321 mm, ρ = 1.5 × 10^{-6} Ω-m, V = 10 V

The resistance of the wire may be given as:

$$R = \frac{\rho l}{A} = \frac{\rho l}{(\pi r^2)} = \frac{1.5\times10^{-6}\times2}{\pi\times(0.321\times10^{-3})^2} = 9.267\ \Omega$$

Now, the current through the wire may be given as:

$$I = \frac{V}{R} = \frac{10}{9.267} = 1.079\ \text{A}$$

Problem 3.39: *Determine the resistance of a hollow cylindrical conductor of length 1 m and inner and outer radii of 1 mm and 2 mm respectively. The resistivity of the material is given as: 2 × 10^{-8} Ω-m.*

Solution: l = 1 m, r_I = 1 mm, r_O = 2 mm, ρ = 2 × 10^{-8} Ω-m

The area of cross section of the hollow conductor may be given as:

$$A = \pi \times (r_O^2 - r_I^2) = \pi \times [(2)^2 - (1)^2] = 3\,\pi\ \text{mm}^2$$

So, the resistance of the hollow cylinder may be given as:

$$R = \frac{\rho l}{A} = \frac{2\times10^{-8}\times1}{3\pi\times10^{-6}} = 2.12\ \text{m}\Omega$$

Problem 3.40: *Determine the electric field in a copper wire having a cross sectional area of 2 mm^2 and carrying a current of 1 A. The resistivity of the copper is 1.7 × 10^{-8} Ω-m.*

Solution: A = 2 mm^2, I = 1 A, ρ = 1.7 × 10^{-8} Ω-m

The current density in the copper conductor may be given as:

$$J = \sigma E$$

So, $E = \dfrac{J}{\sigma} = J \times \rho = \dfrac{I}{A} \times \rho = \dfrac{1}{2\times10^{-6}} \times 1.7 \times 10^{-8} = 8.5 \times 10^{-3}$ V m^{-1}

Problem 3.41: *A given copper wire is stretched to reduce its diameter to half its previous value. Determine its new resistance.*

Solution: $D_2 = \dfrac{1}{2} D_1$, $R_1 = R$

The volume of the wire remains constant and may be given as:

$$\text{Volume} = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1}$$

The ratio of two resistances (before stretching and after stretching) may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1}\times\frac{A_1}{A_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{(\pi D_1^2/4)}{(\pi D_2^2/4)}\right)^2 = \left(\frac{D_1}{D_2}\right)^4$$

or, $$R_2 = \left(\frac{D_1}{D_2}\right)^4 \times R_1 = \left(\frac{D_1}{(D_1/2)}\right)^4 \times R = 16\,R$$

Problem 3.42: *Determine the change in the resistance of a constantan wire when its radius is made half and length is reduced to one fourth of its original length.*

Solution: $r_2 = \dfrac{1}{2} r_1$, $l_2 = \dfrac{1}{4} l_1$

The ratio of two resistances (before and after) may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1}\times\frac{A_1}{A_2} = \frac{l_2}{l_1}\times\frac{\pi r_1^2}{\pi r_2^2} = \frac{(l_1/4)}{l_1}\times\left(\frac{r_1}{r_1/2}\right)^2 = 1$$

So, $R_2 = R_1$

i.e. No change in its resistance.

Problem 3.43: *A wire of resistance 5 Ω is uniformly stretched until its new length becomes 4 times of the original length. Determine its new resistance.*

Solution: $R_1 = 5\ \Omega$, $l_2 = 4\,l_1$

The volume of the wire remains constant and may be given as:

$$\text{Volume} = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_1}{A_2} = \frac{l_2}{l_1}$$

The ratio of two resistances (before stretching and after stretching) may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1}\times\frac{A_1}{A_2} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{4 l_1}{l_1}\right)^2 = 16$$

So, $R_2 = 16 \times R_1 = 16 \times 5 = 80\ \Omega$

Problem 3.44: *A metallic wire of length 1 m is stretched to double its length. Determine the ratio of its initial and final resistances, assuming that there is no change in its density during the stretching.* [CBSE 1993-94]

Solution: $l_1 = 1$ m, $l_2 = 2\,l_1 = 2$ m

Since the density of the material of the wire remains unchanged on stretching, so the volume of the wire remains constant and may be given as:

$$\text{Volume} = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{A_2}{A_1} = \frac{l_1}{l_2}$$

The ratio of two resistances (before stretching and after stretching) may be given as:

$$\frac{R_1}{R_2} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

So, $R_1 : R_2 = 1 : 4$

Problem 3.45: *A wire of certain radius is stretched, so that its radius decreases by a factor n. Determine its new resistance.*

Solution: $r_2 = \dfrac{r_1}{n}$

The volume of the wire remains constant and may be given as:

$$\text{Volume} = A_1 \times l_1 = A_2 \times l_2$$

or, $$\frac{l_2}{l_1} = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

The ratio of two resistances (before stretching and after stretching) may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{r_1}{r_2}\right)^4 = \left(\frac{r_1}{(r_1 / n)}\right)^4 = n^4$$

So, $R_2 = n^4 \times R_1$

Problem 3.46: *A wire 1 m long and 0.13 mm in diameter has a resistance of 4.2 Ω. Determine the resistance of another wire of the same material with a length of 1.5 m and 0.155 mm diameter.*

Solution: $l_1 = 1$ m, $D_1 = 0.13$ mm, $R_1 = 4.2\ \Omega$, $l_2 = 1.5$ m, $D_2 = 0.155$ mm, $\rho_2 = \rho_1 = \rho$

The ratio of resistances of two wires may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{l_2}{l_1} \times \frac{(\pi D_1^2 / 4)}{(\pi D_2^2 / 4)} = \frac{l_2}{l_1} \times \left(\frac{D_1}{D_2}\right)^2$$

So, $$R_2 = \frac{l_2}{l_1} \times \left(\frac{D_1}{D_2}\right)^2 \times R_1 = \frac{1.5}{1} \times \left(\frac{0.13}{0.155}\right)^2 \times 4.2 = 4.432\ \Omega$$

Problem 3.47: *A rheostat has 100 turns of a wire of radius 0.4 mm having a resistivity of 4.2×10^{-7} Ω-m. The diameter of each turn is 3 cm. What is the maximum value of resistance that it can introduce in the circuit.*

Solution: $N = 100$ turns, $r = 0.4$ mm, $\rho = 4.2 \times 10^{-7}$ Ω-m, $D_{\text{Turn}} = 3$ cm

The length of the wire in the rheostat may be given as:

$$l = N \times \pi \times D_{\text{Turn}}$$

The maximum resistance that this rheostat can introduce in the circuit is the resistance of its complete length and may be given as:

$$R = \frac{\rho l}{A} = \frac{\rho \times (N \times \pi \times D_{Turn})}{\pi r^2} = \frac{4.2 \times 10^{-7} \times (100 \times \pi \times 0.03)}{\pi \times (0.4 \times 10^{-3})^2} = 7.875\ \Omega$$

Problem 3.48: *Calculate the amount of copper required to draw a wire of length 10 km having a resistance of 10 Ω. The density and resistivity of the copper are 8.9×10^3 kg m^{-3} and 1.68×10^{-8} Ω-m respectively.*

Solution: $l = 10$ km, $R = 10\ \Omega$, $\rho_d = 8.9 \times 10^3$ kg m^{-3}, $\rho = 1.68 \times 10^{-8}$ Ω-m

The resistance of the copper wire may be given as:

$$R = \frac{\rho l}{A} = 10\ \Omega$$

So, $$A = \frac{\rho l}{10} = \frac{1.68 \times 10^{-8} \times 10 \times 10^3}{10} = 1.68 \times 10^{-5}\ \text{m}^2$$

The required mass of the required copper may be given as:

$$m = \rho_d \times V = \rho_d \times (A \times l) = 8.9 \times 10^3 \times (1.68 \times 10^{-5} \times 10 \times 10^3) = 1495.2\ \text{kg}$$

Problem 3.49: *The size of a carbon block is 1 cm × 1 cm × 50 cm. Determine its resistance. i) between the opposite square faces, ii) between the opposite rectangular faces. The resistivity of the carbon is given as 3.5×10^{-5} Ω-m.*

Solution: Size = 1 cm × 1 cm × 50 cm, $\rho = 3.5 \times 10^{-5}$ Ω-m

Between the opposite square faces:

The area of cross section, $A = 1\ \text{cm} \times 1\ \text{cm} = 1\ \text{cm}^2 = 1 \times 10^{-4}\ \text{m}^2$

The length of the carbon block, $l = 50$ cm = 0.50 m

So, the resistance may be given as:

$$R = \frac{\rho l}{A} = \frac{3.5 \times 10^{-5} \times 0.50}{1 \times 10^{-4}} = 0.175\ \Omega$$

Between the opposite rectangular faces:

The area of cross section, $A = 1\ \text{cm} \times 50\ \text{cm} = 50\ \text{cm}^2 = 50 \times 10^{-4}\ \text{m}^2$

The length of the carbon block, $l = 1$ cm = 0.01 m

So, the resistance may be given as:

$$R = \frac{\rho l}{A} = \frac{3.5 \times 10^{-5} \times 0.01}{50 \times 10^{-4}} = 7 \times 10^{-5}\ \Omega = 70\ \mu\Omega$$

Problem 3.50: *Two wires A and B of same material have their length in the ratio 1 : 5 and diameters in the ratio 3 : 2. If the resistance of the wire B is 180 Ω, determine the resistance of the wire A.*

Solution: $\frac{l_A}{l_B} = \frac{1}{5}$, $\frac{D_A}{D_B} = \frac{3}{2}$, $R_B = 180\ \Omega$

The ratio of resistances of two wires may be given as:

$$\frac{R_A}{R_B} = \frac{(\rho l_A / A_A)}{(\rho l_B / A_B)} = \frac{l_A}{l_B} \times \frac{A_B}{A_A} = \frac{l_A}{l_B} \times \frac{(\pi D_B^2 / 4)}{(\pi D_A^2 / 4)} = \frac{l_A}{l_B} \times \left(\frac{D_B}{D_A}\right)^2$$

So, $$R_\text{A} = \frac{l_A}{l_B} \times \left(\frac{D_B}{D_A}\right)^2 \times R_\text{B} = \frac{1}{5} \times \left(\frac{2}{3}\right)^2 \times 180 = 16\ \Omega$$

Problem 3.51: *A uniform wire is cut into four segments. Each segment is twice as long as the earlier segment. If the shortest segment has a resistance of 4 Ω, determine the resistance of the original wire.*

Solution: $l_2 = 2\ l_1$, $l_3 = 2\ l_2 = 2 \times 2\ l_1 = 4\ l_1$, $l_4 = 2\ l_3 = 2 \times 4\ l_1 = 8\ l_1$, $R_1 = 4\ \Omega$

Total length of the original wire may be given as:

$$l = l_1 + l_2 + l_3 + l_4 = l_1 + 2\ l_1 + 4\ l_1 + 8\ l_1 = 15\ l_1$$

The resistance of a wire is directly proportional to its length, if the area of cross section and the resistivity of the material is same,

i.e. $R \propto l$

So, $$\frac{R_{original\ wire}}{R_1} = \frac{l}{l_1}$$

or, $$R_\text{Original wire} = \frac{l}{l_1} \times R_1 = \frac{15 l_1}{l_1} \times 4 = 60\ \Omega$$

Problem 3.52: *Determine the conductance and conductivity of a wire of resistance 0.01 Ω, area of cross section 10^{-4} m^2 and length 0.1 m.*

Solution: $R = 0.01\ \Omega$, $A = 10^{-4}\ \text{m}^2$, $l = 0.1$ m

The conductance of the wire may be given as:

$$G = \frac{1}{R} = \frac{1}{0.01} = 100\ \text{S}$$

The conductivity of the wire may be given as:

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{0.1}{0.01 \times 10^{-4}} = 10^5\ \text{S m}^{-1}$$

3.10 Carriers of Current: "*The mobile charged particles, which can move freely inside the matter to set up a flow of current inside them, are known as current carriers*". The different types of *current carriers* in various states of the matter are given below.

Current Carriers in Solids: In metallic conductors or in semi conductors, the charge carriers are *free electrons*. These *free electrons* can move within the *lattice structure* of the matter. The *positively charged ions* (due to detachment of free electron from atoms) remain fixed in the *lattice structure* and cannot be moved from their position. In both type of semiconductors (*N*-type as well as *P*-type), the carriers of current are free electrons, although the majority charge carriers in *N*-type semiconductor are known as electrons while the majority charge carriers in *P*-type semiconductors are known as holes. The hole has itself no meaning, a hole is a vacancy of electron only. The actual movement of the *electrons* through theses holes gives rise to the flow of current.

Current Carriers in Liquids: In liquids, the charge carriers are *free electrons* as well as the *positive / negative ions* due to their free movements inside the liquid / electrolyte.

Current Carriers in Gases: In gases also, the charge carriers are *free electrons* as well as the *positive / negative ions* due to their free movements inside the gas.

Current Carriers in Vacuum Tubes: In vacuum tubes, the *electrons* emitted from the heated cathode acts as the charge carriers through the vacuum of the tube.

3.11 Mechanism of Current Flow in a Conductor (*Drift Velocity* and *Relaxation Time*): Metals have a large number of *free electrons,* nearly 10^{27} – 10^{30} electrons per cubic meter, depending on the temperature of the metal. These electrons are in a state of continuous random motion due to their thermal energy in the absence of any *external electric field*. At room temperature the typical value of their random velocities (in random directions) are of the order of 10^5 m/sec. As there is no movement of charge or flow of current, the number of electrons travelling in one direction will be equal to the number of electrons travelling in opposite direction. If $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$, ………. $\vec{u}_N$ are the random velocities of N free electrons, then average velocity of all the electrons may be given as:

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + ... + \vec{u}_N}{N} = 0 \qquad (3.26)$$

So, there is no net flow of charge in any direction.

However, in the presence of an external electric field $\vec{E}$, each electron experiences a force $-e\,\vec{E}$ in a direction opposite to that of the electric field $\vec{E}$ and undergoes an acceleration $\vec{a}$ given by:

$$\vec{a} = \frac{\text{Force}}{\text{Mass}} = -\frac{e\vec{E}}{m_e} \qquad (3.27)$$

Where, m_e is the mass of an electron. The electrons accelerate and collide with the positive metal ions alternately within very-very short time duration repeatedly, during their advancement in the opposite direction of the electric field. Between two successive collisions, an electron gains a velocity component (in addition to its random velocity) in the opposite direction to the electric field $\vec{E}$. However, the gain in velocity lasts for very short time duration and is lost in next collision, the electron starts afresh with a random thermal velocity only after each collision.

If an electron having random thermal velocity $\vec{u}_1$ accelerates for short time duration τ_1 (before it suffers next collision). The velocity attained by the electron may be given as:

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\ \tau_1 \qquad (3.27)$$

Similarly the velocity of other electrons may be given as:

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\ \tau_2 \qquad (3.28)$$

$$\vec{v}_3 = \vec{u}_3 + \vec{a}\ \tau_3 \qquad (3.29)$$

$$\vec{v}_N = \vec{u}_N + \vec{a}\ \tau_N \qquad (3.30)$$

The average velocity $\vec{v}_d$ of all the N electrons will be:

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ... + \vec{v}_N}{N}$$

$$= \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + (\vec{u}_3 + \vec{a}\tau_3) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

$$= \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + ... + \vec{u}_N}{N} + \vec{a} \times \frac{\tau_1 + \tau_2 + \tau_3 + ... + \tau_N}{N}$$

or, $$\vec{v}_d = 0 + \vec{a}\ \tau \qquad (3.31)$$

Where, $\tau = \left(\frac{\tau_1 + \tau_2 + \tau_3 + ... + \tau_N}{N} \right)$ is the average time of collision between two successive collisions.

"*The average time period elapsed between the two successive collisions of electrons with positive ions in the lattice structure of the conductor is known as the* ***relaxation time*** (τ) *of the electrons*". The relaxation time for most of the conductors is of the order of 10^{-4} sec. The velocity gain of the electron during this short duration may be given as:

$$\vec{v}_d = \vec{a}\ \tau = -\frac{e\vec{E}}{m_e} \times \tau \qquad (3.32)$$

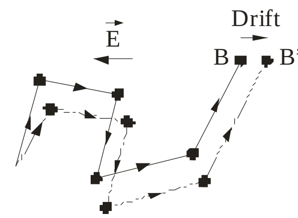

Fig. 3.15

The velocity gained $\vec{v}_d$ is known as ***drift velocity*** of electrons. The *drift velocity* may be defined as, "*the average velocity gained by the free electrons* (*in opposite direction to the electric field*) *of the metal under the influence of the external electric field between two successive collisions.*"

The reader may note that although the *electric field* accelerates an *electron* between two *successive collisions* but no net acceleration is produced. This is due to the reason of repeated successive collisions in very short durations. As a result, the electron acquires a constant average velocity $\vec{v}_d$ in the opposite direction to the *external electric field* $\vec{E}$. So, an electron slowly and steadily drifts in the opposite direction of *external electric field* $\vec{E}$.

3.12 Relation between Electric Current and Drift Velocity (Derivation of Ohm's Law): Let us assume that a potential difference of *V* Volts is applied across a conductor of length *l* and the uniform cross sectional area is *A*. The *electric field* $|\vec{E}|$ set up inside the conductor may be given as:

$$|\vec{E}| = \frac{V}{l} \qquad (3.33)$$

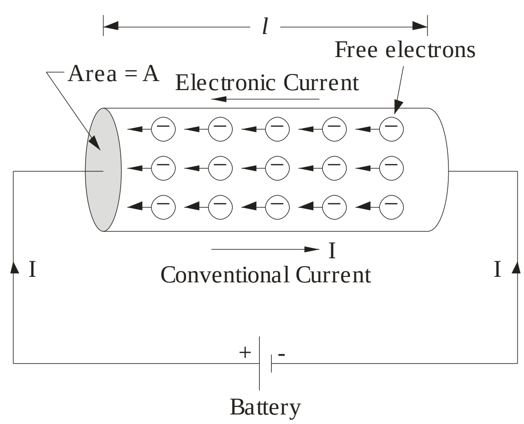

Fig. 3.16

Under the influence of *external electric field* $\vec{E}$, the free electrons begin to drift in the opposite direction of *electric field* with an average *drift velocity* $\vec{v}_d$.

Let the number of free electrons per unit volume or electron density = *n*

and, the charge on each electron = *e*

Number of electrons in length (*l*) of the conductor may be given as:

$$N_l = n \times \text{Volume of the conductor}$$
$$= n \times A\, l \qquad (3.34)$$

Total charge contained in length *l* of the conductor may be given as:

$$q = e \times N_1 = e \times n \times A\, l \qquad (3.35)$$

All the electrons entering in the conductor from the right end will leave from the left end of the conductor after a time *t*, given by:

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{l}{v_d} \qquad (3.36)$$

So, the current flowing through the conductor may be given as:

$$I = \frac{q}{t} = \frac{e \times n \times A\, l}{(l / v_d)} = e \times n \times A \times v_d = e\, n\, A\, v_d \qquad (3.37)$$

This is the desired relationship between the *current* and the *drift velocity*.

The *current density* may be given as:

$$J = \frac{I}{A} = \frac{e n A v_d}{A} = e\, n\, v_d \qquad (3.38.1)$$

or, $$\vec{J} = e\, n\, \vec{v}_d \qquad (3.38.2)$$

The above equation is valid for both positive as well as negative charges.

Deducing The Ohm's Law: When a *potential difference* of *V* Volts is applied across a conductor of length *l*, the *drift velocity* in terms of *V* may be given as:

$$v_d = \frac{e|\vec{E}|}{m_e} \times \tau = \frac{eV}{m_e\, l} \times \tau \qquad (3.39)$$

The relationship between the current and the drift velocity may now be given as:

$$I = e\, n\, A\, v_d = e\, n\, A \times \frac{eV\tau}{m_e\, l}$$

or, $$\frac{V}{I} = \frac{m_e\, l}{n e^2 \tau A} \qquad (3.40)$$

At a fixed temperature and physical conditions, the quantities mass of electron (m_e), the number of free electrons per cubic meter (*n*), the charge on an electron (*e*), the relaxation time (τ) and the area of cross section (*A*) of the conductor all have constant values for a given conductor.

So, $$\frac{V}{I} = \frac{m_e\, l}{n e^2 \tau A} = \text{a constant known as the resistance, } (R)$$

This proves the Ohm's law and now we have the value of the resistance of the conductor as:

$$R = \frac{m_e\, l}{n e^2 \tau A} = \frac{\rho l}{A} \qquad (3.41)$$

So, $$\rho = \frac{m_e}{n e^2 \tau} \qquad (3.42.1)$$

and, $$\sigma = \frac{n e^2 \tau}{m_e} \qquad (3.42.2)$$

We may easily conclude from the above two equations that the *resistivity* or the *conductivity* of the conductor is independent of the dimensions of the conductor, but depends on two parameters:

i) Number of free electrons per unit volume or electron density of the conductor (*a basic property of the material but temperature dependent*).

ii) The relaxation time (τ), *i.e.* the average time between two successive collisions of an electron (*a basic property of the material but temperature dependent*).

Verification of the Vector Form of Ohm's Law (The Relation $\vec{J} = \sigma\, \vec{E}$):

The charge on an electron, $q = -\,e$

and, $\vec{v}_d = -\dfrac{e\vec{E}}{m_e} \times \tau$

and, $I = q\, n\, A\, v_d$

So, $\vec{J} = \dfrac{I}{A} = \dfrac{q\,n\,A\vec{v}_d}{A} = q\;n\;\vec{v}_d = (-e \times n) \times \left(-\dfrac{e\vec{E}}{m_e}\right) \times \tau = \dfrac{ne^2\tau}{m_e} \times \vec{E}$

or, $\vec{J} = \sigma\,\vec{E}$ (3.43)

Cause of Resistance offered to Flow of the Current: *The basic reason of the resistance offered by a conductor* (*or insulator or semiconductor*) *to flow of current is the collisions of free and moving electrons with the positive metal ions fixed in the lattice structure of the material.* When a *potential difference* is applied across a conductor, its *free electrons* get accelerated. On their way of movement, under the influence of *external electric field*, they frequently collide with the *positive metal ions* which are fixed in the *lattice structure*, *i.e.* their motion is opposed and this opposition to the flow of electron is known as the resistance offered by the conductor to flow of the current. Larger the number of collisions per second, smaller is the relaxation time and hence larger will be the *resistivity* of the metal, given by $\left(\rho = \dfrac{m_e}{ne^2\,\tau}\right)$.

The number of collisions made per second by the electrons depends on the arrangement of atoms / ions in the lattice structure of the conductor. So, ***the resistance offered by a conductor to flow of the current depends on the nature of the material*** (copper, aluminum, silver *etc.*) of the conductor.

The resistance offered by a conductor to flow of the current depends on the length of the conductor. A long wire offers more resistance than short wire because there will be more collisions when electron goes a longer way in the conductor.

The resistance offered by a conductor to flow of the current depends on the area of cross section of the conductor. A thick wire offers less resistance than a thin wire as more area of cross section is available for the flow of current (electrons).

Cause of High Resistivity of Nichrome (Ni-Cr Alloy): In general, *alloys have more resistivity then their constituent metals*. In an alloy, *e.g.* in nichrome, Ni^{2+} and Cr^{3+} ions have different charge and size. They occupy random locations relative to each other. An electron therefore passes through a very random medium and is very frequently deflected from its own path. So, there is a small *relaxation time* and hence large resistivity.

Cause of Quick Current / Instantaneous Energizing of the Circuit: The *drift velocity* ($\vec{v}_d$) of the electrons are about 1 mm/sec, but an electric lamp or any other appliance starts up as soon as the switch is turned ON. This is due to the reason that electrons are present everywhere in an electric circuit. When a *potential difference* is applied across an electric circuit by turning ON the switch, an *electric field* is setup throughout the circuit (almost with the speed of light). Electrons in the every part of the circuit begin to drift under the influence of the *electric field* and an electric current set up in the electric circuit immediately. The reader may understands it, as soon as one electron enters from one end, another leaves from the another end.

Problem 3.53: *Determine the number of free electrons in one meter^3 volume of copper, assuming that there is one free electron per atom of the copper. Density of copper is 8.9 × 10^3 kgm^{-3} and atomic weight is 63.5.* (*Avogadro's Number, A = 6.02 × 10^{26} per kg-atom*)

Solution: $\rho_d = 8.9 \times 10^3$ kgm^{-3}, Atomic weight of copper = 63.5 gm,

$A = 6.02 \times 10^{26}$ per kg-atom = 6.02×10^{23} per gm-atom

Mass of 1 m^3 of copper, $M = \rho_d \times Vol. = 8.9 \times 10^3 \times 1 = 8.9 \times 10^3$ kg

Since one free electron per atom is there and the number of atoms in 63.5 gm of copper = A

So, the number of free electrons in 8.9 × 10^3 kg of copper may be given as:

$$n = \frac{A \times 8.9 \times 10^3}{63.5 \times 10^{-3}} = \frac{6.02 \times 10^{23} \times 8.9 \times 10^3}{63.5 \times 10^{-3}} = 8.437 \times 10^{28} \text{ electrons}$$

Problem 3.54: *A copper wire has a resistance of 10 Ω and a cross sectional area of 1 mm^2. A potential difference of 10 V is applied across the wire. Determine the drift velocity of the electrons, if the number of free electrons in per cubic meter of copper is 8 × 10^{28}.* [CBSE 1995-96]

Solution: R = 10 Ω, A = 1 mm^2, V = 10 V, n = 8 × 10^{28} electrons/m^3

The current flowing through the copper wire may be given as:

$$I = \frac{V}{R} = n\,e\,A\,v_d$$

or, $$v_d = \frac{V}{R \times n\,e\,A} = \frac{10}{10 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-6}}$$

$$= 7.813 \times 10^{-5} \text{ m/sec} = 0.07813 \text{ mm/sec}$$

Problem 3.55: *a*) *Estimate the average drift velocity of conducting electrons in a copper wire of cross sectional area 1 × 10^{-7} m^2, carrying a current of 1.5 A. Given: each copper atom contributes one conducting electron per atom, the density of copper is 9 × 10^3 kg/m^3, atomic mass of copper is 63.5 and Avogadro's Number is 6.02 × 10^{23} per mole.*

b) *Compare the drift velocity obtained above with: i) thermal speeds of copper at ordinary temperature, ii) speeds of electrons carrying the current, iii) speed of propagation of electric field along the conductor which causes the drift motion.* [NCERT]

Solution: A = 1 × 10^{-7} m^2, free electron per atom = 1, I = 1.5 A, ρ_d = 9 × 10^3 kg/m^3,

m_{Cu} = 63.5 gm, A = 6.02 × 10^{23} per mole

a) Mass of 1 m^3 of copper, $M = \rho_d \times Vol.$ = 9 × 10^3 ×1 = 9 × 10^3 kg

Since one free electron per atom is there and the number of atoms in 63.5 gm of copper = A

So, the number of free electrons in 9 × 10^3 kg of copper may be given as:

$$n = \frac{A \times 9 \times 10^3}{63.5 \times 10^{-3}} = \frac{6.02 \times 10^{23} \times 9 \times 10^3}{63.5 \times 10^{-3}} = 8.532 \times 10^{28} \text{ electrons}$$

The current flowing through the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

or, $$v_d = \frac{I}{n\,e\,A} = \frac{1.5}{8.532 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-7}}$$

$$= 1.099 \times 10^{-3} \text{ m/sec} = 1.099 \text{ mm/sec}$$

b) The thermal speed of a copper atom at any temperature T may be given as:

$$v_{rms} = \sqrt{\frac{2 k_B T}{m}}$$

The ordinary room temperature, T = 27 °C = 27 + 273 = 300 K

The Boltzmann Constant, k_B = 1.38 × 10^{-23} JK^{-1}

The mass of a copper atom, $m_{\text{Cu}} = \dfrac{M}{N} = \dfrac{63.5\times10^{-3}}{6.02\times10^{23}}$ kg

So, $v_{\text{rms}} = \sqrt{\dfrac{2k_B T}{m_{Cu}}} = \sqrt{\dfrac{2\times1.38\times10^{-23}\times300\times6.02\times10^{23}}{63.5\times10^{-3}}}$ = 280.17 m/sec

The ratio of drift velocity to thermal speed of a copper atom may be given as:

$$\frac{v_d\,(electron)}{v_{rms}\,(Cu\ atom)} = \frac{1.099\times10^{-3}}{343.14} = 3.203 \times 10^{-6}$$

The speed, of electrons carrying the current, is the fastest speed of an electron inside the copper which it may have. We know that the maximum kinetic energy of electron in copper corresponds to a temperature of $T_0 = 10^5$ K.

So, $\dfrac{1}{2}\times m_e\, v_F^2 = k_{\text{B}}\, T$

or, $v_{\text{F}} = \sqrt{\dfrac{2k_B T}{m_e}} = \sqrt{\dfrac{2\times1.38\times10^{-23}\times10^5}{9.1\times10^{-31}}}$ = 1.742 × 10^6 m/sec

The ratio of drift velocity to the speed of electrons carrying the current may be given as:

$$\frac{v_d\,(electron)}{v_F\,(electron)} = \frac{1.099\times10^{-3}}{1.742\times10^{6}} = 6.308 \times 10^{-10}$$

The speed of the electric field (electromagnetic wave) along the conductor is same as the speed of light, *i.e.* 3 × 10^8 m/sec. So, the ratio of the drift velocity to the speed of electric field may be given as:

$$\frac{v_d\,(electron)}{v(electric\ \ field)} = \frac{1.099\times10^{-3}}{3\times10^{8}} = 3.663 \times 10^{-12}$$

Problem 3.56: *Determine the electric field in a copper wire of cross-sectional area 2 mm^2 carrying a current of 1 A. The conductivity of the copper is 6.25 × 10^7 Sm^{-1}.*

Solution: A = 2 mm^2, I = 1 A, σ = 6.25 × 10^7 Sm^{-1}

The current density in a copper wire may be given as:

$$J = \frac{I}{A} = \sigma\, E$$

So, $E = \dfrac{I}{\sigma\times A} = \dfrac{1}{6.25\times10^{7}\times2\times10^{-6}}$ = 8 × 10^{-3} V/m

Problem 3.57: *A potential difference of 100 V is applied to the ends of a copper wire of length one meter. Determine the average drift velocity of the electrons. Compare it with the thermal velocity at 27°C. Given conductivity of copper σ = 5.81 × 10^7 Sm^{-1} and number density of conduction electrons n = 8.5 × 10^{28} per meter^3.*

Solution: V = 100 V, l = 1 m, T = 27 °C, σ = 5.81 × 10^7 Sm^{-1}, n = 8.5 × 10^{28} m^{-3}

The current density in the copper wire may be given as:

$$J = \sigma\, E = n\, e\, v_{\text{d}}$$

$$v_d = \frac{\sigma E}{ne} = \frac{\sigma}{ne} \times \frac{V}{l} = \frac{5.81 \times 10^7}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}} \times \frac{100}{1} = 0.427 \text{ m/sec}$$

The thermal speed of the electrons at any temperature T may be given as:

$$v_{rms} = \sqrt{\frac{2 k_B T}{m_e}}$$

The ordinary room temperature, T = 27 °C = 27 + 273 = 300 K

The Boltzmann Constant, $k_B = 1.38 \times 10^{-23}$ JK^{-1}

So, $$v_{rms} = \sqrt{\frac{2 k_B T}{m_e}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}} = 9.5388 \times 10^4 \text{ m/sec}$$

The ratio of drift velocity to thermal speed of electrons may be given as:

$$\frac{v_d\,(electron)}{v_{rms}\,(electron)} = \frac{0.427}{9.5388 \times 10^4} = 4.476 \times 10^{-6}$$

Problem 3.58: *Determine the time of relaxation between collisions and free path of electrons in copper at room temperature. Given the resistivity of copper $\rho = 1.7 \times 10^{-8}$ Ω-m, the number density of electrons $n = 8.5 \times 10^{28}$ m^{-3}, the mass of electron $m_e = 9.1 \times 10^{-31}$ kg, the charge on an electron $e = 1.6 \times 10^{-19}$ C and the drift velocity $v_d = 1.6 \times 10^{-4}$ m/sec.*

Solution: $\rho = 1.7 \times 10^{-8}$ Ω-m, $n = 8.5 \times 10^{28}$ m^{-3}, $m_0 = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $v_d = 1.6 \times 10^{-4}$ m/sec

The resistivity of the copper may be given as:

$$\rho = \frac{m_e}{n e^2 \tau}$$

So, $$\tau = \frac{m_e}{n e^2 \rho} = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}} = 2.46 \times 10^{-14} \text{ sec}$$

The mean free path of the electron may be given as:

Mean free path = $v_d \times \tau = 1.6 \times 10^{-4} \times 2.46 \times 10^{-14} = 3.936 \times 10^{-18}$ m

Problem 3.59: *An aluminum wire of diameter 0.24 cm is connected in series with a copper wire of diameter 0.16 cm. The wires carry an electric current of 10 A. Determine: i) current density in the aluminum wire, ii) drift velocity of electrons in the copper wire. Given that: Number of electrons per cubic meter volume of the copper as $n = 8.4 \times 10^{28}$.*

Solution: D_{Al} = 0.24 cm, D_{Cu} = 0.16 cm, I = 10 A, $n = 8.4 \times 10^{28}$ m^{-3}

The current density in the aluminum wire may be given as:

$$J_{Al} = \frac{I}{A} = \frac{I}{(\pi D_{Al}^2 / 4)} = \frac{10 \times 4}{\pi \times (0.24 \times 10^{-2})^2} = 2.2 \times 10^6 \text{ A/m}^2 = 2.2 \text{ A/mm}^2$$

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

or, $$v_d = \frac{I}{neA} = \frac{I}{ne(\pi D_{Cu}^2 / 4)} = \frac{10 \times 4}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (0.16 \times 10^{-2})^2}$$

or, $v_d = 3.7 \times 10^{-4}$ m/sec = 0.37 mm/sec

Problem 3.60: *A current of 1 A is flowing through a copper wire of length 0.1 m and cross sectional area of 1×10^{-6} m^2. i) If the specific resistance of copper is 1.7×10^{-8} Ω-m, determine the potential difference between the ends of the wire. ii) Determine the current density in the wire. iii) If there is one free electron per atom in the copper, determine the drift velocity of electrons. Given that: Density of copper $\rho_d = 8.9 \times 10^3$ kg/m^3, atomic weight $m_{Cu} = 63.5$ and Avogadro's Number is 6.02×10^{26} per kg-atom.*

Solution: $I = 1$ A, $l = 0.1$ m, $A = 1 \times 10^{-6}$ m^2, $\rho = 1.7 \times 10^{-8}$ Ω-m, $\rho_d = 8.9 \times 10^3$ kg/m^3,

$m_{Cu} = 63.5$ gm, Av. No. = 6.02×10^{26} per kg-atom = 6.02×10^{23} per gm-atom

The resistance of the wire may be given as:

$$R = \frac{\rho l}{A} = \frac{1.7\times10^{-8}\times0.1}{1\times10^{-6}} = 1.7 \times 10^{-3}\ \Omega = 1.7\ \text{m}\Omega$$

So, the potential difference across the copper wire may be given as:

$$V = I\,R = 1 \times 1.7 \times 10^{-3} = 1.7 \times 10^{-3}\ \text{V} = 1.7\ \text{mV}$$

The current density in the wire may be given as:

$$J = \frac{I}{A} = \frac{1}{1\times10^{-6}} = 10^6\ \text{A/m}^2$$

Mass of 1 m^3 of copper, $M = \rho_d \times Vol. = 8.9 \times 10^3 \times 1 = 8.9 \times 10^3$ kg

Since one free electron per atom is there and number of atoms in 63.5 gm of copper = Av. No.

So, the number of free electrons in 8.9×10^3 kg of copper may be given as:

$$n = \frac{\text{Av. No.}\times8.9\times10^3}{63.5\times10^{-3}} = \frac{6.02\times10^{23}\times8.9\times10^3}{63.5\times10^{-3}} = 8.437 \times 10^{28}\ \text{electrons}$$

The current through the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

or, $$v_d = \frac{I}{neA} = \frac{1}{8.437\times10^{28}\times1.6\times10^{-19}\times1\times10^{-6}} = 7.408 \times 10^{-5}\ \text{m/sec}$$

Problem 3.61: *The free electrons of a copper wire of cross sectional area 10^{-6} m^2 acquire a drift velocity of 10^{-4} m/sec, when a certain potential difference is applied across the wire. Determine the current flowing in the wire, if density of free electrons in the copper is 8.5×10^{28} electrons/m^3.*

Solution: $A = 10^{-6}$ m^2, $v_d = 10^{-4}$ m/sec, $n = 8.5 \times 10^{28}$ electrons/m^3

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d = 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6} \times 10^{-4} = 1.36\ \text{A}$$

Problem 3.62: *A current of 5 A is flowing through a metallic wire of cross sectional area 4×10^{-6} m^2. If the density of charge carriers in the wire is 5×10^{26} m^{-3}, determine the drift velocity of the electrons.*

Solution: $I = 5$ A, $A = 4 \times 10^{-6}$ m^2, $n = 5 \times 10^{26}$ electrons/m^3

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

So, $$v_d = \frac{I}{neA} = \frac{5}{5\times10^{26}\times1.6\times10^{-19}\times4\times10^{-6}} = 0.015625 \text{ m/sec} = 15.625 \text{ mm/sec}$$

Problem 3.63: *A current of 1.8 A is flowing through a metallic wire of cross sectional area 0.5 mm^2. Determine the current density in the wire. If the density of free electrons in the wire is 8.8 × 10^{28} m^{-3}, determine the drift velocity of the electrons.*

Solution: I = 1.8 A, A = 0.5 mm^2, n = 8.8 × 10^{28} electrons/m^3

The current density in the wire may be given as:

$$J = \frac{I}{A} = \frac{1.8}{0.5\times10^{-6}} = 3.6\times10^{6} \text{ A/m}^2$$

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

So, $$v_d = \frac{I}{neA} = \frac{1.8}{8.8\times10^{28}\times1.6\times10^{-19}\times0.5\times10^{-6}} = 2.557\times10^{-4} \text{ m/sec} = 0.2557 \text{ mm/sec}$$

Problem 3.64: *The resistivity of copper at room temperature is 1.7 × 10^{-8} Ω-m. If the density of free electrons in the copper is 8.4 × 10^{28} m^{-3}, determine the relaxation time for the free electrons of copper. Given: m_e = 9.1 × 10^{-31} kg and e = 1.6 × 10^{-19} C.*

Solution: ρ = 1.7 × 10^{-8} Ω-m, n = 8.4 × 10^{28} electrons/m^3, m_e = 9.1 × 10^{-31} kg, e = 1.6 × 10^{-19} C

The resistivity of the copper may be given as:

$$\rho = \frac{m_e}{ne^2\tau}$$

So, $$\tau = \frac{m_e}{ne^2\rho} = \frac{9.1\times10^{-31}}{8.4\times10^{28}\times(1.6\times10^{-19})^2\times1.7\times10^{-8}} = 2.489\times10^{-14} \text{ sec}$$

Problem 3.65: *A copper wire of diameter 1 mm carries a current of 0.2 A. The copper has 8.4 × 10^{28} atoms/meter^3. Determine the drift velocity of electrons in copper, assuming that one charge carrier of 1.6 × 10^{-19} C is associated with each atom of the copper.*

Solution: D = 1 mm, I = 0.2 A, n = 8.4 × 10^{28} electrons/m^3, n = 8.4 × 10^{28} kg, e = 1.6 × 10^{-19} C

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

So, $$v_d = \frac{I}{neA} = \frac{I}{ne\times(\pi D^2/4)} = \frac{0.2\times4}{8.4\times10^{28}\times1.6\times10^{-19}\times\pi\times(1\times10^{-3})^2}$$

$$= 1.895\times10^{-5} \text{ m/sec}$$

Problem 3.66: *A current of 2 A is flowing through a wire of length 4 m and cross sectional area of 1 mm^2. If each cubic meter of the wire contains 10^{29} free electrons, determine the average time taken by an electron to cross the length of the wire.*

Solution: I = 2 A, l = 4 m, A = 1 mm^2, n = 10^{29} electrons/m^3

The current in the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

So, $v_d = \dfrac{I}{neA} = \dfrac{2}{10^{29} \times 1.6 \times 10^{-19} \times 1 \times 10^{-6}}$ = 1.25 × 10^{-4} m/sec

The time taken by an electron to cross the length of wire may be given as:

$$t = \frac{l}{v_d} = \frac{4}{1.25 \times 10^{-4}} = 3.2 \times 10^4 \text{ sec} = \frac{3.2 \times 10^4}{60 \times 60} = 8.889 \text{ hrs.}$$

Problem 3.67: *A charge of 10 C flows through a wire in 5 minutes. The radius of the wire is 1 mm. It contains 5 × 10^{22} electrons/cm^3. Determine the current and drift velocity.*

Solution: q = 10 C, t = 5 minutes, r = 1 mm, n = 5 × 10^{22} electrons/cm^3 = 5 × 10^{28} electrons/m^3

The current through the wire may be given as:

$$I = \frac{q}{t} = \frac{10}{5 \times 60} = 0.033333 \text{ A} = 33.333 \text{ mA}$$

or, $n\,e\,A\,v_d = 0.033333$

or, $v_d = \dfrac{0.033333}{neA} = \dfrac{0.033333}{ne \times \pi r^2} = \dfrac{0.033333}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (1 \times 10^{-3})^2}$

$= 1.326 \times 10^{-6}$ m/sec

Problem 3.68: *A copper wire of diameter 0.15 cm is connected in series with an aluminum wire of diameter 0.30 cm. A current of 10 A is flowing through them. Determine: i) current density in the copper wire, ii) drift velocity of the free electrons in the aluminum wire. The number of free electrons per unit volume of the aluminum wire is 10^{28} m^{-3}.*

Solution: D_{Cu} = 0.15 cm, D_{Cu} = 0.30 cm, I = 10 A, $n = 10^{28}$ m^{-3}

The current density in the copper wire may be given as:

$$J_{Cu} = \frac{I}{A} = \frac{I}{(\pi D_{Cu}^2/4)} = \frac{10 \times 4}{\pi \times (0.15 \times 10^{-2})^2} = 5.659 \times 10^6 \text{ A/m}^2 = 5.659 \text{ A/mm}^2$$

The current in the aluminum wire may be given as:

$I = n\,e\,A\,v_d$

or, $v_d = \dfrac{I}{neA} = \dfrac{I}{ne(\pi D_{Al}^2/4)} = \dfrac{10 \times 4}{10^{28} \times 1.6 \times 10^{-19} \times \pi \times (0.30 \times 10^{-2})^2}$

$= 8.842 \times 10^{-4}$ m/sec = 0.8842 mm/sec

Problem 3.69: *A current of 30 A is flowing through a wire of cross sectional area of 2 mm^2. Determine the drift velocity of the electrons, assuming one free electron per atom. Also determine the rms velocity of electrons at this temperature, assuming the temperature of the wire to be 27°C. Compare the drift velocity and rms velocity. Given that: Boltzmann's Constant = 1.38 × 10^{-23} JK^{-1}, density of copper = 8.9 gm cm^{-3}, atomic weight of copper = 63.*

Solution: I = 30 A, A = 2 mm^2, T = 27 °C, $k = 1.38 \times 10^{-23}$ Jk^{-1}, m_{Cu} = 63 gm,

ρ_d = 8.9 gm cm^{-3} = 8.9 × 10^3 kg m^{-3}

Mass of 1 m^3 of copper, $M = \rho_d \times Vol.$ = 8.9 × 10^3 × 1 = 8.9 × 10^3 kg

Since one free electron per atom is there and the number of atoms in 63 gm of copper = Av. No.

So, the number of free electrons in 8.9 × 10^3 kg of copper may be given as:

$$n = \frac{\text{Av. No.} \times 8.9 \times 10^3}{63 \times 10^{-3}} = \frac{6.02 \times 10^{23} \times 8.9 \times 10^3}{63 \times 10^{-3}} = 8.504 \times 10^{28} \text{ electrons}$$

The current through the copper wire may be given as:

$$I = n\,e\,A\,v_d$$

So, $$v_d = \frac{I}{neA} = \frac{30}{8.504 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}} = 1.102 \times 10^{-3} \text{ m/sec} = 1.102 \text{ mm/sec}$$

The thermal speed of an electron at any temperature T may be given as:

$$v_{rms} = \sqrt{\frac{2k_B T}{m_e}}$$

The room temperature, $T = 27\ °C = 27 + 273 = 300$ K

The Boltzmann Constant, $k_B = 1.38 \times 10^{-23}\ JK^{-1}$

So, $$v_{rms} = \sqrt{\frac{2k_B T}{m_e}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}} = 9.5388 \times 10^4 \text{ m/sec}$$

The ratio of drift velocity to thermal speed of electrons may be given as:

$$\frac{v_d\ (electron)}{v_{rms}\ (electron)} = \frac{1.102 \times 10^{-3}}{9.5388 \times 10^4} = 1.155 \times 10^{-8}$$

Problem 3.70: *Determine the drift velocity of electrons in silver wire of length 1 m, having cross sectional area of 3.14×10^{-6} m^2 and carrying a current of 10 A. Given: free charge carriers = 1 electron per atom, atomic mass of silver = 108, density of silver = 10.5×10^3 kg m^{-3}, charge on the electron = 1.6×10^{-19} C, Avogadro's number = 6.02×10^{23}.*

Solution: $l = 1$ m, $A = 3.14 \times 10^{-6}$ m^2, $I = 10$ A, free charge carriers = 1 electron/atom
$m_{Ag} = 108$ gm, $\rho_D = 10.5 \times 10^3$ kg/m^3, Av. No. = 6.02×10^{23}

Mass of 1 m^3 of silver, $M = \rho_d \times Vol. = 10.5 \times 10^3 \times 1 = 10.5 \times 10^3$ kg

Since one free electron per atom is there and number of atoms in 108 gm of copper = Av. No.

So, the number of free electrons in 10.5×10^3 kg of copper may be given as:

$$n = \frac{\text{Av. No.} \times 10.5 \times 10^3}{108 \times 10^{-3}} = \frac{6.02 \times 10^{23} \times 10.5 \times 10^3}{108 \times 10^{-3}} = 5.853 \times 10^{28} \text{ electrons}$$

The current through the copper wire may be given by the relationship:

$$I = n\,e\,A\,v_d$$

So, $$v_d = \frac{I}{neA} = \frac{10}{5.853 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times 10^{-6}} = 3.4 \times 10^{-4} \text{ m/sec} = 0.34 \text{ mm/sec}$$

Problem 3.71: *When a potential difference of 1.5 V is applied across a wire of length 0.2 m and area of cross section 0.3 mm^2, a current of 2.4 A flows through the wire. If the number density of free electrons in the wire is 8.4×10^{28} m^{-3}, determine the average relaxation time. Given that mass of electron = 9.1×10^{-31} kg and charge on electron = 1.6×10^{-19} C.*

Solution: $V = 1.5$ V, $l = 0.2$ m, $A = 0.3$ mm^2, $I = 2.4$ A, $n = 8.4 \times 10^{28}$ m^{-3},

$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C

Resistance of the wire may be given as:

$$R = \frac{V}{I} = \frac{1.5}{2.4} = 0.625\ \Omega = \frac{m_e\, l}{n e^2 \tau A}$$

So, $$\tau = \frac{m_e\, l}{0.625 \times n e^2 A} = \frac{9.1 \times 10^{-31} \times 0.2}{0.625 \times 8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 0.3 \times 10^{-6}} = 4.51 \times 10^{-16}\ \text{sec}$$

3.13 Mobility of Charge Carriers: "*The mobility of a charge carrier may be defined as the drift velocity acquired by the electron in a unit electric field*".

So, $$\mu = \frac{v_d}{|\vec{E}|} \tag{3.44}$$

As, the drift velocity may be given as:

$$v_d = \frac{q|\vec{E}|}{m_e} \times \tau$$

So, $$\mu = \frac{1}{|\vec{E}|} \times \frac{q|\vec{E}|}{m_e} \times \tau = \frac{q}{m_e} \times \tau \tag{3.45}$$

For an electron, $$\mu_e = \frac{e}{m_e}\tau \tag{3.46}$$

For a hole, $$\mu_h = \frac{e}{m_h}\tau \tag{3.47}$$

The *mobility* of both the *electrons* and the *holes* are positive, although their *drift velocities* are opposite to each other.

SI unit of mobility = $\frac{v_d\ (\mathrm{m\,s^{-1}})}{|\vec{E}|\ (\mathrm{V\,m^{-1}})} = \mathrm{m^2\ V^{-1}\ s^{-1}}$

Unit of mobility used in Practice = $\mathrm{cm^2\ V^{-1}\ s^{-1}}$

$$1\ \mathrm{m^2\ V^{-1}\ s^{-1}} = 10^4\ \mathrm{cm^2\ V^{-1}\ s^{-1}} \tag{3.48}$$

Relation between Electric Current and Mobility for a Conductor:

In a metallic conductor, the *electric current* flows due to its *free electrons* and may be given as:

$$I = n\, e\, A\, v_d = n\, e\, A\, \mu_e\, |\vec{E}| \tag{3.49}$$

Relation between Electric Current and Mobility for a Semiconductor:

In a semiconductor, the *electric current* flows due to its *free electrons* as well as due to the *holes* and may be given as:

$$I = I_e + I_h = n\, e\, A\, v_e + p\, e\, A\, v_h = n\, e\, A\, \mu_e\, |\vec{E}| + p\, e\, A\, \mu_h\, |\vec{E}|$$

$$= e\, A\, |\vec{E}|\ (n\, \mu_e + p\, \mu_h) \tag{3.50}$$

Where, n and p are *electron density* and *hole density* respectively, in a semiconductor.

Conductivity of a Semiconductor: According to *Ohm's law*:

$$I = \frac{V}{R} = \frac{|\vec{E}|\, l}{(\rho\, l / A)} = \sigma\, |\vec{E}|\, A = e\, A\, |\vec{E}|\, (n\, \mu_e + p\, \mu_h)$$

So, $\sigma = e\,(n\, \mu_e + p\, \mu_h)$ (3.51)

Problem 3.72: *A potential difference of 6 V is applied across a conductor of length 0.12 m. Determine the drift velocity of electrons, if the electron mobility is 5.6 × 10^{-6} m^2 V^{-1} s^{-1}.*

Solution: $V = 6$ V, $l = 0.12$ m, $\mu_e = 5.6 \times 10^{-6}$ m^2 V^{-1} s^{-1}

The drift velocity may be given as:

$$v_d = \mu_e\, |\vec{E}| = \mu_e \times \frac{V}{l} = 5.6 \times 10^{-6} \times \frac{6}{0.12} = 2.8 \times 10^{-4} \text{ m/sec} = 0.28 \text{ mm/sec}$$

Problem 3.73: *The number density of electrons in the copper is 8.5 × 10^{28} m^{-3}. Determine the current flowing through a copper wire of length 0.2 m, area of cross section 1 mm^2, when connected to a battery of 3 V. The electron mobility = 4.5 × 10^{-6} m^2 V^{-1} s^{-1} and charge on electron = 1.6 × 10^{-16} C.*

Solution: $n = 8.5 \times 10^{28}$ m^{-3}, $l = 0.2$ m, $A = 1$ mm^2, $V = 3$ V,

$\mu_e = 4.5 \times 10^{-6}$ m^2 V^{-1} s^{-1}, $e = 1.6 \times 10^{-19}$ C

The current flowing through the conductor may be given as:

$$I = n\, e\, A\, v_d = n\, e\, A\, \mu_e\, |\vec{E}| = n\, e\, A\, \mu_e \times \frac{V}{l}$$

$$= 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-6} \times 4.5 \times 10^{-6} \times \frac{3}{0.2} = 0.918 \text{ A}$$

Problem 3.74: *The number density of electrons in a semiconductor is 0.45 × 10^{12} m^{-3} and the number density of hole is 5 × 10^{20} m^{-3}. Determine its conductivity. Given: electron mobility = 0.135 m^2 V^{-1} s^{-1} and hole mobility = 0.048 m^2 V^{-1} s^{-1} and e = 1.6 × 10^{-19} C.*

Solution: $n_e = 0.45 \times 10^{12}$ m^{-3}, $n_h = 5 \times 10^{20}$ m^{-3}, $\mu_e = 0.135$ m^2 V^{-1} s^{-1},

$\mu_h = 0.048$ m^2 V^{-1} s^{-1}, $e = 1.6 \times 10^{-19}$ C

The conductivity of the semiconductor may be given as:

$$\sigma = e\,(n\, \mu_e + p\, \mu_h) = 1.6 \times 10^{-19} \times (0.45 \times 10^{12} \times 0.135 + 5 \times 10^{20} \times 0.048) = 3.84 \text{ S m}^{-1}$$

Problem 3.75: *A potential difference of 4.5 V is applied across a conductor of length 0.1 m. If the drift velocity of electrons is 1.5 × 10^{-4} m/sec, determine the electron mobility.*

Solution: $V = 4.5$ V, $l = 0.1$ m, $v_d = 1.5 \times 10^{-4}$ m/sec

The electron mobility may be given as:

$$\mu_e = \frac{v_d}{|\vec{E}|} = v_d \times \frac{l}{V} = 1.5 \times 10^{-4} \times \frac{0.1}{4.5} = 3.333 \times 10^{-6} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Problem 3.76: *The number density of electrons in the copper is 8.5 × 10^{28} m^{-3}. A current of 1 A flows through a copper wire of length 0.24 m and area of cross section 1.2 mm^2, when connected to a battery of 3 V. Determine the electron mobility.*

Solution: $n = 8.5 \times 10^{28}$ m^{-3}, $I = 1$ A, $l = 0.24$ m, $A = 1.2$ mm^2, $V = 3$ V

The current through the copper wire may be given as:

$$I = n\,e\,A\,v_d = n\,e\,A \times \mu_e \times |\vec{E}| = n\,e\,A \times \mu_e \times \frac{V}{l}$$

So, $$\mu_e = \frac{I \times l}{n\,e\,AV} = \frac{1 \times 0.24}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.2 \times 10^{-6} \times 3} = 4.9 \times 10^{-6}\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

Problem 3.77: *The mobility of electrons and holes in a sample of intrinsic germanium at room temperature are $0.54\ m^2\ V^{-1}\ s^{-1}$ and $0.18\ m^2\ V^{-1}\ s^{-1}$ respectively. If the electron and hole densities are equal to $3.6 \times 10^{19}\ m^{-3}$, determine the conductivity of germanium.*

Solution: $\mu_e = 0.54\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, $\mu_h = 0.18\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, $n = p = 3.6 \times 10^{19}\ \text{m}^{-3}$

The conductivity of the germanium may be given as:

$$\sigma = e\,(n\,\mu_e + p\,\mu_h) = 1.6 \times 10^{-19} \times 3.6 \times 10^{19} \times (0.54 + 0.18) = 4.1472\ \text{S m}^{-1}$$

3.14 Temperature Dependency of Resistivity: The *resistivity* of any material depends on the *number density of charge carriers* and the *mean collision time* (τ, the *relaxation time*) between them.

We know that the *resistivity* of the material may be given as: $\rho = \dfrac{m_e}{n e^2 \tau}$.

Metals (Conductors): The *number density* (n) of *free electrons,* in metals, is almost constant and independent of the temperature. As, the temperature of the metal increases, the thermal speed of *electrons* as well as *to and fro oscillations* of the *positive metal ions* about their mean position increases. Consequently, the *free electrons* collide more frequently with the *metal ions* oscillating with greater amplitude about their mean positions. So, the *mean collision time* (τ) decreases and hence, *the resistivity of a metal* $\left(\rho \propto \dfrac{1}{\tau}\right)$ *increases and consequently the conductivity decreases with the increasing temperature of the metal.*

For most of the metals, *resistivity increases linearly* with the increase in temperature for small temperature differences. So, the *resistivity* of the metal at any temperature (T) may be given as:

$$\rho_t = \rho_0\,[1 + \alpha\,(T - T_0)] \qquad (y = mx + c) \qquad (3.52)$$

Where, ρ_0 is the resistivity at a lower temperature T_0 (usually 20°C) and α is the *coefficient of resistivity*.

So, $$\alpha = \frac{\rho_t - \rho_0}{\rho_0 (T - T_0)} = \frac{1}{\rho_0} \times \frac{d\rho}{dT} \qquad (3.53)$$

Hence, the ***temperature coefficient of resistivity*** (α) may be defined as the *increase in resistivity per unit resistivity per degree rise in temperature*.

The SI unit of *temperature coefficient of resistivity* (α) is °C^{-1}. The value of α is positive for metals, as the *resistivity always increases for a metal with increase in temperature.*

The temperature dependency of the *resistivity* (ρ) at low temperature is non-linear. The *resistivity* of a pure metal increases as a higher power of temperature at very low temperatures, as shown for the copper in the Fig. 3.17 (a).

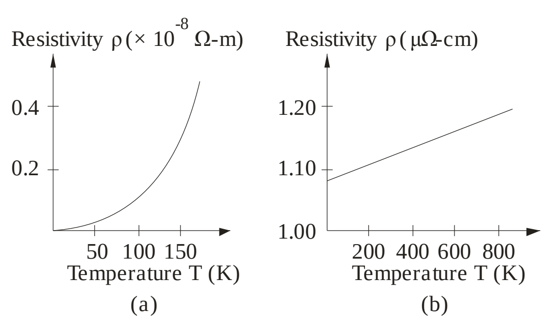

Fig. 3.17

Alloys, generally have a high *resistivity*. The *resistivity* of nichrome has weak temperature dependence, as shown in the Fig. 3.17 (b), while that of manganin is almost independent of temperature. At absolute zero temperature, a pure metal has negligibly small *resistivity*, while an alloy (like nichrome) has some *residual resistivity*.

The *resistance dependency equation* may also be written as:

$$R_t = R_0 [1 + \alpha (T - T_0)] = R_0 (1 + \alpha t) \qquad (3.54)$$

Where, R_t = the resistance at t °C

R_0 = the resistance at lower temperature (usually 20 °C)

t = the rise in temperature above lower temperature (usually 20 °C).

Semiconductors and Insulators: The *relaxation time* (τ), in semiconductors and insulators, is almost constant and independent of temperature. The number density of *free electrons* (n) increases exponentially, with the increasing temperature of semiconductors and insulators. Consequently, the *conductivity* of the semiconductors and insulators increases or *resistivity* decreases exponentially with the increase in temperature. The *number density of free electrons* at temperature T may be given as:

$$n(T) = n_0 \, e^{-(E_g / k_B T)} \qquad (3.55)$$

Where, k_B is Boltzmann constant and E_g is the *energy gap* (positive energy) between *conduction band* and the *valence band* of the material.

As, we know that the *resistivity* of the material is inversely proportional to the *number density of the free electrons* (n),

i.e. $\rho \propto \dfrac{1}{n}$ or, $n \propto \dfrac{1}{\rho}$ (3.56)

So, we may write:

$$\frac{1}{\rho(T)} = \frac{1}{\rho_0} \times e^{-(E_g / k_B T)}$$

or, $$\rho(T) = \rho_0 \, e^{(E_g / k_B T)} \qquad (3.57)$$

The reader may easily interpret that the *resistivity* of the semiconductors and insulators rapidly increases with decrease in temperature, becoming infinitely large as $T \rightarrow 0$.

The value of $k_B T$, at room temperature (27°C) for semiconductors and insulators, may be given as:

$$k_B T = 1.38 \times 10^{-23} \times (27 + 273) = 4.14 \times 10^{-21} \text{ V} = 0.0414 \times 10^{-19} \text{ V}$$
$$= 0.025875 \times 1.6 \times 10^{-19} \text{ V} \approx 0.03 \text{ eV}$$

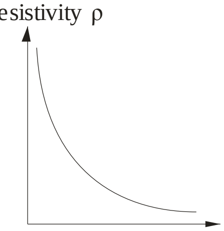

Fig. 3.18

The given substance is a semiconductor or an insulator depends on the value of energy gap (E_g) only.

i) If $E_g \leq 1$ eV, the resistivity at room temperature is not very high and the given substance is a ***semiconductor***.

ii) If $E_g > 1$ eV, the resistivity at room temperature is very high (~ 10^3 Ω-m) and the given substance is an ***insulator***.

The *coefficient of resistivity* (α) is negative for carbon, insulators and semiconductors, *i.e.* their *resistivity* decreases with increases in temperature, as shown in the Fig. 3.18.

Electrolytes: The *viscous forces* and hence the *inter-ionic attraction* (between solute-solute, solute-solvent, solvent-solvent) decreases between the ions of the electrolyte inside the solution with increase in the temperature of the electrolyte. So, the ions can move more freely inside the solution at greater temperatures. Hence, *conductivity of the electrolyte increases* and the *resistivity of the electrolyte decreases* with increase in temperature of the electrolyte.

Use of Alloys in Manufacturing of Standard Resistors: Alloys like constantan or menganin are used in manufacturing of standard resistor coils because of the following reasons:

i) These alloys have high value of *resistivity*.

ii) They have very small *temperature coefficient* (α) *of resistivity*. So, their resistance remains almost constant and independent of the temperature rise of the resistor.

iii) They are least affected by atmospheric conditions like air, moisture, oxidation (rusting) *etc*.

iv) Their contact potential drop with copper is small.

Problem 3.78: *i*) *Determine the temperature at which the resistance of a copper conductor becomes double of its resistance at 0 °C.* (*Given:* α *for copper = 3.9 × 10^{-3} °C^{-1}*)

ii) *Does this temperature hold good for all copper conductors regardless of shape and size?*

Solution: $t_0 = 0$ °C, $t_t = t$ °C, $R_t = 2\,R_0$

The expression for the resistance in terms of temperature coefficient may be given as:

$$R_t = R_0\,(1 + \alpha\, t)$$

or, $$t = \frac{(R_t - R_0)}{R_0 \times \alpha} = \frac{(2R_0 - R_0)}{R_0 \times 3.9 \times 10^{-3}} = 256.41\ °\text{C}$$

Since, the temperature coefficient (α) is related to the resistivity of the material (which is independent of the size and shape of the conductor's material), so this temperature holds good for all copper conductors regardless of shape and size.

Problem 3.79: *The resistance of platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is found to be 5.975 Ω. Determine the temperature of the hot bath.* [NCERT]

Solution: $R_0 = 5\ \Omega$ (at 0 °C), $R_{steam} = 5.39\ \Omega$ (at 100 °C), $R_t = 5.975\ \Omega$

At steam point (*t* = 100 °C):

The expression for the resistance in terms of temperature coefficient may be given as:

$$R_{steam} = R_0\,(1 + \alpha\, t_{steam})$$

or, $$\alpha = \frac{(R_{steam} - R_0)}{R_0 \times t_{steam}} = \frac{(5.39 - 5)}{5 \times 100} = 7.8 \times 10^{-4}\ °\text{C}^{-1}$$

At *t* °C corresponding to 5.975 Ω:

$$R_t = R_0\,(1 + \alpha\, t)$$

or, $$t = \frac{(R_t - R_0)}{R_0 \times \alpha} = \frac{(5.975 - 5)}{5 \times 7.8 \times 10^{-4}} = 250\ °\text{C}$$

Problem 3.80: *A nichrome heating element connected to a 220 V supply draws an initial current of 2.2 A, which settles down after a few seconds to 2 A. Determine the steady state temperature of the heating element. The room temperature is 30 °C and the average temperature coefficient of resistance of nichrome is 1.7 × 10^{-4} °C^{-1}.*

Solution: $V = 220$ V, $I_i = 2.2$ A, $I_f = 2$ A, $t_{room} = 30$ °C, $\alpha = 1.7 \times 10^{-4}$ °C^{-1}

The resistance of the nichrome heating element at room temperature may be given as:

$$R_{room} = \frac{V}{I_i} = \frac{220}{2.2} = 100\ \Omega = R_1$$

The resistance of the nichrome heating element at steady state temperature may be given as:

$$R_t = \frac{V}{I_f} = \frac{220}{2} = 110\ \Omega = R_1\,[1 + \alpha\,(t - 30)]$$

or, $$[1 + \alpha\,(t - 30)] = \frac{110}{R_1} = \frac{110}{100} = 1.1$$

or, $$t = \frac{1.1 - 1}{\alpha} + 30 = \frac{0.1}{1.7 \times 10^{-4}} + 30 = 618.235\ °\text{C}$$

Problem 3.81: *An electric toaster uses nichrome wire as its heating element. Its resistance at room temperature (27 °C) is found to be 75.3 Ω, when a negligibly small current flows through the heating element. When the toaster is connected to a 230 V supply, the current settles after a few seconds to a steady state value of 2.68 A. Determine the steady state temperature of the heating element. The temperature coefficient of resistance of nichrome wire averaged over the temperature range involved is 1.7×10^{-4} °C^{-1}.* [NCERT]

Solution: $t_{room} = 27$ °C, $R_1 = 75.3\ \Omega$, $V = 230$ V, $I_f = 2.68$ A, $\alpha = 1.7 \times 10^{-4}$ °C^{-1}

The resistance of the nichrome heating element at steady state temperature may be given as:

$$R_t = \frac{V}{I_f} = \frac{230}{2.68} = 85.821\ \Omega = R_1\,[1 + \alpha\,(t - 27)]$$

or, $$[1 + \alpha\,(t - 27)] = \frac{230}{2.68 \times R_1} = \frac{230}{2.68 \times 75.3} = 1.1397$$

or, $$t = \frac{1.1397 - 1}{\alpha} + 27 = \frac{0.1397}{1.7 \times 10^{-4}} + 27 = 848.76\ °\text{C}$$

Problem 3.82: *The resistance of a tungsten filament at 150 °C is 133 Ω. Determine its resistance at 500 °C. The temperature coefficient of resistance of tungsten wire averaged over the temperature range involved is 4.5×10^{-3} °C^{-1}.*

Solution: $t_1 = 150$ °C, $R_{150} = 133\ \Omega$, $t_2 = 500$ °C, $\alpha = 4.5 \times 10^{-3}$ °C^{-1}

The resistance of the tungsten filament at 150 °C may be given as:

$$R_{150} = R_0\,(1 + \alpha\,t_1) = R_0 \times (1 + 4.5 \times 10^{-3} \times 150) = 1.675\,R_0 \qquad (3.58)$$

The resistance of the tungsten filament at 500 °C may be given as:

$$R_{500} = R_0\,(1 + \alpha\,t_2) = R_0 \times (1 + 4.5 \times 10^{-3} \times 500) = 3.25\,R_0 \qquad (3.59)$$

Equation (3.59) / (3.58):

$$\frac{R_{500}}{R_{150}} = \frac{3.25\,R_0}{1.675\,R_0}$$

or, $$R_{500} = \frac{3.25}{1.675} \times R_{150} = = \frac{3.25}{1.675} \times 133 = 258.06\ °\text{C}$$

Problem 3.83: *The resistance of a conductor at 20 °C is 3.15 Ω and at 100 °C is 3.75 Ω. Determine the temperature coefficient of resistance of the conductor. Also determine the resistance of the conductor at 0 °C.*

Solution: $R_1 = 3.15\ \Omega$ (at $t_1 = 20$ °C), $R_2 = 3.75\ \Omega$ (at $t_2 = 100$ °C), $t_0 = 0$ °C

The resistance of the conductor at 20 °C may be given as:

$$R_{20} = 3.15\ \Omega = R_0\,(1 + \alpha\, t_1) = R_0 \times (1 + \alpha \times 20) \qquad (3.60)$$

The resistance of conductor at 100 °C may be given as:

$$R_{100} = 3.75\ \Omega = R_0\,(1 + \alpha\, t_2) = R_0 \times (1 + \alpha \times 100) \qquad (3.61)$$

Equation (3.61) / (3.60):

$$\frac{R_0\,(1+\alpha\times 100)}{R_0\,(1+\alpha\times 20)} = \frac{3.75}{3.15} = 1.19$$

or, $(1 + 100\,\alpha) = 1.19\,(1 + 20\,\alpha)$

or, $(100 - 20 \times 1.19) \times \alpha = (1.19 - 1)$

or, $\alpha = \dfrac{0.19}{(100 - 20\times 1.19)} = 2.493 \times 10^{-3}\ °C^{-1}$

Putting in equation (3.60):

$$R_0 = \frac{3.15}{(1+20\,\alpha)} = \frac{3.15}{(1+20\times 2.493\times 10^{-3})} = 3\ \Omega$$

Problem 3.84: *A standard resistor marked 2 Ω is found to have a resistance of 2.118 Ω at 30 °C. Determine the temperature at which the marking is correct. The temperature coefficient of the resistance of the material of the coil is $4.2 \times 10^{-3}\ °C^{-1}$.*

Solution: $R_1 = 2.118\ \Omega$ (at $t_1 = 30$ °C), $R_2 = 2\ \Omega$ (at $t_2 = t$ °C) $\alpha = 4.2 \times 10^{-3}\ °C^{-1}$

The resistance of the conductor at 30 °C may be given as:

$$R_{30} = 2.118 = R_0\,(1 + \alpha\, t_1) = R_0 \times (1 + 4.2 \times 10^{-3} \times 30) = 1.126\, R_0 \qquad (3.62)$$

The resistance of conductor at t °C may be given as:

$$R_t = 2 = R_0\,(1 + \alpha\, t_2) = R_0 \times (1 + 4.2 \times 10^{-3} \times t) \qquad (3.63)$$

Equation (3.63) / (3.62):

$$\frac{R_0\,(1+4.2\times 10^{-3}\times t)}{1.126\,R_0} = \frac{2}{2.118}$$

or, $t = \dfrac{1}{4.2\times 10^{-3}} \times \left(\dfrac{2\times 1.126}{2.118} - 1\right) = 15.06$ °C

Problem 3.85: *A potential difference of 200 V is applied to a coil at a temperature of 15 °C which results in a current of 10 A. Determine the mean temperature of the coil when the current has fallen to 5 A, the applied voltage being same as before. Given:* $\alpha = \dfrac{1}{234}\ °C^{-1}$.

Solution: $V = 200$ V, $I_1 = 10$ A (at $t_1 = 15$ °C), $I_2 = 5$ A (at $t_2 = t$ °C) $\alpha = \dfrac{1}{234}\ °C^{-1}$

The resistance of the conductor at 15 °C may be given as:

$$R_{15} = \frac{200}{10} = 20 = R_0\,(1 + \alpha\, t_1) = R_0 \times \left(1+\frac{15}{234}\right) = 1.064\, R_0 \qquad (3.64)$$

The resistance of conductor at t °C may be given as:

$$R_t = \frac{200}{5} = 40 = R_0\,(1 + \alpha\, t_2) = R_0 \times \left(1 + \frac{t}{234}\right) \tag{3.65}$$

Equation (3.65) / (3.64):

$$\frac{R_0\,(1+t/234)}{R_0\,(1+15/234)} = \frac{40}{20} = 2$$

or, $$t = 234 \times \left[2 \times \left(1 + \frac{15}{234}\right) - 1\right] = 264\ °\mathrm{C}$$

Problem 3.86: *The resistances of iron and copper wires at 20 °C are 3.9 Ω and 4.1 Ω respectively. Determine the temperature at which resistances of both the wires will be equal. Temperature coefficient of resistivity for iron is 5×10^{-3} K^{-1} and for copper is 4×10^{-3} K^{-1}. Neglect thermal expansion.*

Solution: $R_{Fe.1}$ = 3.9 Ω (at t = 20 °C), $R_{Cu.1}$ = 4.1 Ω (at t = 20 °C), $R_{Fe.2} = R_{Cu.2}$,

$\alpha_{Fe} = 5 \times 10^{-3}\ \mathrm{K}^{-1}$, $\alpha_{Cu} = 4 \times 10^{-3}\ \mathrm{K}^{-1}$

The resistance of the iron wire at t °C may be given as:

$$R_{Fe.2} = R_{Fe.1}\,[1 + \alpha_{Fe}\,(t - 20)]$$

The resistance of the copper wire at t °C may be given as:

$$R_{Cu.2} = R_{Cu.1}\,[1 + \alpha_{Cu}\,(t - 20)]$$

Since both the resistances are equal at this temperature,

So, $$R_{Fe.1}\,[1 + \alpha_{Fe}\,(t - 20)] = R_{Cu.1}\,[1 + \alpha_{Cu}\,(t - 20)]$$

or, $$3.9 \times [1 + 5 \times 10^{-3} \times (t - 20)] = 4.1 \times [1 + 4 \times 10^{-3} \times (t - 20)]$$

or, $$[3.9 \times 5 - 4.1 \times 4] \times 10^{-3} \times (t - 20) = 4.1 - 3.9 = 0.2$$

or, $$t = \frac{0.2}{[3.9 \times 5 - 4.1 \times 4] \times 10^{-3}} + 20 = 84.52\ °\mathrm{C}$$

Problem 3.87: *A metal wire of diameter 2 mm and length 100 m has a resistance of 0.5475 Ω at 20 °C and 0.805 Ω at 150 °C. Determine: i) the temperature coefficient of resistance, ii) resistance of the wire at 0 °C, iii) resistivity at 0 °C and 20 °C.*

Solution: D = 2 mm, l = 100 m, R_1 = 0.5475 Ω (at t = 20 °C), R_2 = 0.805 Ω (at t = 150 °C)

The resistance of the wire at 20 °C may be given as:

$$R_{20} = R_0\,(1 + \alpha \times 20) = 0.5475 \tag{3.66}$$

The resistance of the wire at 150 °C may be given as:

$$R_{150} = R_0\,(1 + \alpha \times 150) = 0.805 \tag{3.67}$$

Equation (3.67) / (3.66):

$$\frac{R_0\,(1+150\,\alpha)}{R_0\,(1+20\,\alpha)} = \frac{0.805}{0.5475} = 1.4703$$

or, $$1 + 150\,\alpha = 1.4703 + 29.406\,\alpha$$

So, $$\alpha = \frac{(1.4703 - 1)}{(150 - 29.406)} = 3.9 \times 10^{-3}\ °\mathrm{C}^{-1}$$

The resistance at 0 °C may be obtained by putting the value of α in the equation (3.66):

$$R_0 = \frac{0.5475}{(1+20\times3.9\times10^{-3})} = 0.5433\ \Omega$$

The resistivity at 0 °C may be given as:

$$\rho_0 = \frac{R_0\,A}{l} = \frac{R_0\,(\pi D^2/4)}{l} = \frac{0.5433\times\pi\times(2\times10^{-3})^2}{4\times100} = 1.707\times10^{-8}\ \Omega\text{-m}$$

The resistivity at 20 °C may be given as:

$$\rho_{20} = \rho_0\,(1+\alpha\,t) = 1.707\times10^{-8}\times(1+3.9\times10^{-3}\times20) = 1.84\times10^{-8}\ \Omega\text{-m}$$

Problem 3.88: *A platinum wire has a resistance of 10 Ω at 0 °C and of 20 Ω at 273 °C. Determine its temperature coefficient of resistance.*

Solution: $R_0 = 10\ \Omega$, $R_{273} = 20\ \Omega$

The resistance of the platinum wire at a temperature t may be given as:

$$R_t = R_0\,(1+\alpha\,t)$$

or, $$\alpha = \frac{(R_t - R_0)}{R_0\,t} = \frac{(20-10)}{10\times273} = \frac{1}{273}\ ^\circ\text{C}^{-1}$$

Problem 3.89: *A standard resistor marked 3 Ω is found to have a true resistance of 3.115 Ω at 300 K. Determine the temperature at which the marking of the resistor is correct. Temperature coefficient of resistance of the material of the wire is $4.2\times10^{-3}\ ^\circ C^{-1}$.*

Solution: $R_t = 3\ \Omega$, $R_1 = 3.115\ \Omega$ (at t = 300 K = 27 °C), $\alpha = 4.2\times10^{-3}\ ^\circ\text{C}^{-1}$

The resistance of the wire at 27 °C may be given as:

$$R_{27} = R_0\,(1+\alpha\,t) = R_0\,(1+4.2\times10^{-3}\times27) = 1.1134\,R_0 = 3.115 \quad (3.68)$$

The resistance of the wire at t °C may be given as:

$$R_t = R_0\,(1+\alpha\,t) = R_0\,(1+4.2\times10^{-3}\times t) = 3 \quad (3.69)$$

Equation (3.69) / (3.68):

$$\frac{R_0\,(1+4.2\times10^{-3}\times t)}{1.1134\,R_0} = \frac{3}{3.115}$$

or, $$(1+4.2\times10^{-3}\times t) = \frac{3}{3.115}\times1.1134 = 1.0723$$

or, $$t = \frac{(1.0723-1)}{4.2\times10^{-3}} = 17.21\ ^\circ\text{C} = 17.21 + 273 = 290.21\ \text{K}$$

Problem 3.90: *The resistance of a silver wire at 0 °C is 1.25 Ω. Determine the temperature at which the resistance of the wire will be doubled. The temperature coefficient of resistance for silver is $3.75\times10^{-3}\ ^\circ C^{-1}$.*

Solution: $R_0 = 1.25\ \Omega$, $R_t = 2\,R_0$, $\alpha = 3.75\times10^{-3}\ ^\circ\text{C}^{-1}$

The temperature of the silver wire at a temperature t may be given as:

$$R_t = R_0\,(1+\alpha\,t)$$

or, $$2\,R_0 = R_0\,(1+3.75\times10^{-3}\times t)$$

So, $$t = \frac{(2-1)}{3.75\times10^{-3}} = 266.667\ ^\circ\text{C}$$

Problem 3.91: *The resistance of a platinum wire used in a platinum resistance thermometer at 0 °C is 3 Ω and at 100 °C is 3.75 Ω. Its resistance under a measurement is 3.15 Ω. Determine the temperature under measurement.*

Solution: $R_0 = 3\ \Omega$ (at 0 °C), $R_{100} = 3.75\ \Omega$ (at 100 °C), $R_t = 3.15\ \Omega$

At the temperature of 100 °C:

The expression for the resistance in terms of temperature coefficient may be given as:

$$R_{100} = R_0\,(1 + \alpha\, t)$$

or, $$\alpha = \frac{(R_{100} - R_0)}{R_0 \times t} = \frac{(3.75 - 3)}{3 \times 100} = 2.5 \times 10^{-3}\ °C^{-1}$$

At *t* °C corresponding to 3.15 Ω:

$$R_t = R_0\,(1 + \alpha\, t)$$

or, $$t = \frac{(R_t - R_0)}{R_0 \times \alpha} = \frac{(3.15 - 3)}{3 \times 2.5 \times 10^{-3}} = 20\ °C$$

Problem 3.92: *The temperature coefficient of a resistance wire is 1.25 × 10^{-3} °C^{-1}. At 300 K its resistance is found to be 1 Ω. Determine the temperature at which the resistance of the wire will become 2 Ω.* [IIT 1980]

Solution: $\alpha = 1.25 \times 10^{-3}\ °C^{-1}$, $R_{27} = 1\ \Omega$ (at 300 K = 27 °C), $R_t = 2\ \Omega$

The resistance of the wire at 27 °C may be given as:

$$R_{27} = R_0\,(1 + \alpha\, t) = R_0\,(1 + 1.25 \times 10^{-3} \times 27) = 1.03375\, R_0 = 1 \qquad (3.70)$$

The resistance of the wire at *t* °C may be given as:

$$R_t = R_0\,(1 + \alpha\, t) = R_0\,(1 + 1.25 \times 10^{-3} \times t) = 2 \qquad (3.71)$$

Equation (3.71) / (3.70):

$$\frac{R_0\,(1 + 1.25 \times 10^{-3} \times t)}{1.03375\, R_0} = \frac{2}{1}$$

or, $$(1 + 1.25 \times 10^{-3} \times t) = 2 \times 1.03375 = 2.0675$$

or, $$t = \frac{(2.0675 - 1)}{1.25 \times 10^{-3}} = 854\ °C = 854 + 273 = 1127\ K$$

Problem 3.93: *The temperature coefficient of resistivity of copper is 4 × 10^{-3} °C^{-1}. Determine the temperature of a 5 m long copper wire of diameter 0.2 mm at 100 °C, if the resistivity of the copper at 0 °C is 1.7 × 10^{-8} Ω-m.*

Solution: $\alpha = 4 \times 10^{-3}\ °C^{-1}$, $l = 5$ m, $D = 0.2$ mm, $t = 100$ °C, $\rho_0 = 1.7 \times 10^{-8}\ \Omega$-m

The resistivity at 100 °C may be given as:

$$\rho_{100} = \rho_0\,(1 + \alpha\, t) = 1.7 \times 10^{-8} \times (1 + 4 \times 10^{-3} \times 100) = 2.38 \times 10^{-8}\ \Omega\text{-m}$$

So, the resistance of the wire at 100 °C may be given as:

$$R_{100} = \frac{\rho_{100}\, l}{A} = \frac{\rho_{100}\, l}{(\pi D^2 / 4)} = \frac{2.38 \times 10^{-8} \times 5 \times 4}{\pi \times (0.2 \times 10^{-3})^2} = 3.788\ \Omega$$

3.15 Limitations of Ohm's Law (Ohmic and Non-Ohmic Conductors): *Ohm's law* is obeyed by many conductors under certain conditions but it is not fundamental or universal law for all the substances.

Ohmic Conductors: *The conductors which obey Ohm's law are known as Ohmic Conductors*. The linear relationship between *voltage* (V) and *current* (I) holds good, *i.e.* $I \propto V$. The resistance of the conductor $\left(R = \dfrac{V}{I}\right)$ is independent of the *current* flowing through the conductor. The *current* (I) gets reversed in the direction on reversing the *potential difference* applied across the conductor, and the magnitude of *current* changes linearly with the voltage (V). So, *the V – I characteristics for ohmic conductors is a straight line passing through the origin*. A metallic conductor with small currents flowing through them or an electrolyte like copper sulphate ($CuSO_4$) solution with copper electrodes are ohmic conductors, as shown by their $V – I$ characteristic in the Fig. 3.19 and 3.20.

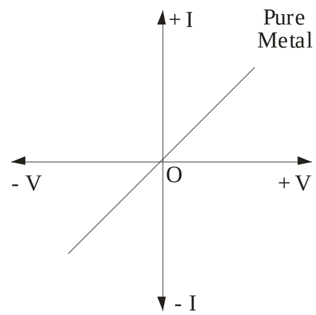

Fig. 3.19

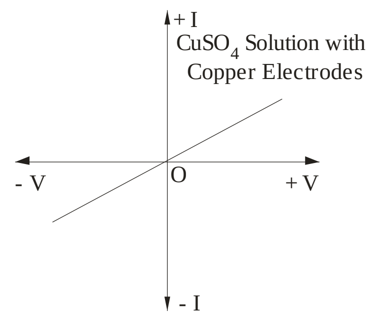

Fig. 3.20

Non-Ohmic Conductors: *The conductors which do not obey Ohm's law and deviate from the linear relationship of V and I are known as Non-Ohmic Conductors*. The resistance of such conductors / semiconductors is not constant even at a given temperature, or better to mention here that their resistance is current dependent. *Non-Ohmic* situations can be categorized as given below:

i) The $V – I$ relationship is non-linear.

i*i*) The $V – I$ characteristic does not pass through the origin.

iii) The $V – I$ relationship depends on the sign of *voltage* (V) applied for the same magnitude of V.

iv) The $V – I$ relationship is not a unique curve for the conductor / semiconductor.

***i*) Non-Ohmic Nature of Conductors:** The metallic conductors obey *Ohm's law* fairly well at smaller values of *current*. But the conductors get heat up, due to *heating effect of current*, when larger amount of *current* flows through them. This causes the temperature of the conductor to rise, and due to *temperature coefficient of resistivity* the increased temperature of the conductor results in increased *resistivity* of the conductor. So, the resistance of the conductors increases at larger values of the *current* flowing through them. The $V – I$ characteristic for the conductors no longer remains linear as shown in the Fig. 3.21, *i.e. conductors become non-ohmic at larger values of current*, as this is the deviation in the behavior of conductors from *Ohm's law*.

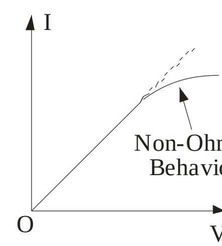

Fig. 3.21

***ii*) Water Voltammeter:** A *back emf* (V_0) sets up in a *Water Voltammeter* due to the liberation of the hydrogen at the *cathode* and oxygen at the *anode*. No current can flow through the *Water Voltammeter* until the *applied voltage* across it exceeds the *back emf* (V_0). So, the $V – I$ characteristic is a straight line but not passing through the origin, as shown in the Fig. 3.22. Hence, *the electrolyte* (water acidified with dilute H_2SO_4) *is a non-ohmic conductor*, as this is the deviation in the behavior of conductors from *Ohm's law*.

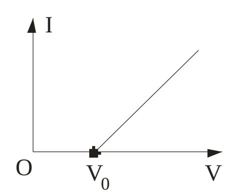

Fig. 3.22

***iii*) *P-N* Junction Diode:** It consists of a junction of *P-type semiconductor* and *N-type semiconductor*. When a *voltage* is applied across the *P-N junction*, the resulting *current I* is shown in the Fig. 3.23. The reader may observe that the $V – I$ characteristic is not linear for a *P-N junction diode*. Also, the behavior offered by the material to the flow of *current* is different for two *polarities* of the *voltage* applied. The *reverse current* (– I) is very small for very high *reverse voltage* (– V), this mode of conduction is known as *reverse biased P-N junction*. The value of *forward current* (+I) is sufficiently high for lower values of *forward voltage* (+V), this mode of conduction is known as *forward biased P-N junction*. So, the *junction diode* allows the *current* to flow in one direction only, *i.e.* it acts as a *rectifier*. Hence, the *junction diode* is non ohmic conductor, as this is the deviation in the behavior of conductors from *Ohm's law.*

***iv*) Thyristor:** It consists of four alternate layers of *P-type and N-type semiconductors*. The $V - I$ characteristic for a *thyristor* is shown in Fig. 3.24. The reader may easily observe that: *a*) $V - I$ characteristic is non-linear, *b*) $V - I$ characteristic has different pattern for *positive* and *negative* values of *voltage* applied across the *thyristor*, *c*) there are two or more values of *current* for same value of *potential difference* across the *thyristor*, in certain regions, *i.e.* the $V - I$ characteristic is not unique. The region *AB* has a peculiar behavior; the *current* through the device is increasing with the decreasing voltage, *i.e.* α is negative in this region *AB*.

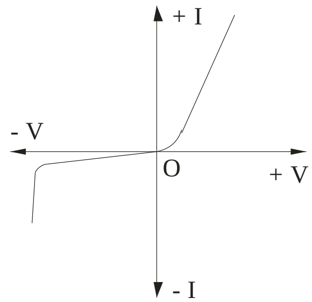

Fig. 3.23

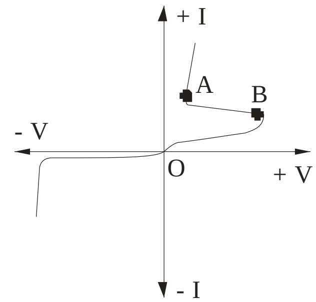

Fig. 3.24

***v*) Gallium Arsanide (GaAs):** The $V - I$ characteristic for the semiconductor Gallium Arsanide (GaAs) is shown in the Fig. 3.25. It exhibits non-linear behavior. After a certain value of *potential difference* applied across it, the *current* decreases as the voltage increases. It denotes that ΔV is positive but ΔI is negative for this region, hence the effective resistance $\left(R = \dfrac{\Delta V}{\Delta I} \right)$ in this region is negative.

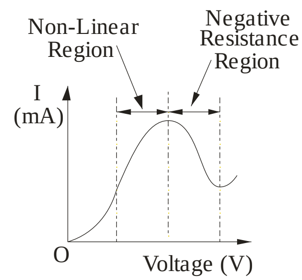

Fig. 3.25

3.16 Superconductivity: It has been observed during 1911 by Prof. *Kammerlingh Onnes* at the University of Leiden (Holland), that the *resistivity of mercury suddenly drops to zero at a temperature of 4.2 K* (which is in close proximity of *absolute zero temperature*) and the mercury becomes a *superconductor.*

The phenomenon of complete loss of resistivity by certain metals and alloys, when they are cooled below a certain temperature is known as '***Superconductivity***'. The temperature at which a substance undergoes a transition from *normal conductor* to *superconductor* in a *zero magnetic field* is known as '***transition temperature***' or '***critical temperature***' (T_C).

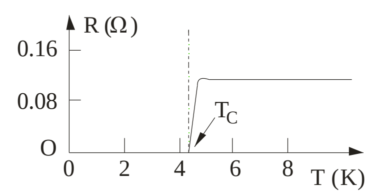

Fig. 3.26

A current once setup in a *superconductor* persists for a very long time without any apparent change in its magnitude.

Cause of Superconductivity: It is believed that a weak attractive force acts on the electrons, near the *transition temperature*, which brings them closer to form coupled pairs. Also, the ions fixed in the *lattice structure* have almost zero internal energy due to which their amplitude of vibrations about their mean position becomes zero. Such coupled pairs are not deflected by almost zero ionic vibrations and so move without collisions with the *ions* of the metal fixed in *lattice structure.*

Meissener Effect: If a conductor is cooled in a *magnetic field* to a temperature below the *transition* (*critical*) *temperature*, then the lines of *magnetic field* (*B*) are pushed out of the specimen, as shown in the Fig. 3.27. So, *magnetic field* (*flux density*, *B*) becomes zero inside a superconducting specimen. Hence, this effect indicates that the specimen becomes *perfectly diamagnetic* as soon as the *superconductivity* appears in a specimen.

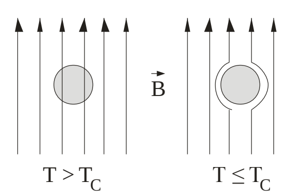

Fig. 3.27

High T_C Superconductivity: A *current*, once setup in a *superconducting loop*, can persist for years without any applied *emf* in the loop. This peculiar and important property of *superconductors* can have numerous important practical applications. But the serious problem in their use is that the *superconductivity* appears in certain substances at extremely low temperatures, which is very costly to maintain around the substance. The scientists are striving all over the world to construct the alloys, which would be superconducting at room temperature. *Superconductivity* at around 125 K has already been achieved, but still it is quite low temperature (as, 125 K = 125 – 273 = – 148 °C) which is very costly to maintain around a superconducting material. Efforts are still being made to improve upon this temperature.

Various Applications of Superconducting Materials: Various possible applications of superconductors are given below:

i) They may be used for producing high *magnetic fields* for a longer time, required for the research work in high energy physics.

ii) For storage data in the memory of high speed computers.

iii) They may be used for the construction of very sensitive galvanometers for measurement purpose.

iv) They may be used for creating high *magnetic field* for longer duration in *magnetic levitation trains* (*trains moving in the air above a rail due to magnetism*).

v) They may be very-very useful in the transmission of *electrical energy* over the long distances without any line losses [loss of electrical energy in long transmission lines as heating or ($I^2 R$) loss] due to their *zero resistivity*.

3.17 Combination of Resistors in Series and in Parallel: A number of several *resistors* may be connected, for getting some useful work done for a specific purpose, in *series* or in *parallel* or in a combination of these two (*series-parallel* combination). We may have to replace these *series* combination or *parallel* combination or *series-parallel* combination by a single *equivalent resistance* for calculation purposes to know about the satisfactory working of the circuit designed for a specific purpose.

"*If a combination of two or more resistors in any electrical circuit may be replaced by a single resistance, such that there is no change in the current flowing through the circuit and the potential difference across the terminals of the combination, then the single resistance is known as the* ***equivalent resistance*** *of the combination*".

Resistors in Series: *If a number of resistors are connected end to end, so that the current flowing through each of them is same and current flows from one resistor to next resistor then to next resistor and so on till the end, the combination is known as the series combination of the resistors*.

The *series* combination of three resistors (R_1, R_2 and R_3) connected across a d.c. supply source (battery of V Volts) is shown in Fig. 3.28. The *current* flowing through all the three resistors is same and denoted as I. However, the *potential differences* across three resistors are different and marked as V_1 across R_1, V_2 across R_2 and V_3 across R_3. According to *Ohm's law*, the *potential differences* across three individual resistors may be given as:

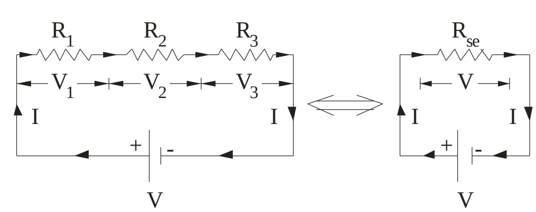

Fig. 3.28

$$V_1 = I R_1, \qquad V_2 = I R_2, \qquad V_3 = I R_3 \qquad (3.72)$$

If R_{se} is the equivalent resistance of combination of three *series* connected resistors (as shown in the figure), then *potential difference* across the *equivalent resistor* R_{se} may be given as:

$$V = I R_{se} \qquad (3.73)$$

The reader may observe from the Fig. 3.28, that the supply voltage V is equal to the sum of *potential differences* V_1, V_2 and V_3.

So, $V = V_1 + V_2 + V_3$

or, $I\,R_{se} = I\,R_1 + I\,R_2 + I\,R_3 = I \times (R_1 + R_2 + R_3)$

or, $R_{se} = (R_1 + R_2 + R_3)$ (3.74)

So, the *equivalent resistance* of *N* resistors connected in *series* may be given as:

$$R_{se} = (R_1 + R_2 + R_3 + \ldots\ldots\ldots + R_N) \quad (3.75)$$

Few rules for *series* combination of the resistors may be concluded here:

i) The *current* flowing through each *series* connected resistor is same.

ii) Total *potential drop* across the combination of *series* connected resistors is equal to the sum of *potential differences* across the individual resistors.

iii) The *potential drops across individual resistors are in the same ratio as of their resistances*.

iv) The *equivalent resistance* of the *series* combination of resistors is equal to the *sum of individual resistances*.

v) The *equivalent resistance* of the *series* combination is even *larger than the largest resistance in the combination*.

Resistors in Parallel: *If a number of resistors are connected in between two points, so that the potential difference across each of them is same and each resistor provides a unique path to flow of current, the combination is known as the parallel combination of the resistors.*

The *parallel* combination of three resistors (R_1, R_2 and R_3) connected between the points *A* and *B* and across a d.c. supply source (battery of *V* Volts) is shown in Fig. 3.29. The *current* flowing through all the three resistors are unique (say I_1 through R_1, I_2 through R_2 and I_3 through R_3). However, the *potential difference* across three resistors is same and is equal to the *potential difference* of the d.c. supply source applied across the terminals *A* and *B*. According to *Ohm's law*, the *currents* flowing through three individual resistors may be given as:

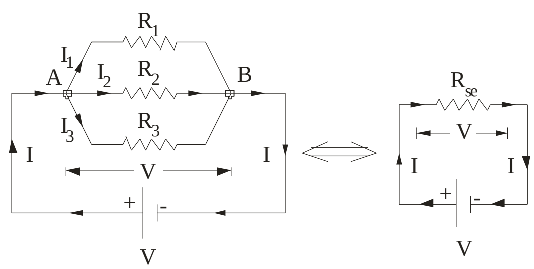

Fig. 3.29

$$I_1 = \frac{V}{R_1}, \qquad I_2 = \frac{V}{R_2}, \qquad I_3 = \frac{V}{R_3} \quad (3.76)$$

If R_{sh} is the equivalent resistance of combination of three resistors connected in *parallel* (as shown in the figure), then current flowing through the equivalent resistor R_{sh} may be given as:

$$I = \frac{V}{R_{eq}} \quad (3.77)$$

The reader may observe from the Fig. 3.29, that the current *I* is equal to the sum of individual currents I_1, I_2 and I_3.

So, $I = I_1 + I_2 + I_3$

or, $$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \times \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or, $$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad (3.78)$$

So, the *equivalent resistance* of *N* resistors connected in parallel may be given as:

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_N}\right) \quad (3.79.1)$$

or, $$R_{eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_N}\right)} \quad (3.79.2)$$

The *equivalent resistance* of two resistors connected in parallel may be given as:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{(R_1 + R_2)}{R_1 R_2}$$

So, $$R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)} \quad (3.80)$$

Few rules for *parallel* combination of the resistors may be concluded here:

i) The *potential difference* across each *parallel* connected resistor is same.

ii) Total *current* flowing through the combination of *parallel* connected resistors is equal to the sum of *current* flowing through the individual resistors.

iii) The *current flowing through individual resistors is in the inverse ratio of their resistances*.

iv) The reciprocal of *equivalent resistance* of the *parallel* combination of resistors is equal to the *sum of reciprocal of individual resistances*.

v) The *equivalent resistance* of the *parallel* combination is even *smaller than the smallest resistance in the combination.*

Problem 3.94: *A wire of resistance 4R is bent in the form of a circle, as shown in the Fig. 3.30 (a). Determine the effective resistance between the ends of its diameter.* [CBSE 1997-98]

Solution: Resistance of the wire = $4R$

The resistance of two halves of the wire bent in the circular shape may be given as:

$$R_1 = R_2 = \frac{4R}{2} = 2R$$

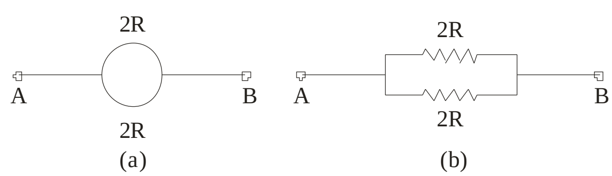

Fig. 3.30

The reader may observe that both the halves are connected in parallel as shown in the Fig. 3.30 (b), so the equivalent resistance may be given as:

$$R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{2R \times 2R}{2R + 2R} = \frac{4R^2}{4R} = R$$

Problem 3.95: *Find the value of current I in the circuit shown in the Fig. 3.31.* [CBSE 2002-03, IIT 1983]

Solution: $R_1 = R_2 = R_1 = 30\ \Omega$, $V = 2$ V

The equivalent resistance of the circuit across the points *A* and *B* may be given as:

$$R_{eq} = 30 \parallel (30 + 30) = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

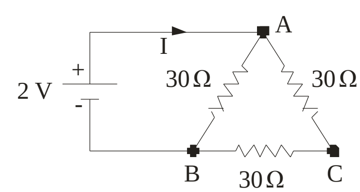

Fig. 3.31

So, the current *I* in the circuit may be given as:

$$I = \frac{V}{R_{eq}} = \frac{2}{20} = 0.1 \text{ A}$$

Problem 3.96: *Determine the voltage drop across the resistance R_1 in the circuit shown in the Fig. 3.32 with $E = 60$ V, $R_1 = 18\ \Omega$, $R_2 = R_4 = 10\ \Omega$ and $R_3 = 5\ \Omega$.* [CBSE 1993-94]

Solution: The equivalent resistance of the circuit as seen from the d.c. supply source (battery) may be given as:

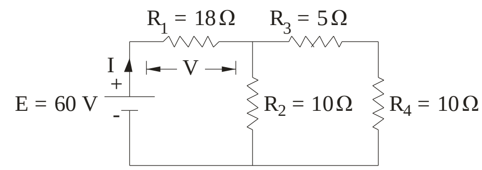

Fig. 3.32

$$R_{eq} = [(R_3 + R_4) \parallel R_2] + R_1$$
$$= [(5 + 10) \parallel 10] + 18$$
$$= \frac{15 \times 10}{15 + 10} + 18 = 24\ \Omega$$

So, the current supplied by the d.c. supply source may be given as:

$$I = \frac{E}{R_{eq}} = \frac{60}{24} = 2.5 \text{ A}$$

Hence, the potential difference across the resistance R_1 may be given as:

$$V = I \times R_1 = 2.5 \times 18 = 45 \text{ V}$$

Problem 3.97: *A letter 'A' consists of a uniform wire of resistance 1 Ω/cm. The sides of the letter are each 20 cm long and the cross piece in the middle is 10 cm long while the apes angle is 60°. Determine the resistance of the letter across the two ends of the legs.* [Himachal 1997-98]

Solution: The reader may easily conclude that the length of the sections:

$$l_{AB} = l_{BC} = l_{CD} = l_{DE} = l_{BD} = 10 \text{ cm}$$

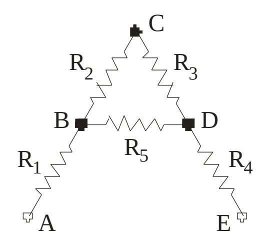

Fig. 3.33

So, the resistances of these sections may be given as:

$$R_{AB} = R_{BC} = R_{CD} = R_{DE} = R_{BD} = 10 \times 1 = 10\ \Omega$$

Now, the equivalent resistance of the letter '*A*' across its ands (*A* and *E*) may be given as:

$$R_{AE} = R_1 + [(R_2 + R_3) \parallel R_5] + R_4 = 10 + [(10 + 10) \parallel 10] + 10$$

or, $$R_{AE} = 10 + \frac{20 \times 10}{20 + 10} + 10 = 10 + 6.667 + 10 = 26.667\ \Omega$$

Problem 3.98: *A set of n identical resistors, each of resistance R ohm, when connected in series have an equivalent resistance of X ohm, and when the resistors are connected in parallel, their equivalent resistance is Y ohm. Derive the expression for the relation of R, X and Y.* [CBSE 1995-96]

Solution: The equivalent resistance of n resistors, each of $R\ \Omega$, connected in series may be given as:

$$R_{eq} = X = R + R + R + \ldots\ldots + R \qquad (n\text{-times})$$

or, $$X = n\,R \qquad (3.81)$$

The equivalent resistance of n resistors, each of $R\ \Omega$, connected in parallel may be given as:

$$R_{eq} = Y = \frac{1}{\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots\ldots + \frac{1}{R}\right)_{\text{n times}}} = \frac{R}{n} \qquad (3.82)$$

Multiplying equations (3.81) and (3.82):

$$XY = nR \times \frac{R}{n} = R^2$$

or, $XY = R^2$

Problem 3.99: *A parallel combination of three resistors takes a current of 7.5 A from a 30 V supply. If the two resistors are 10 Ω and 12 Ω, determine the value of third resistor.*

[Punjab 1990-91, Haryana 1993-94]

Solution: $I = 7.5$ A, $V = 30$ V, $R_1 = 10\ \Omega$, $R_2 = 12\ \Omega$

The equivalent resistance of R_1, R_2 and R_3 connected in parallel may be given as:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10} + \frac{1}{12} + \frac{1}{R_3}$$

The current drawn by this combination from a d.c. voltage source (30 V) may be given as:

$$\text{I} = \frac{V}{R_{eq}} = V \times \frac{1}{R_{eq}} = 30 \times \left(\frac{1}{10} + \frac{1}{12} + \frac{1}{R_3}\right) = 7.5$$

or, $$\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{R_3}\right) = \frac{7.5}{30} = \frac{1}{4}$$

or, $$\frac{1}{R_3} = \frac{1}{4} - \frac{1}{10} - \frac{1}{12} = \frac{15 - 6 - 5}{60} = \frac{4}{60} = \frac{1}{15}$$

or, $R_3 = 15\ \Omega$

Problem 3.100: *When a current of 0.5 A is passed through two resistors connected in series, the potential difference across the ends of the series arrangement is 12.5 V. On connecting them in parallel and passing a current of 1.5 A, the potential difference across the ends of parallel arrangement is 6 V. Determine the two resistances.*

Solution: $I_1 = 0.5$ A, $V_1 = 12.5$ V (for series arrangement),

$I_2 = 1.5$ A, $V_2 = 6$ V (for parallel arrangement)

The ohm's law for series arrangement (of R_1 and R_2) may be given as:

$$V_1 = I_1 \times R_{\text{eq (series)}} = I_1 \times (R_1 + R_2)$$

or, $12.5 = 0.5 \times (R_1 + R_2)$

or, $R_1 + R_2 = 25$ (3.83)

The ohm's law for parallel arrangement (of R_1 and R_2) may be given as:

$$V_2 = I_2 \times R_{\text{eq (parallel)}} = I_2 \times \frac{R_1 R_2}{R_1 + R_2}$$

or, $$6 = 1.5 \times \frac{R_1 R_2}{R_1 + R_2} = 1.5 \times \frac{R_1 R_2}{25}$$

or, $$R_1 R_2 = 6 \times \frac{25}{1.5} = 100 \qquad (3.84)$$

Now, $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4 R_1 R_2 = (25)^2 - 4 \times 100 = 225$

or, $R_1 - R_2 = 15$ (3.85)

Equation (3.83) + (3.85):

$$2\,R_1 = 40 \qquad \text{or,} \qquad R_1 = \frac{40}{2} = 20\ \Omega$$

Putting in equation (3.84):

$$R_2 = \frac{100}{20} = 5\ \Omega$$

Problem 3.101: *Two square metal plates A and B are of same thickness and material. The side of B is twice that of A. Two metal plates are connected in series, as shown in the Fig. 3.34. Determine the ratio R_A / R_B of resistances of two plates.*

Solution: $l_B = 2 \times l_A$

The ratio of the resistances of two plates *A* and *B* may be given as:

$$\frac{R_A}{R_B} = \frac{\left(\dfrac{\rho\, l_A}{A_A}\right)}{\left(\dfrac{\rho\, l_B}{A_B}\right)} = \frac{\rho\, l_A}{A_A} \times \frac{A_B}{\rho\, l_B} = \frac{l_A}{l_A \times d} \times \frac{l_B \times d}{l_B} = 1$$

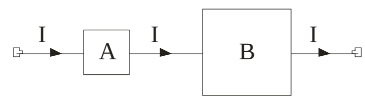

Fig. 3.34

So, $R_A : R_B = 1 : 1$

Problem 3.102: *Three conductors of conductance's G_1, G_2 and G_3 are connected in series. Determine the equivalent conductance of the combination.*

Solution: The equivalent resistance of three series connected resistors may be given as:

$$R_{eq} = R_1 + R_2 + R_3$$

So, $$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} = \frac{G_2\,G_3 + G_3\,G_1 + G_1\,G_2}{G_1\,G_2\,G_3}$$

or, $$G_{eq} = \frac{G_1\,G_2\,G_3}{G_1\,G_2 + G_2\,G_3 + G_3\,G_1}$$

Problem 3.103: *A cooper rod of length 20 cm and cross sectional area 2 mm^2 is joined with a similar aluminum rod as shown in the Fig. 3.38. Determine the resistance of the combination across two extreme ends of the combination. Resistivity of the copper = 1.7×10^{-8} Ω-m, and the resistivity of aluminum = 2.6×10^{-8} Ω-m.*

Solution: $l_{Cu} = l_{Al} = 20$ cm, $\qquad A_{Cu} = A_{Al} = 2$ mm^2,

$\rho_{Cu} = 1.7 \times 10^{-8}$ Ω-m, $\qquad \rho_{Al} = 2.6 \times 10^{-8}$ Ω-m

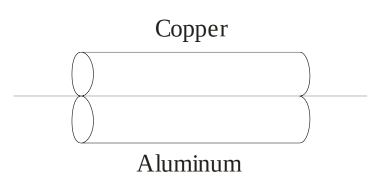

Fig. 3.35

Since both the rods are kept in parallel, the equivalent resistance of the combination may be given as:

$$R_{eq} = R_{Cu} \parallel R_{Al} = \frac{\rho_{Cu}\, l_{Cu}}{A_{cu}} \parallel \frac{\rho_{Al}\, l_{Al}}{A_{Al}}$$

or, $$R_{eq} = \frac{1.7 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} \parallel \frac{2.6 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} = 1.7 \times 10^{-3} \parallel 2.6 \times 10^{-3}$$

$$= (1.7 \parallel 2.6)\ \text{m}\Omega = \frac{1.7 \times 2.6}{1.7 + 2.6} = 1.028\ \text{m}\Omega$$

Problem 3.104: *A wire of uniform cross section and length 'l' has a resistance of 16 Ω. It is cut into four equal parts. Each part is stretched uniformly to length 'l' and all the four stretched parts are then connected in parallel. Determine the equivalent resistance of combination so formed. Assume that stretching of wires does not cause any change in the density of their material.* [CBSE 1996-97]

Solution: $R = 16\ \Omega$, $l_2 = \dfrac{l_1}{4}$ (all four equal parts), $l_3 = l$ (all four equal parts are stretched)

The resistance of each section after cutting into four equal parts may be given as:

$$R_2 = \frac{R}{4} = \frac{16}{4} = 4\ \Omega$$ and Area of cross section is unaltered, *i.e.* $A_2 = A_1$

The volume of each part remains constant. If there is no change in density after its stretching, the volume of the wire after stretching and before stretching may be given as:

$$\text{Volume} = A_3\, l_3 = A_2\, l_2$$

or, $$\frac{A_2}{A_3} = \frac{l_3}{l_2}$$

The ratio of the resistances before stretching and after stretching may be given as:

$$\frac{R_{stretched}}{R_{unstretched}} = \frac{(\rho l_3 / A_3)}{(\rho l_2 / A_2)} = \frac{l_3}{l_2} \times \frac{A_2}{A_3} = \left(\frac{l_3}{l_2}\right)^2$$

or, $$R_{\text{stretched}} = \left(\frac{l_3}{l_2}\right)^2 \times R_{\text{Unstretched}} = \left(\frac{l_3}{l_2}\right)^2 \times R_2 = \left[\frac{l_1}{(l_1/4)}\right]^2 \times 4 = 64\ \Omega$$

The equivalent resistance of four wires of 64 Ω in parallel may be given as:

$$R_{\text{eq}} = \frac{R_{stretched}}{4} = \frac{64}{4} = 16\ \Omega$$

Problem 3.105: *Determine the equivalent resistance across the terminals: i) A and B, ii) A and D, iii) A and C, in the network shown in the Fig. 3.36.* [IIT]

Solution: The equivalent resistance across the points *A* and *B* may be given as:

$$R_{\text{AB}} = [\{(3 + 7) \parallel 10\} + 5] \parallel 10$$

$$= \left(\frac{10\times10}{10+10} + 5\right) \parallel 10 = 10 \parallel 10 = \frac{10\times10}{10+10} = 5\ \Omega$$

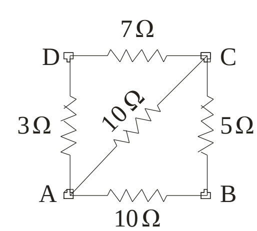

Fig. 3.36

The equivalent resistance across the points *A* and *D* may be given as:

$$R_{\text{AD}} = [\{(10 + 5) \parallel 10\} + 7] \parallel 3$$

$$= \left(\frac{15\times10}{15+10} + 7\right) \parallel 3 = 13 \parallel 3 = \frac{13\times3}{13+3} = \frac{39}{16} = 2.4375\ \Omega$$

The equivalent resistance across the points *A* and *C* may be given as:

$$R_{\text{AC}} = [(3 + 7) \parallel 10 \parallel (10 + 5)] = 10 \parallel 10 \parallel 15$$

$$= \frac{1}{\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{15}\right)} = \frac{30}{(3+3+2)} = \frac{15}{4} = 3.75\ \Omega$$

Problem 3.106: *Determine the effective resistance across the terminals A and B in the network shown in the Fig. 3.37.*

Solution: The equivalent resistance across the terminals A and B may be given as:

$$R_{eq} = ([\{([\{(3+3) \parallel 6\} + 3] \parallel 6) + 3\} \parallel 6] + 3) \parallel 3$$

$$= ([\{([\left(\frac{6\times 6}{6+6}+3\right)] \parallel 6) + 3\} \parallel 6] + 3) \parallel 3$$

$$= ([\{(6 \parallel 6) + 3\} \parallel 6] + 3) \parallel 3$$

$$= ([\left\{\frac{6\times 6}{6+6}+3\right\} \parallel 6] + 3) \parallel 3$$

$$= ([6 \parallel 6] + 3) \parallel 3 = \left(\frac{6\times 6}{6+6}+3\right) \parallel 3$$

$$= 6 \parallel 3 = \frac{6\times 3}{6+3} = 2\ \Omega$$

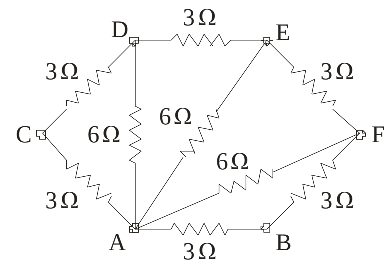

Fig. 3.37

Problem 3.107: *Determine the effective resistance of the network shown in the Fig. 3.41 across the points A and B, when: i) the switch S is open, ii) the switch S is closed.*

Solution: The equivalent resistance across the terminals A and B, when switch S is open, may be given as:

$$R_{eq} = (6+12) \parallel (12+6) = \frac{18\times 18}{18+18} = 9\ \Omega$$

The equivalent resistance across the terminals A and B, when switch S is closed, may be given as:

$$R_{eq} = (6 \parallel 12) + (12 \parallel 6) = \frac{6\times 12}{6+12} + \frac{6\times 12}{6+12}$$

$$= 4 + 4 = 8\ \Omega$$

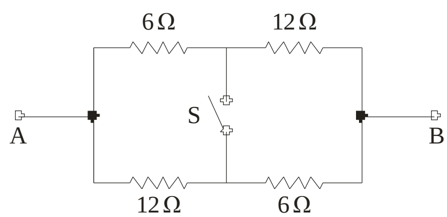

Fig. 3.38

Problem 3.108: *Determine the current measured by the ammeter A in the circuit shown in the Fig. 3.39 (a).* [CBSE 1999-2000]

Solution: The circuit shown in the Fig. 3.39 (a) may be redrawn as shown in the Fig. 3.39 (b).

The reader may easily observe that the equivalent resistance of the network as seen by the d.c. supply source (battery) may be given as:

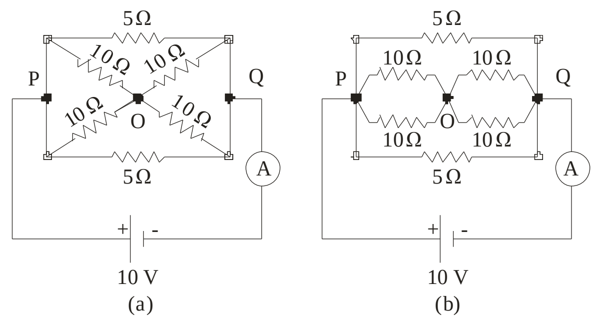

Fig. 3.39

$$R_{eq} = 5 \parallel [(10 \parallel 10) + (10 \parallel 10)] \parallel 5 = 5 \parallel \left[\frac{10\times 10}{10+10} + \frac{10\times 10}{10+10}\right] \parallel 5$$

$$= 5 \parallel [5+5] \parallel 5 = \frac{1}{\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{5}\right)} = \frac{10}{(2+1+2)} = 2\ \Omega$$

So, the current flowing through the circuit, which ammeter will measure, may be given as:

$$I = \frac{V}{R_{eq}} = \frac{10}{2} = 5 \text{ A}$$

Problem 3.109: *Determine the potential difference across the points A and B in the circuit shown in the Fig. 3.40, the internal resistances of the cells are negligible.*

Solution: The effective emf working in the loop (the emf of two sources is opposite to each other) may be given as:

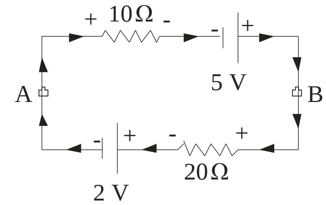

Fig. 3.40

$E = 5 - 2 = 3$ V (source of emf 5 V is dominating)

So, the current in the loop may be given as:

$$I = \frac{3}{(10+20)} = 0.1 \text{ A}$$

So, the potential difference across the 20 Ω resistance may be given as:

$$V_{20\,\Omega} = 0.1 \times 20 = 2 \text{ V}$$

And, the potential difference across the points *A* and *B* may be given as:

$$V_{AB} = 2 + 2 = 4 \text{ V}$$

Also, the potential difference across the 10 Ω resistance may be given as:

$$V_{10\,\Omega} = 0.1 \times 10 = 1 \text{ V}$$

And, the potential difference across the points *A* and *B* may be given as:

$$V_{AB} = -1 + 5 = 4 \text{ V}$$

Problem 3.110: *Determine the potential difference across the capacitor in the circuit shown in the Fig. 3.41* (*a*).

Solution: A capacitor, supplied by the d.c. current once charged to full voltage, behaves as open circuit. So, the network may be redrawn as shown in the Fig. 3.41 (b) for its response on steady state.

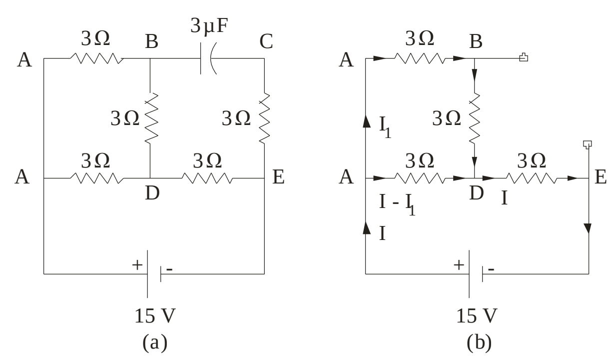

Fig. 3.41

The equivalent circuit of the network as seen by the d.c. source (battery) may be given as:

$$R_{eq} = [(3 + 3) \parallel 3] + 3$$

$$= \frac{6 \times 3}{6+3} + 3 = 2 + 3 = 5\ \Omega$$

The current supplied by the d.c. source (battery) may be given as:

$$I = \frac{V}{R_{eq}} = \frac{15}{5} = 3 \text{ A} = \text{The current through the branch } DE$$

The current through the branch *ABD* may be given as (Inverse current division rule):

$$I_1 = \frac{3}{3+6} \times 3 = 1 \text{ A} \quad \text{(Inverse current division rule)}$$

The potential difference across the capacitor may be given as:

$V_{\text{Capacitor}}$ = Potential Difference across the points BE

= Potential Difference across the points ($BD + DE$)

$= I_1 \times 3 + I \times 3 = (I_1 + I) \times 3 = (1 + 3) \times 3 = 12$ V

Problem 3.111: *Determine the potential difference across the points A and B in the circuit shown in the Fig. 3.42.* [CBSE 1993-94]

Solution: Since two branches are in parallel across the d.c. source (battery), the current through each branch may be given as:

$$I_1 = I_2 = \frac{V}{R_{branch}} = \frac{10}{(1+3)} = 2.5 \text{ A}$$

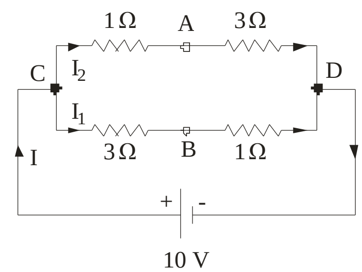

Fig. 3.42

If potential at point D is assumed as zero, the potential at point A and the potential at point B may, respectively, be given as:

$$V_A = I_1 \times 3 = 2.5 \times 3 = 7.5 \text{ V}$$

and, $V_B = I_2 \times 1 = 2.5 \times 1 = 2.5$ V

So, the potential difference across the points A and B may be given as:

$$V_{AB} = V_A - V_B = 7.5 - 2.5 = 5 \text{ V}$$

Problem 3.112: *In the network shown in the Fig. 3.43 (a), $R_1 = 100\ \Omega$, $R_2 = R_3 = 50\ \Omega$, $R_4 = 75\ \Omega$ and $E = 4.75$ V. Determine the equivalent resistance of the network and current through each resistor.*

Solution: $R_1 = 100\ \Omega$, $R_2 = R_3 = 50\ \Omega$, $R_4 = 75\ \Omega$, $E = 4.75$ V

The reader may easily observe that three resistors R_2, R_3 and R_4 are connected in parallel across two points, so the network may be redrawn as shown in the Fig. 3.43 (b).

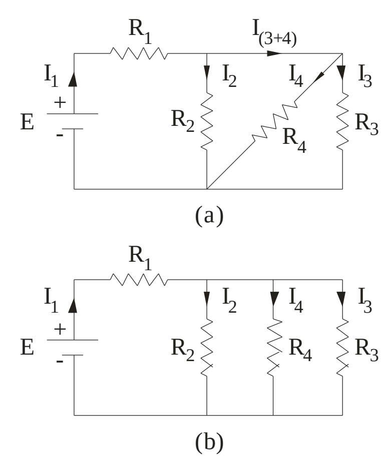

Fig. 3.43

The equivalent resistance of the network as seen by the d.c. source (battery) may be given as:

$$R_{eq} = (R_2 \parallel R_3 \parallel R_4) + R_1 = \frac{1}{\left(\frac{1}{50} + \frac{1}{50} + \frac{1}{75}\right)} + 100$$

$$= \frac{150}{(3+3+2)} + 100 = \frac{75}{4} + 100 = \frac{475}{4}\ \Omega$$

The current supplied by the d.c. source (I_1) may be given as:

$$I_1 = \frac{E}{R_{eq}} = \frac{4.75}{(475/4)} = 0.04 \text{ A} = 40 \text{ mA}$$

The currents I_2, I_3 and I_4 may respectively be given as (Inverse current division rule):

$$I_2 = \frac{\left(\frac{1}{50}\right)}{\left(\frac{1}{50} + \frac{1}{50} + \frac{1}{75}\right)} \times 40 = \frac{150}{50 \times (3+3+2)} \times 40 = 15 \text{ mA}$$

$$I_3 = \frac{\left(\frac{1}{50}\right)}{\left(\frac{1}{50}+\frac{1}{50}+\frac{1}{75}\right)} \times 40 = \frac{150}{50\times(3+3+2)} \times 40 = 15 \text{ mA}$$

$$I_4 = \frac{\left(\frac{1}{75}\right)}{\left(\frac{1}{50}+\frac{1}{50}+\frac{1}{75}\right)} \times 40 = \frac{150}{75\times(3+3+2)} \times 40 = 10 \text{ mA}$$

Problem 3.113: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.45 (a).*

Solution: The reader may easily observe from the Fig. 3.43 (a) that all the three resistors are connected between the points A and B, so the network may be redrawn as shown in the Fig. 3.44 (b).

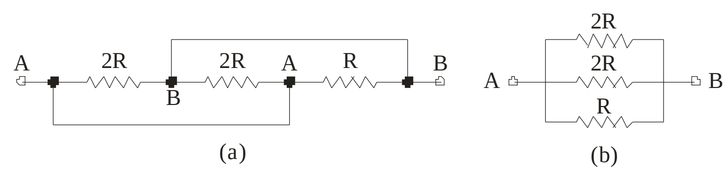

Fig. 3.45

Now the equivalent resistance across the points A and B may be given as:

$$R_{eq} = \frac{1}{\left(\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}\right)} = \frac{2R}{(1+1+2)} = \frac{R}{2}$$

Problem 3.114: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.45 (a).*

Solution: The given network may be drawn as shown in the Fig. 3.45 (b).

The reader may easily observe from the redrawn network, that the equivalent resistance across the points A and B may be given as:

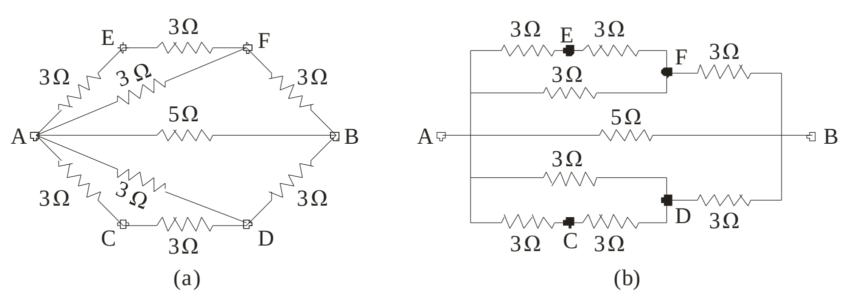

Fig. 3.45

$$R_{eq} = [\{(3+3) \parallel 3\} + 3] \parallel 5 \parallel [\{(3+3) \parallel 3\} + 3]$$

$$= \left[\frac{6\times 3}{6+3}+3\right] \parallel 5 \parallel \left[\frac{6\times 3}{6+3}+3\right] = 5 \parallel 5 \parallel 5$$

$$= \frac{1}{\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\right)} = \frac{5}{3}\ \Omega$$

Problem 3.115: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.46 (a).*

Solution: The reader may observe that the branches *CD* and *DE* will carry no currents due to zero potential difference between the points *C-D* and *D-E* (part of *balanced Wheatstone bridges*). So, the network may be redrawn by removing branches *CD* and *DE* from the network, as shown in the Fig. 3.46 (b).

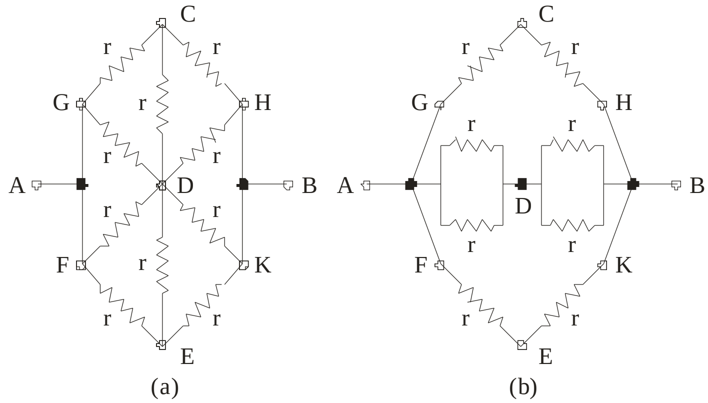

Fig. 3.46

The equivalent resistance across the points *A* and *B* may be given as:

$$R_{\text{eq}} = (r + r) \parallel [(r \parallel r) + (r \parallel r)] \parallel (r + r)$$

$$= (2r) \parallel \left[\frac{r \times r}{r+r} + \frac{r \times r}{r+r}\right] \parallel (2r) = (2r) \parallel \left[\frac{r}{2} + \frac{r}{2}\right] \parallel (2r)$$

$$= (2r) \parallel (r) \parallel (2r) = \frac{1}{\left(\frac{1}{2r} + \frac{1}{r} + \frac{1}{2r}\right)} = \frac{2r}{(1+2+1)} = \frac{r}{2}$$

Problem 3.116: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.47 (a).*

Solution: The reader may observe that if the resistance between points *CD* and *FE* are split into two equal halves as shown in the Fig. 3.47 (b), there is a line of symmetry (C_1-C-C_2) in the circuit shown in the Fig. 3.47 (b). This is due to the reason that the potentials at these points are equal. So, the circuit may be split in two equal halves, one of which is shown in the Fig. 3.47 (c).

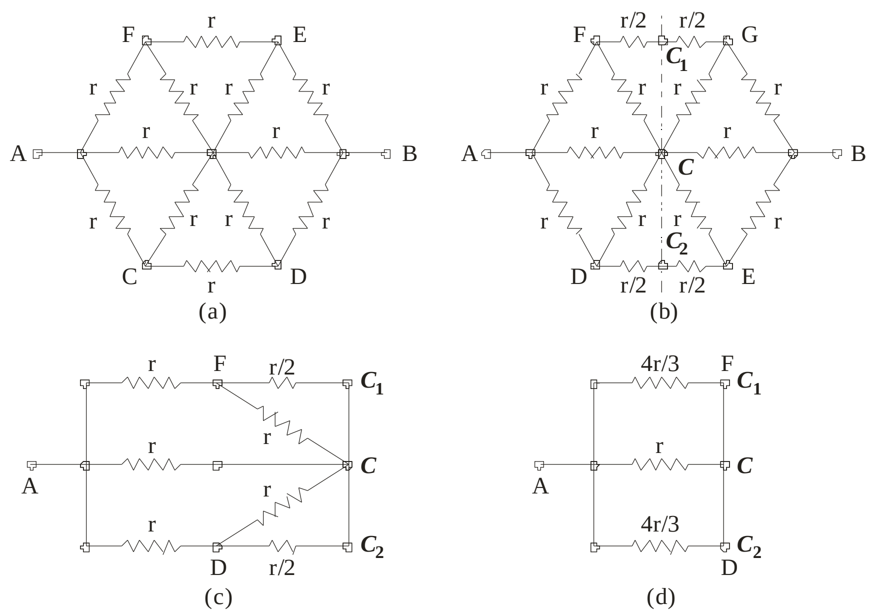

Fig. 3.47

Now, the equivalent resistance of one half of the circuit, shown in the Fig. 3.47 (c), across the points *A* and *C* may be given as:

$$R_{eq\,(1/2)} = \left[r + \left(\frac{r}{2} \parallel r\right)\right] \parallel [r] \parallel \left[r + \left(\frac{r}{2} \parallel r\right)\right]$$

or, $$R_{eq\,(1/2)} = \left[r + \frac{\frac{r}{2} \times r}{\frac{r}{2} + r}\right] \parallel [r] \parallel \left[r + \frac{\frac{r}{2} \times r}{\frac{r}{2} + r}\right] = \left[r + \frac{r}{3}\right] \parallel [r] \parallel \left[r + \frac{r}{3}\right]$$

$$= \left[\frac{4r}{3}\right] \parallel [r] \parallel \left[\frac{4r}{3}\right] = \frac{1}{\left(\frac{3}{4r} + \frac{1}{r} + \frac{3}{4r}\right)} = \frac{4r}{(3+4+3)}$$

$$= 0.4\,r$$

So, the equivalent resistance of the complete network across the points A and B may be given as:

$$R_{eq} = 2 \times R_{eq\,(1/2)} = 2 \times 0.4\,r = 0.8\,r$$

Problem 3.117: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.48* (*a*).

Solution: The reader may observe that the resistors connected between the points *AC*, *AD*, *BC* and *BD* are forming a *balanced Wheatstone Bridge*, as the potentials at points *C* and *D* are equal. So, the current passing through the resistor across the points *CD* is zero, and it may be open without affecting the network.

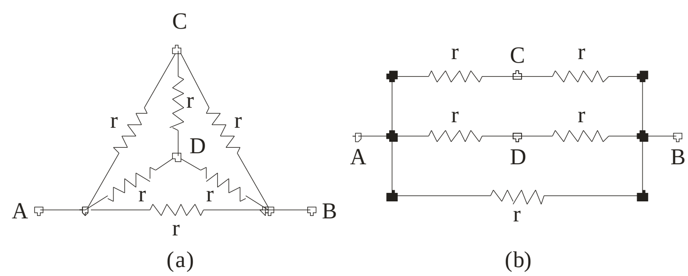

Fig. 3.48

Now the network may be redrawn as shown in the Fig. 3.48 (b), and the equivalent resistance across the points A and B may be given as:

$$R_{eq} = (r + r) \parallel (r + r) \parallel r = \frac{1}{\left(\frac{1}{2r} + \frac{1}{2r} + \frac{1}{r}\right)} = \frac{2r}{(1+1+2)} = \frac{r}{2}$$

Problem 3.118: *Determine the equivalent resistance across the points P and Q in the circuit shown in the Fig. 3.49* (*a*).

Solution: The reader may observe that two resistances of same value (r) are in parallel across each pair of points (*AB*, *BC* and *CA*). So, the equivalent resistance across each pair of points may be given as:

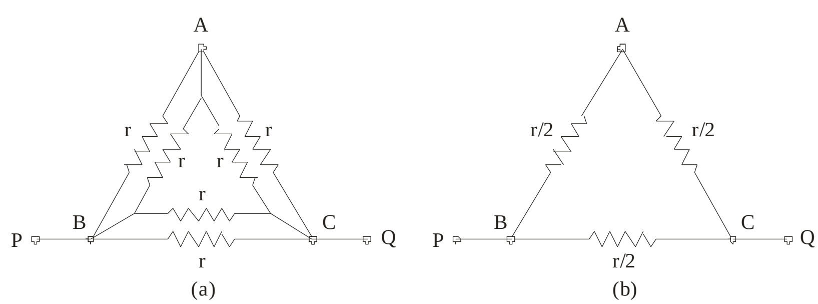

Fig. 3.49

$$R_{AB} = R_{BC} = R_{CA} = \frac{r \times r}{r + r} = \frac{r}{2}$$

Now, the network may be redrawn as shown in the Fig. 3.49 (b), and the equivalent resistance across the points *P* and *Q* may be given as:

$$R_{\text{eq}} = \left(\frac{r}{2}+\frac{r}{2}\right) \parallel \frac{r}{2} = \frac{r\times\dfrac{r}{2}}{r+\dfrac{r}{2}} = \frac{r}{3}$$

Problem 3.119: *How can the resistors of 2 Ω, 3 Ω and 6 Ω be connected, so as to give an equivalent resistance of 4 Ω?*

Solution: The reader may easily conclude that the required resistance is more than 2 Ω and 3 Ω but less than 6 Ω. So, the resistor of 6 Ω is to be connected either in parallel with 3 Ω or in parallel with 2 Ω. Let us check the first option that the resistors of 6 Ω and 3 Ω are connected in parallel and then this parallel combination is connected in series with the resistor of 2 Ω as shown in the Fig. 3.50.

The equivalent resistance across the points *A* and *B* may be given as:

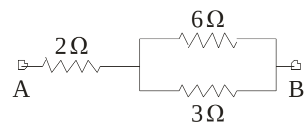

Fig. 3.50

$$R_{\text{eq}} = \frac{6\times 3}{6+3} + 2 = 2 + 2 = 4\ \Omega$$

Problem 3.120: *Three resistors, each of 30 Ω, are to be connected in some combination, so as to obtain the equivalent resistance of : i) 90 Ω, ii) 10 Ω iii) 45 Ω. Determine the configuration of each combination.*

Solution: The reader may easily conclude that the required resistance of 90 Ω may simply be obtained by connecting all three resistors in series, as shown in the Fig. 3.51 (a). The equivalent resistance of all three resistors connected in series may be given as:

$$R_{\text{eq}} = 30 + 30 + 30 = 90\ \Omega$$

The reader may also conclude that the required resistance of 10 Ω may simply be obtained by connecting all three resistors in parallel, as shown in the Fig. 3.51 (b). The equivalent resistance of all three resistors connected in parallel may be given as:

$$R_{\text{eq}} = \frac{1}{\left(\dfrac{1}{30}+\dfrac{1}{30}+\dfrac{1}{30}\right)} = \frac{30}{(1+1+1)} = 10\ \Omega$$

The reader may also conclude that the required resistance of 45 Ω may easily be obtained by connecting two resistors in parallel ($R_{\text{parallel}} = \dfrac{30}{2} = 15\ \Omega$) and then one resistor in series with it ($R_{\text{eq}} = 30 + 15 = 45\ \Omega$) , as shown in the Fig. 3.51 (c). The equivalent resistance of two resistors connected in parallel and then this combination in series with the third resistor may be given as:

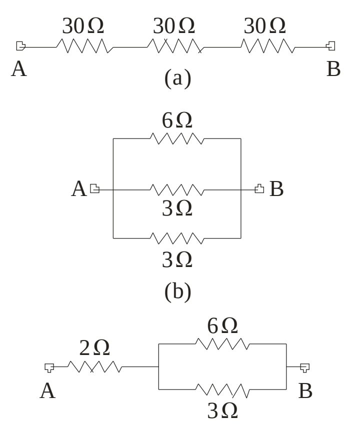

Fig. 3.51

$$R_{\text{eq}} = \frac{1}{\left(\dfrac{1}{30}+\dfrac{1}{30}\right)} + 30 = \frac{30}{(1+1)} + 30 = 45\ \Omega$$

Problem 3.121: *A 5 Ω resistor is connected in series with a parallel combination of ‘n’ resistors of 6 Ω each. The equivalent resistance is 7 Ω. Determine the value of n.*

Solution: The arrangement of the resistors in the network is shown in the Fig. 3.52. The equivalent resistance across the points A and B may be given as:

$$R_{eq} = 5 + \frac{1}{\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+........+(n-times)\right)}$$

$$= 5 + \frac{6}{[1+1+1+........+(n-times)]}$$

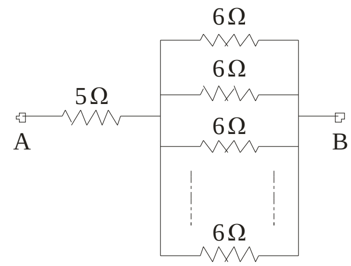

Fig. 3.52

or, $7 = 5 + \dfrac{6}{n}$

So, $n = \dfrac{6}{(7-5)} = 3$

Problem 3.122: *A uniform wire of resistance 2.2 Ω has a length of 2 m. Determine the length of the similar wire which when connected in parallel with the 2 m long wire, will result in a overall resistance of 2 Ω.*

Solution: $R_{2\text{ m}} = 2.2\ \Omega$, $l = 2$ m, $R_{eq} = 2\ \Omega$

The resistance per unit length of the wire may be given as:

$$r = \frac{R_{2m}}{l} = \frac{2.2}{2} = 1.1\ \Omega/\text{m}$$

Let the length of wire required is L.

The resistance of the wire with length L may be given as:

$$R_L = 1.1\ L$$

Now the equivalent resistance of the parallel combination of two wires may be given as:

$$R_{eq} = R_{2\text{ m}} \parallel R_L = \frac{2.2\times 1.1L}{2.2+1.1L} = 2$$

or, $2.2 + 1.1\ L = \dfrac{2.2\times 1.1L}{2} = 1.21$ L

or, $L = \dfrac{2.2}{(1.21-1.1)} = 20$ m

Problem 3.123: *A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal halves and two halves are connected in parallel across a d.c. source (battery) of 3 V. Determine the current supplied by the d.c. source.* [CBSE 2008-09]

Solution: $R = 15\ \Omega$, $l_2 = 2\ l_1$, $V = 3$ V

If the wire is stretched gradually the density of the material of wire and hence the volume of the material of wire remains unaltered. So, the unaltered volume under two conditions may be given as:

$$Volume = A_1 \times l_1 = A_2 \times l_2$$

or, $\dfrac{A_1}{A_2} = \dfrac{l_2}{l_1}$

The ratio of the resistances after stretching may be given as:

$$\frac{R_2}{R_1} = \frac{(\rho l_2 / A_2)}{(\rho l_1 / A_1)} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{2l_1}{l_1}\right)^2 = 4$$

So, $R_2 = 4 \times R_1 = 4 \times 15 = 60\ \Omega$

The resistances of two halves of the wire may be given as:

$$R_{1/2} = \frac{60}{2} = 30\ \Omega$$

The equivalent resistance of two halves connected in parallel may be given as:

$$R_{eq} = R_{1/2} \parallel R_{1/2} = \frac{30 \times 30}{30 + 30} = 15\ \Omega$$

The current supplied by the d.c. source may be given as:

$$I = \frac{V}{R_{eq}} = \frac{3}{15} = 0.2\ \text{A}$$

Problem 3.124: *The total resistance of two resistors when connected in series is 9 Ω and when connected in parallel is 2 Ω. Determine the resistances of two resistors.* [Punjab 1999-2000]

Solution: $R_{eq\,(series)} = 9\ \Omega$, $R_{eq\,(parallel)} = 2\ \Omega$

The equivalent resistance of two wires connected in series may be given as:

$$R_{eq\,(series)} = R_1 + R_2 = 9 \tag{3.86}$$

The equivalent resistance of two wires connected in parallel may be given as:

$$R_{eq\,(parallel)} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{9} = 2$$

or, $$R_1 R_2 = 2 \times 9 = 18 \tag{3.87}$$

So, $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4R_1 R_2 = (9)^2 - 4 \times 18 = 9$

or, $$R_1 - R_2 = 3 \tag{3.88}$$

Equation (3.86) + (3.88):

$$2\,R_1 = 9 + 3 = 12 \qquad \text{or,} \qquad R_1 = \frac{12}{2} = 6\ \Omega$$

Putting the value of R_1 in equation (3.87):

$$R_2 = \frac{18}{6} = 3\ \Omega$$

Problem 3.125: *Two wires a and b, each of length 40 m and area of cross section 10^{-7} m², are connected in series and a potential difference of 60 V is applied across the ends of the combination. Their resistances are respectively 40 Ω and 20 Ω. Determine for each wire, i) specific resistance, ii) electric field, iii) current density.*

Solution: $l_a = l_b = 40$ m, $A_a = A_b = 10^{-7}$ m², $R_a = 40\ \Omega$, $R_b = 20\ \Omega$, $V = 60$ V

The specific resistances of two wires may respectively be given as:

$$\rho_a = \frac{R_a\, A_a}{l_a} = \frac{40\times10^{-7}}{40} = 1 \times 10^{-7}\ \Omega\text{-m}$$

and, $$\rho_b = \frac{R_b\, A_b}{l_b} = \frac{20\times10^{-7}}{40} = 5 \times 10^{-8}\ \Omega\text{-m}$$

The potential difference across two wires may respectively be given as:

$$V_a = \frac{R_a}{R_a + R_b} \times V = \frac{40}{40+20} \times 60 = 40\ \text{V}$$

and, $$V_b = \frac{R_b}{R_a + R_b} \times V = \frac{20}{40+20} \times 60 = 20\ \text{V}$$

So, the electric field in two wires may be given as:

$$\left|\vec{E}_a\right| = \frac{V_a}{l_a} = \frac{40}{40} = 1\ \text{V/m} \qquad \text{and,} \qquad \left|\vec{E}_b\right| = \frac{V_b}{l_b} = \frac{20}{40} = 0.5\ \text{V/m}$$

The current flowing through each of the wire may be given as:

$$I_a = I_b = \frac{V}{R_{eq}} = \frac{60}{40+20} = 1\ \text{A}$$

So, the current density in two wires may be given as:

$$J_a = J_b = \frac{I}{A} = \frac{1}{10^{-7}} = 1 \times 10^{7}\ \text{A/m}^2$$

Problem 3.126: *Three resistances, each of 4 Ω, are connected in the form of an equilateral triangle. Determine equivalent resistance across any two vertices of the triangle.*

Solution: $R_1 = R_2 = R_3 = 4\ \Omega$

Three resistances connected in the form of an equilateral triangle are shown in the Fig. 3.53. The equivalent resistance across any two vertices of the triangle may be given as:

$$R_{eq} = (4 + 4) \parallel 4 = \frac{8\times4}{8+4} = 2.667\ \Omega$$

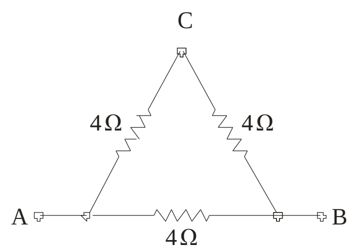

Fig. 3.53

Problem 3.127: *The ratio of two resistances is given as 1 : 4. If the equivalent resistance of these two resistors connected in parallel is 20 Ω, determine the value of two resistors.* [Punjab 1999-2000]

Solution: $R_1 : R_2 = 1 : 4, \qquad R_{eq\,(parallel)} = 20\ \Omega$

The reader may clearly observe that, $R_2 = 4\,R_1$

The equivalent resistance of two resistors connected in parallel may be given as:

$$R_{eq\,(parallel)} = \frac{R_1\, R_2}{R_1 + R_2} = \frac{R_1 \times 4\,R_1}{R_1 + 4\,R_1} = 20$$

or, $$\frac{4}{5}\, R_1 = 20 \qquad \text{or,} \qquad R_1 = 20 \times \frac{5}{4} = 25\ \Omega$$

and, $R_2 = 4 \times R_1 = 4 \times 25 = 100\ \Omega$

Problem 3.128: *Four resistors, of 12 Ω each, are connected in parallel. Three such combinations are then connected in series. Determine the total equivalent resistance. If a d.c. source (battery) of 9 V emf and negligible internal resistance is connected across the terminals of the network of the resistors, determine the current flowing through each of the resistor.* [Haryana 2001-02]

Solution: The desired network is drawn in the Fig. 3.54. The equivalent resistance of the network as seen by the d.c. source (battery) may be given as:

$$R_{\text{eq}} = 3 \times (12 \parallel 12 \parallel 12 \parallel 12) = 3 \times \frac{1}{\left(\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}\right)} = 3 \times \frac{12}{(1+1+1+1)} = 9\ \Omega$$

The current supplied by the d.c. source may be given as:

$$I = \frac{V}{R_{eq}} = \frac{9}{9} = 1\ \text{A}$$

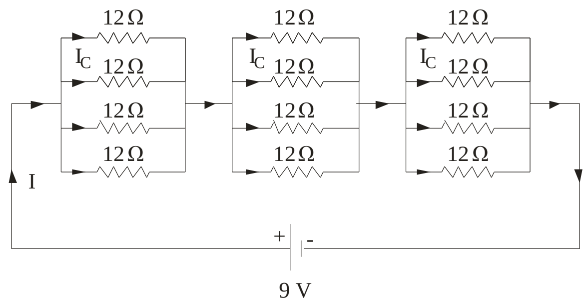

Fig. 3.54

The four resistances connected in parallel will divide this current of 1 A equally among themselves. So, the current through each resistor may be given as:

$$I_{\text{C}} = \frac{1}{4} = 0.25\ \text{A}$$

Problem 3.129: *Five resistors are connected as shown in the Fig. 3.55. Determine the equivalent resistance across the points B and C.* [Punjab 2000-01]

Solution: The equivalent resistance across the points *B* and *C* may be given as:

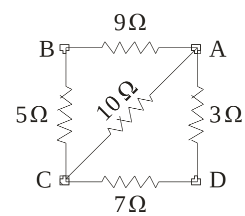

Fig. 3.55

$$R_{\text{eq}} = [\{(3+7) \parallel 10\} + 9] \parallel 5 = \left[\frac{10\times10}{10+10}+9\right] \parallel 5$$

$$= [5+9] \parallel 5 = \frac{14\times5}{14+5} = \frac{70}{19}\ \Omega$$

Problem 3.130: *The resistance of the rheostat shown in the Fig. 3.56 is 30 Ω. Neglecting the meter resistance, determine the minimum and maximum current through the ammeter as the resistance of the rheostat is varied.*

Solution: $R_{\text{h}} = 30\ \Omega$

The minimum and maximum resistances of the network shown in the Fig. 3.56 may respectively be given as:

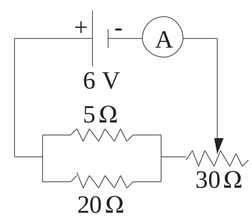

Fig. 3.56

$$R_{\text{eq (min)}} = (5 \parallel 20) + 0 = \frac{5\times20}{5+20} = 4\ \Omega$$

and, $$R_{\text{eq (max)}} = (5 \parallel 20) + 30 = \frac{5\times20}{5+20} + 30 = 34\ \Omega$$

So, the minimum and maximum current through the ammeter may respectively be given as:

$$I_{\text{min}} = \frac{V}{R_{eq\,(\text{max})}} = \frac{6}{34} = 0.1765\ \text{A} \qquad \text{and,} \qquad I_{\text{max}} = \frac{V}{R_{eq\,(\text{min})}} = \frac{6}{4} = 1.5\ \text{A}$$

Problem 3.131: *If the reading of the ammeter A_1 is 2.4 A in the Fig. 3.57, determine the readings of the ammeters A_2 and A_3. Neglect the resistances of the ammeters.*

Solution: $I_1 = 2.4$ A

Since, the currents are been divided in the parallel resistances in the inverse ratio of their resistances. So, the current I_1 may be given by the relationship:

$$I_1 = \frac{30}{20+30} \times I = 2.4$$

or, $$I = \frac{50}{30} \times 2.4 = 4 \text{ A}$$

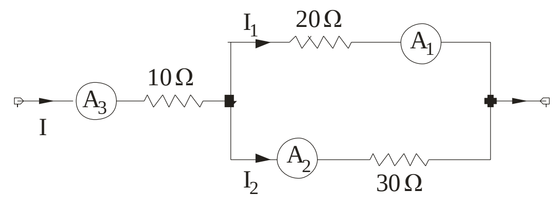

Fig. 3.57

So, the reading of the ammeter A_3 may be given as:

$$I = 4 \text{ A}$$

Now, the reading of the ammeter A_2 may be given as:

$$I_2 = \frac{20}{20+30} \times I = \frac{20}{50} \times 4 = 1.6 \text{ A}$$

Problem 3.132: *Determine the current in the 5 Ω resistor in the circuit shown in the Fig. 3.58, when the switch S is: i) open, ii) closed.*

Solution: The current through the resistor of value 5 Ω, with switch S open, may be given as:

$$I_{\text{S Open}} = \frac{V}{R_{eq}} = \frac{3}{5+10} = 0.2 \text{ A}$$

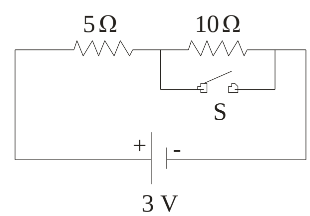

Fig. 3.58

When the switch S is closed, the resistor of value 10 Ω will get short circuited and no current will through it, all the current will flow through the switch S. So, the current through the resistor of value 5 Ω, with switch S closed, may be given as:

$$I_{\text{S Closed}} = \frac{V}{R_{eq}} = \frac{3}{5} = 0.6 \text{ A}$$

Problem 3.133: *The letter A consists of a uniform wire of resistance 1 Ω/cm. The sides of letter A are 40 cm long and the cross piece 10 cm long divides the sides in the ratio 1 : 3 from the apex. Determine the resistance of the letter between the two ends of the legs.* [Punjab 1997-98]

Solution: The letter A is drawn in the Fig. 3.59, as per the specification given in the problem.

The resistances of various parts of the letter A may be given as:

$$R_1 = R_2 = \left(\frac{3}{1+3} \times 40\right) \times 1 = 30 \ \Omega$$

and, $$R_3 = R_4 = \left(\frac{1}{1+3} \times 40\right) \times 1 = 10 \ \Omega$$

and, $$R_5 = 10 \times 1 = 10 \ \Omega$$

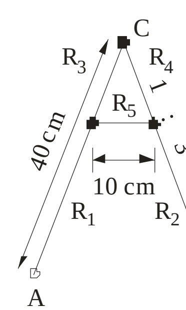

Fig. 3.59

Now the equivalent resistance across the points A and B may be given as:

$$R_{eq} = R_1 + [(R_3 + R_4) \parallel R_5] + R_2 = 30 + [(10 + 10) \parallel 10] + 30$$

or, $$R_{eq} = 30 + \frac{20\times10}{20+10} + 30 = 30 + 6.667 + 30 = 66.667\ \Omega$$

Problem 3.134: *Determine the equivalent resistance across the points A and B in each of the network shown in the Fig. 3.60.*

Solution: *a*) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = 8 + (8 \parallel 8) = 8 + \frac{8\times8}{8+8}$$

$$= 8 + 4 = 12\ \Omega$$

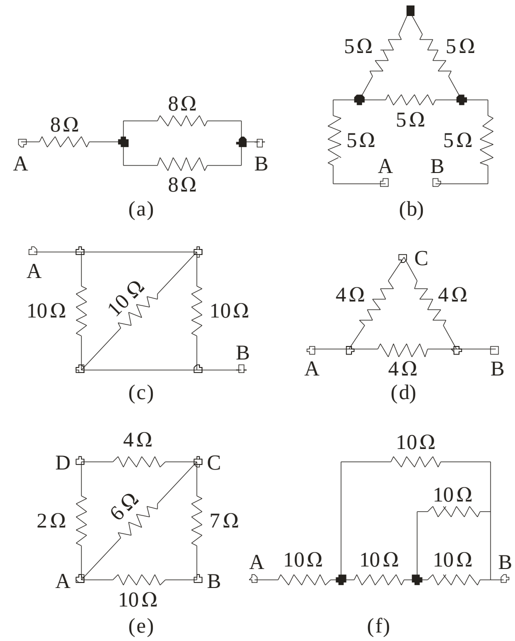

Fig. 3.60

b) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = 5 + [(5 + 5) \parallel 5] + 5$$

$$= 5 + \frac{10\times5}{10+5} + 5 = 5 + \frac{10}{3} + 5$$

$$= \frac{10+30}{3} = \frac{40}{3}\ \Omega$$

c) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = 10 \parallel 10 \parallel 10$$

$$= \frac{1}{\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)} = \frac{10}{(1+1+1)} = \frac{10}{3}\ \Omega$$

d) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = (4 + 4) \parallel 4 = \frac{8\times4}{8+4} = \frac{8}{3}\ \Omega$$

e) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = [\{(2 + 4) \parallel 6\} + 7] \parallel 10 = \left[\frac{6\times6}{6+6}+7\right] \parallel 10 = 10 \parallel 10 = \frac{10\times10}{10+10} = 5\ \Omega$$

f) The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = [\{(10 \parallel 10) + 10\} \parallel 10] + 10$$

or, $$R_{eq} = \left[\left\{\frac{10\times10}{10+10}+10\right\}\parallel 10\right] + 10 = \frac{15\times10}{15+10} + 10 = 6 + 10 = 16\ \Omega$$

Problem 3.135: *Determine the equivalent resistance across the points A and B in each of the network shown in the Fig. 3.61.*

Solution: *a*) All the four branches having resistors 2 Ω, (2 + 2) Ω, (2 + 2) Ω and 2 Ω are connected across the points *A* and *B*. So, all four branches are connected in parallel. The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = 2 \parallel 4 \parallel 4 \parallel 2 = \frac{1}{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{2}\right)} = \frac{4}{(2+1+1+2)} = \frac{2}{3}\ \Omega$$

b) All the three resistors (R) are connected across the points A and B. So, three resistors are connected in parallel, and the equivalent resistance across the points A and B may be given as:

$$R_{eq} = R \parallel R \parallel R$$

$$= \frac{1}{\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)}$$

$$= \frac{R}{(1+1+1)} = \frac{R}{3}\ \Omega$$

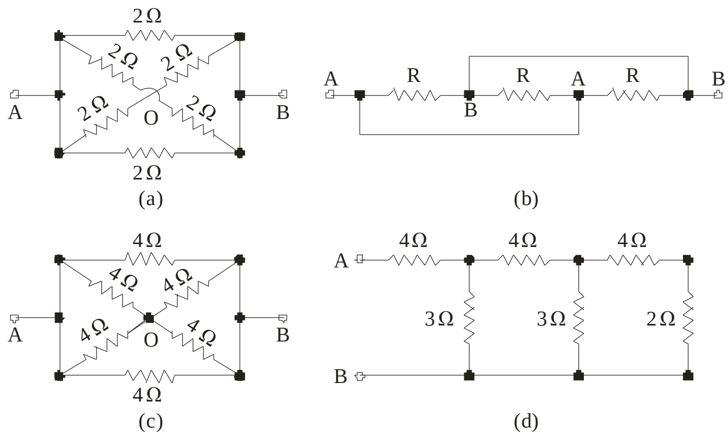

Fig. 3.61

c) The equivalent resistance across the points A and B may be given as:

$$R_{eq} = 4 \parallel [(4 \parallel 4) + (4 \parallel 4)] \parallel 4 = 4 \parallel \left[\frac{4\times 4}{4+4}+\frac{4\times 4}{4+4}\right] \parallel 4 = 4 \parallel [2+2] \parallel 4$$

$$= \frac{1}{\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)} = \frac{4}{(1+1+1)} = \frac{4}{3}\ \Omega$$

d) The equivalent resistance across the points A and B may be given as:

$$R_{eq} = ([\{(4+2) \parallel 3\} + 4] \parallel 3) + 4 = \left(\left[\frac{6\times 3}{6+3}+4\right] \parallel 3\right) + 4$$

$$= ([2+4] \parallel 3) + 4 = \left(\frac{6\times 3}{6+3}\right) + 4 = 2 + 4 = 6\ \Omega$$

Problem 3.136: *Determine the equivalent resistance across the points A and B in each of the network shown in the Fig. 3.62.*

Solution: *a*) Three branches having resistors (R) are connected across the points B and C. So, three resistances are connected in parallel and then one resistor is connected in series with the parallel combination. The equivalent resistance across the points A and B may be given as:

$$R_{eq} = (R \parallel R \parallel R) + R$$

or, $$R_{eq} = \frac{1}{\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)} + R$$

$$= \frac{R}{(1+1+1)} + R$$

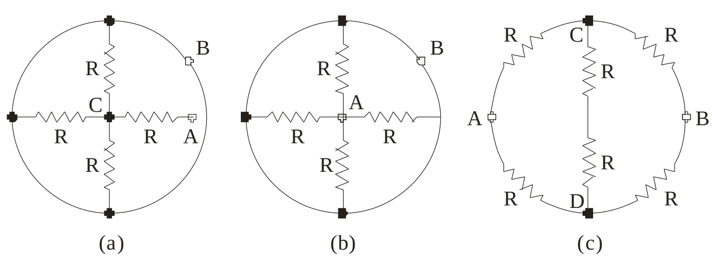

Fig. 3.62

or, $R_{eq} = \frac{R}{3} + R = \frac{4}{3} R$

b) All the four resistances are connected across the points *A* and *B*. So, all the four resistances are connected in parallel. The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = \frac{1}{\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)}$$

$$= \frac{R}{(1+1+1+1)} = \frac{R}{4}$$

c) This is a *balanced Wheatstone Bridge*, as the potential at the points *C* and *D* are equal. So, the central branch across the points *C* and *D* may be open circuited without any effect on the network. The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = (R + R) \parallel (R + R) = \frac{2R \times 2R}{2R + 2R} = R$$

Problem 3.137: *Determine the potential difference across the points A and B in the network shown in the Fig. 3.63.*

Solution: The equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = 2.5 + (6 \parallel 6 \parallel 3) = 2.5 + \frac{1}{\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{3}\right)}$$

$$= 2.5 + \frac{6}{(1+1+2)} = 2.5 + 1.5 = 4\ \Omega$$

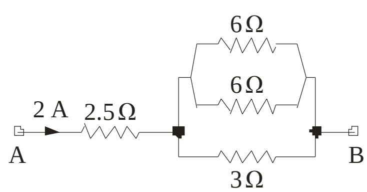

Fig. 3.63

So, the potential difference across the points *A* and *B* may be given as:

$$V = I \times R_{eq} = 2 \times 4 = 8\ \text{V}$$

Problem 3.138: *A voltmeter reads 30 V, when connected across the terminals of the resistor 400 Ω in the network shown in the Fig. 3.64. Determine the reading of voltmeter, when it is connected across the terminals of the resistor 300 Ω.* [IIT 1990]

Solution: Let the internal resistance of the voltmeter is *R* Ω.

Since the potential drop across the combination of 400 Ω and voltmeter in parallel is 30 V (half of the supply source, *i.e.* 60 V), the equivalent resistance of the parallel combination of 400 Ω and resistance *R* must also be equal to 300 Ω (the other resistance in the circuit in series).

$$\frac{400 \times R}{400 + R} = 300$$

or, $4R = 3 \times (400 + R)$

or, $R = 1200\ \Omega$

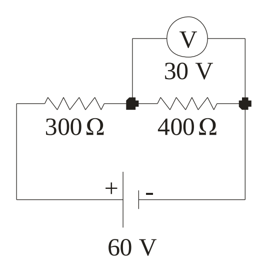

Fig. 3.64

When the voltmeter is connected across the resistor of 300 Ω, the equivalent resistance of the parallel combination may be given as:

$$R_{eq} = \frac{300 \times 1200}{300 + 1200} = 240\ \Omega$$

So, the reading of the voltmeter may be given as:

$$V_{300\,\Omega} = \frac{240}{240+400} \times 60 = 22.5\text{ V}$$

Problem 3.139: *Determine the equivalent resistance between the points A and B in the Fig. 3.65 (a).*

Solution: The given network may be redrawn as shown in the Fig. 3.67 (b). Now, the equivalent resistance across the points A and B may be given as:

$$R_{eq} = [(3 \parallel 6) + 8] \parallel 30$$

$$= \left[\frac{3\times 6}{3+6} + 8\right] \parallel 30$$

$$= [2+8] \parallel 30 = \frac{10\times 30}{10+30} = 7.5\ \Omega$$

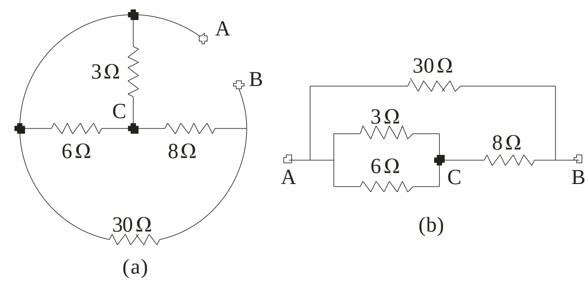

Fig. 3.65

Problem 3.140: *Determine the potential difference between the points A and B of the network shown in the Fig. 3.66.* [Punjab 1992-93]

Solution: Since two parallel branches in the network are having identical resistances (3 + 2 = 5 Ω). The current 2 A will be divided equally among the two parallel branches (1 A each).

The potential at the point A is equal to the potential drop across the points AD.

So, $V_A = 1 \times 3 = 3$ V

Similarly, the potential at the point B is equal to the potential drop across the points BD.

So, $V_B = 1 \times 2 = 2$ V

Now, $V_{AB} = V_A - V_B = 3 - 2 = 1$ V

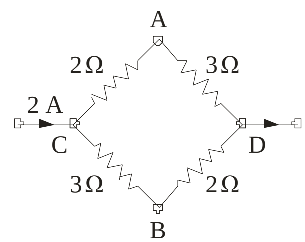

Fig. 3.66

Problem 3.141: *Determine the voltage drop across the resistance R_1 in the circuit given in the Fig. 3.67 with $E = 90$ V, $R_1 = R_2 = 5$ kΩ and $R_3 = R_4 = 10$ kΩ.* [CBSE 1993-94]

Solution: $E = 90$ V, $R_1 = R_2 = 5$ kΩ, $R_3 = R_4 = 10$ kΩ

The equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = [(R_3 + R_4) \parallel R_2] + R_1 = [(10+10) \parallel 5] + 5$$

$$= \frac{20\times 5}{20+5} + 5 = 4 + 5 = 9\text{ k}\Omega$$

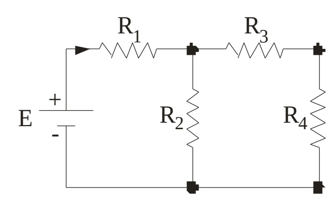

Fig. 3.67

The current supplied by the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{90}{9\times 10^3} = 10\text{ mA}$$

So, the potential drop across the resistance R_1 may be given as:

$$V_{R1} = I \times R_1 = 10 \times 10^{-3} \times 5 \times 10^3 = 50\text{ V}$$

Problem 3.142: *Determine the equivalent resistance between points: i) A and B, ii) A and C of the network shown in the Fig. 3.68 (a).*

Solution: *i*) The given network may be redrawn as shown in the Fig. 3.68 (b), for determining the equivalent resistance across the points *A* and *B*. Now, the equivalent resistance across the points *A* and *B* may be given as:

$$R_{AB} = 10 + [(10 + 10 + 10) \parallel 10] + 10$$

$$= 10 + \frac{30 \times 10}{30 + 10} + 10$$

$$= 10 + 7.5 + 10 = 27.5\ \Omega$$

i) The given network may be redrawn as shown in the Fig. 3.68 (c), for determining the equivalent resistance across the points *A* and *C*. Now, the equivalent resistance across the points *A* and *C* may be given as:

$$R_{AC} = 10 + [(10 + 10) \parallel (10 + 10)] + 10$$

$$= 10 + \frac{20 \times 20}{20 + 20} + 10$$

$$= 10 + 10 + 10 = 30\ \Omega$$

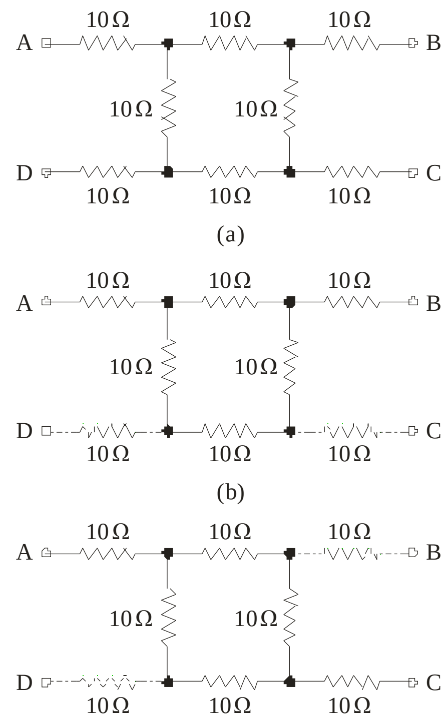

Fig. 3.68

Problem 3.143: *A combination of four resistances is shown in the network drawn in Fig. 3.69. Calculate the potential difference between the points A and B, and the values of the currents flowing in different resistances.*

Solution: The current passing through the series combination of (4 + 10 + 4) Ω may be given as:

$$I_1 = \frac{9}{9 + (4 + 10 + 4)} \times 2.4 = 0.8\ \text{A}$$

The current passing through the 9 Ω resistor may be given as:

$$I_2 = \frac{(4 + 10 + 4)}{9 + (4 + 10 + 4)} \times 2.4 = 1.6\ \text{A}$$

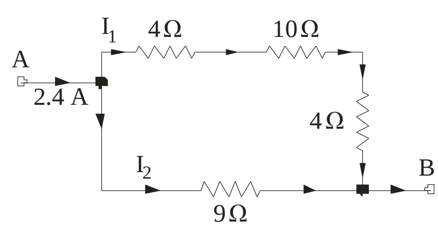

Fig. 3.69

The potential difference across the points *A* and *B* may be given as:

$$V_{AB} = I_1 \times (4 + 10 + 4) = I_2 \times 9$$

$$= 0.8 \times (4 + 10 + 4) = 1.6 \times 9$$

$$= 14.4\ \text{V}$$

Problem 3.144: *Three ammeters; X, Y and Z, are shown in the network drawn in the Fig. 3.70. The ammeter Y reads a current of 0.5 A. i) Determine the readings of the ammeters X and Z. ii) Determine the equivalent resistance of the network.*

Solution: $I_Y = 0.5$ A

The current flowing through the ammeter *Y* may be given by the relationship:

$$I_Y = \frac{3}{6 + 3} \times I_X = 0.5$$

So, $I_X = \dfrac{6+3}{3} \times 0.5 = 1.5$ A

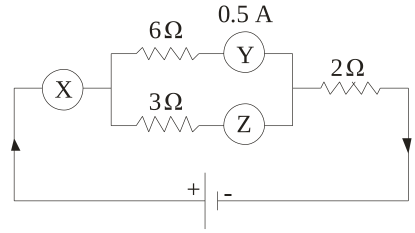

Fig. 3.70

Now, the current flowing through the ammeter Z may be given as:

$$I_Z = \frac{6}{6+3} \times 1.5 = 1 \text{ A}$$

And, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (3 \parallel 6) + 2 = \frac{6\times 3}{6+3} + 2 = 2 + 2 = 4\ \Omega$$

Problem 3.145: *The battery, in the network shown in the Fig. 3.71, has an emf of 12 V and an internal resistance of $\frac{5R}{11}$. If the ammeter shown in the network reads 2 A, determine the value of R.*

Solution: All the three resistors (R, $2R$ and $3R$) are connected across the points A and B, so they are connected in parallel. The equivalent resistance of the network (along with the internal resistance of the battery) as seen by the d.c. source (battery) may be given as:

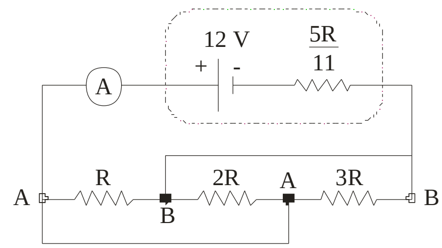

Fig. 3.71

$$R_{eq} = (R \parallel 2R \parallel 3R) + \frac{5R}{11}$$

$$= \frac{1}{\left(\frac{1}{R}+\frac{1}{2R}+\frac{1}{3R}\right)} + \frac{5R}{11}$$

$$= \frac{6R}{(6+3+2)} + \frac{5R}{11} = \frac{6R}{11} + \frac{5R}{11} = \frac{11R}{11} = R$$

The current flowing through the ammeter may be given as:

$$I = \frac{V}{R_{eq}} = \frac{12}{R} = 2 \text{ A}$$

So, $R = \dfrac{12}{2} = 6\ \Omega$

Problem 3.146: *Determine the value of current flowing through the 18 Ω resistor in the network shown in the Fig. 3.72.*

Solution: The equivalent resistance of the network as seen by the d.c. source (battery) may be given as:

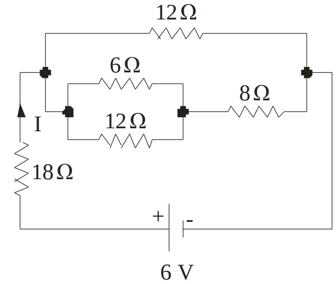

Fig. 3.72

$$R_{eq} = [\{(6 \parallel 12) + 8\} \parallel 12] + 18$$

or, $$R_{eq} = \left[\left\{\frac{6\times 12}{6+12} + 8\right\} \parallel 12\right] + 18$$

$$= [\{4 + 8\} \parallel 12] + 18 = \frac{12\times 12}{12+12} + 18 = 24\ \Omega$$

So, the current flowing through the 18 Ω resistor may be given as:

$$I = \frac{V}{R_{eq}} = \frac{6}{24} = 0.25 \text{ A}$$

Problem 3.147: *Determine the equivalent resistance of the network and the reading of the ammeter shown in the network drawn in the Fig. 3.73 (a).*

Solution: The network may be redrawn as shown in the Fig. 3.73 (b). Now, the equivalent resistance of the network as seen by the d.c. source (battery) may be given as:

$$R_{eq} = [(5 + 7) \parallel 6 \parallel 8] + 1$$

$$= \frac{1}{\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{8}\right)} + 1$$

$$= \frac{24}{(2+4+3)} + 1$$

$$= \frac{24}{9} + 1 = \frac{33}{9} = \frac{11}{3}\ \Omega$$

The current flowing through the ammeter may be given as:

$$I = \frac{V}{R_{eq}} = \frac{11}{(11/3)} = 3 \text{ A}$$

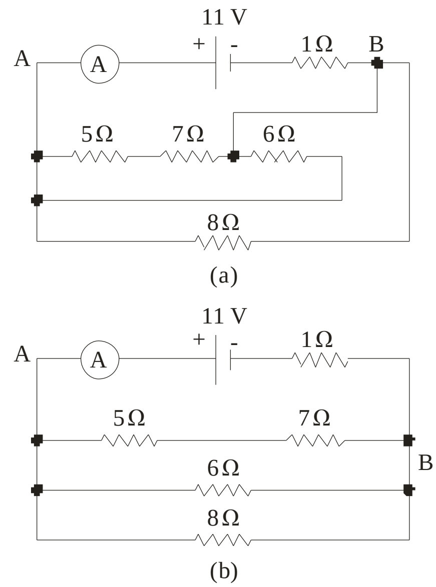

Fig. 3.73

3.18 Internal Resistance of a Cell (Battery): When a d.c. supply source (*battery; consisting of several cells in series*) is connected across a circuit / network (having several resistances to do some useful work). A *current* flows from the *positive terminal* of the battery towards the *negative terminal* of the battery through the circuit. However, the *current* flows from *negative terminal* to the *positive terminal*, inside the battery to maintain the continuous flow of *current* in the circuit. The *current* inside the battery flows due to movement of positive ions towards positive plates of the battery and movement of electrons / negative ions towards negative plates of the battery. The movement of the electrons / ions through the electrolyte is opposed by the electrolyte, due to this a resistance is offered to the flow of *current* inside the battery which is known as the *internal resistance of the battery*, shown by the resistance (*r*) in the circuit given in the Fig. 3.74.

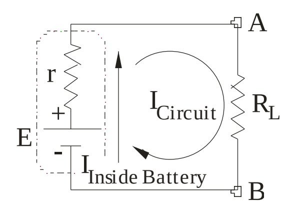

Fig. 3.74

So, "***The resistance offered by the electrolyte of a cell to flow of current inside the battery between its electrodes is known as the internal resistance of the battery***".

The *internal resistance* of the battery depends on the following factors:

i) The *internal resistance* of the battery depends on the *nature of the electrolyte*.

ii) The *internal resistance* of the battery is directly proportional to the *concentration of the electrolyte*.

iii) The *internal resistance* of the battery is directly proportional to the *distance between two electrodes of the battery*.

iv) The *internal resistance* of the battery is inversely proportional to the *area of the electrodes immersed in the electrolyte*.

v) The *internal resistance* of the battery increases with the decrease in *temperature of the electrolyte of the battery*.

vi) The *internal resistance* of the battery increases with the decrease in *specific gravity of the electrolyte* (*ageing of the battery, as it gets older*).

3.19 Relationship between Internal Resistance, EMF and Terminal Voltage (Potential Difference across the terminals) of the Battery: An open circuited d.c. voltage source (battery) has its *terminal voltage* (V_t) equal to its *internal emf* (E), as shown in the Fig. 3.75 (a). This is because of the fact that there is no current flowing through the battery and hence no current in the *internal resistance* of the battery. So, the voltage drop across the *internal resistance* is zero and hence the *voltage* available across the terminals of the battery may be given as:

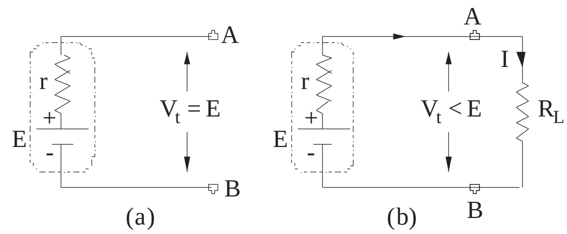

Fig. 3.75

$$V_t = E \qquad (3.89)$$

On the other hand, when a battery supplies a *current* to a circuit (load), the *current* flowing through the battery and hence through the internal resistance is not zero, shown in the Fig. 3.75 (b). There is a *voltage drop* in the *internal resistance* (*r*) of the battery, and the *voltage* available across the terminals of the battery is less than its *internal emf* and may be given as:

$$V_t = E - I \times r \qquad (3.90)$$

The *current* flowing through the circuit may be given as:

$$I = \frac{E}{r + R_L} \qquad (3.91)$$

So, the *terminal voltage* may be given as:

$$V_t = E - \frac{E}{r + R_L} \times r = E \times \left(1 - \frac{r}{r + R_L}\right) = \frac{R_L}{r + R_L} \times E \qquad (3.92.1)$$

$$= E \times \left[1 - \frac{1}{1 + (R_L / r)}\right] \qquad (3.92.2)$$

So, smaller is the *internal resistance* of the battery, greater is the *terminal voltage* of the battery on load and hence *better is the quality of the battery*.

Points to be noted carefully:

i) A d.c. voltage source (battery) has an *open circuit* (*No Load*) *voltage* equal to its *internal emf*.

$$V_t\ (No\ Load) = E$$

ii) A d.c. voltage source (battery) on load has a *terminal voltage* smaller than its *internal emf*. Greater is the *terminal voltage* available on load, *better is the quality of the battery*.

$$V_t\ (On\ Load) < E$$

Problem 3.148: *If a current of 3 A is drawn from a battery for 5 minutes, 900 J of work is done by the current, determine the emf of the battery.*

Solution: $I = 3$ A, $t = 5$ minutes, $W = 900$ J

The charge supplied by the battery may be given as:

$$q = I \times t = 3 \times 5 \times 60 = 900 \text{ C}$$

The emf / potential of the battery may be defined as the work done in flowing per unit charge across the circuit, so the emf / potential of the battery may be given as:

$$E = \frac{W}{q} = \frac{900}{900} = 1 \text{ V}$$

Also, the emf of an ideal source giving zero internal resistance may be given as:

$$E = V = I\,R = I \times \frac{W}{I^2\,t} = 3 \times \frac{900}{(3)^2 \times 5 \times 60} = 1 \text{ V}$$

Problem 3.149: *A voltmeter having resistance 998 Ω is connected across a cell of emf 2 V (internal resistance 2 Ω). Determine the potential difference across the voltmeter, potential difference across the cell and the percentage error in the reading of the voltmeter.*

Solution: $R_V = 998\ \Omega$, $E = 2$ V, $r = 2\ \Omega$

The current flowing through the circuit may be given as:

$$I = \frac{E}{r + R_V} = \frac{2}{2 + 998} = 0.002 \text{ A} = 2 \text{ mA}$$

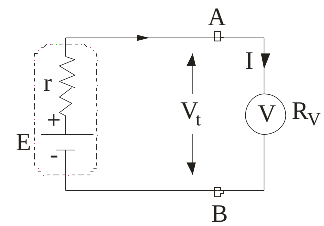

Fig. 3.76

The potential difference across the voltmeter and across the cell will be same (the reader may observe from the Fig. 3.76) and may be given as:

$$V_V = I \times R_V = 0.002 \times 998 = 1.996 \text{ V}$$

and, $V_t = E - I \times r = 2 - 0.002 \times 2 = 1.996$ V

The percentage error in the reading may be given as:

$$\% \text{ error} = \frac{2 - 1.996}{2} \times 100\% = 0.2\%$$

Problem 3.150: *The voltmeter reads 1.5 V, when the switch 'S' in circuit shown in the Fig. 3.77 is open. When the switch 'S' is closed the voltmeter reads 1.35 V and the ammeter reads 1.5 A. Determine the emf and internal resistance of the cell.*

Solution: $V_{S\ Open} = 1.5$ V, $V_{S\ Closed} = 1.35$ V, $I_{S\ Closed} = 1.5$ A

The emf of the battery is open circuit voltage of the battery when the current through the circuit is zero. So, emf of the cell may be given as:

$$E = V_{S\ Open} = 1.5 \text{ V}$$

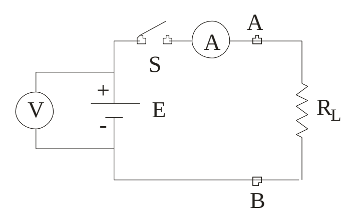

Fig. 3.77

The internal resistance of the cell may be given by the relationship:

$$V_t = E - I \times r$$

or, $$r = \frac{E - V_t}{I} = \frac{1.5 - 1.35}{1.5} = 0.1\ \Omega$$

Problem 3.151: *A cell of emf 2 V and internal resistance 0.1 Ω is connected to a 3.9 Ω external resistance. Determine the potential difference across the terminals of the cell.* [CBSE 2000-01]

Solution: $E = 2$ V, $r = 0.1\ \Omega$, $R_L = 3.9\ \Omega$

The potential difference across the cell terminals / external resistance may be given as:

$$V_t = I \times R_L = \frac{R_L}{r + R_L} \times E = \frac{3.9}{0.1 + 3.9} \times 2 = 1.95 \text{ V}$$

Problem 3.152: *The reading on a high resistance voltmeter is 2.2 V, when it is connected across a cell. The reading of the voltmeter drops to 1.8 V, when a resistance of 5 Ω is connected across the terminals of the cell. Determine the internal resistance of the cell.*

Solution: $E = 2.2$ V, $V_t = 1.8$ V, $R_L = 5\ \Omega$

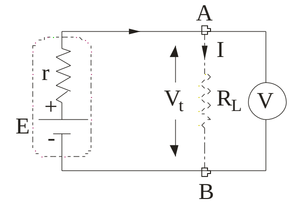

Fig. 3.78

The potential difference across the cell terminals / external resistance may be given as:

$$V_t = I \times R_L = \frac{R_L}{r + R_L} \times E = 1.8 \text{ V}$$

So, $$r = \frac{R_L}{1.8} \times E - R_L = \frac{5}{1.8} \times 2.2 - 5 = 1.111\ \Omega$$

Problem 3.153: *A dry cell of emf 1.6 V and internal resistance 0.1 Ω is connected to a resistor of R Ω. The current supplied by the cell is 2 A. Determine the voltage drop across the resistor R.* [CBSE 1992-93]

Solution: $E = 1.6$ V, $r = 0.1\ \Omega$, $I = 2$ A

The potential drop across the cell terminals / external resistance (R) may be given as:

$$V_t = E - I \times r = 1.6 - 2 \times 0.1 = 1.4 \text{ V}$$

Problem 3.154: *A battery of emf 'E' and internal resistance 'r' supplies a current of 0.5 A to an external resistor of 12 Ω; and supplies a current of 0.25 A to an external resistance of 25 Ω. Determine: i) internal resistance of the battery, ii) emf of the battery.* [CBSE 2001-02]

Solution: $I_1 = 0.5$ A, $R_{L1} = 12\ \Omega$, $I_2 = 0.25$ A, $R_{L2} = 25\ \Omega$

External Resistance (R_{L1}) of 12 Ω:

The current supplied by the battery may be given as:

$$I_1 = \frac{E}{r + R_{L1}} = \frac{E}{r + 12} = 0.5$$

or, $$\frac{E}{0.5} = r + 12$$

or, $$2E - r = 12 \tag{3.93}$$

External Resistance (R_{L2}) of 25 Ω:

The current supplied by the battery may be given as:

$$I_2 = \frac{E}{r + R_{L2}} = \frac{E}{r + 25} = 0.25$$

or, $$\frac{E}{0.25} = r + 25$$

or, $$4E - r = 25 \tag{3.94}$$

Equation (3.94) – (3.93):

$$2E = 13 \qquad \text{or,} \qquad E = \frac{13}{2} = 6.5 \text{ V}$$

Putting the value of E in equation (3.93):

$$2E - r = 12 \qquad \text{or,} \qquad r = 2E - 12 = 2 \times 6.5 - 12 = 1\ \Omega$$

Problem 3.155: *A battery of emf 3 V and internal resistance 'r' is connected in series with a resistor of 55 Ω through an ammeter of resistance 1 Ω. The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of r.* [CBSE 1994-95, Haryana 2001-02]

Solution: $E = 3$ V, $R_L = 55\ \Omega$, $R_A = 1\ \Omega$, $I = 50$ mA

The current supplied by the battery may be given as:

$$I = \frac{E}{r + R_L + R_A} = \frac{3}{r + 55 + 1} = 0.05$$

So, $r = \dfrac{3}{0.05} - (55 + 1) = 4\ \Omega$

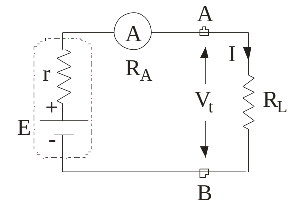

Fig. 3.79

Problem 3.156: *a*) *A car has a fresh storage battery of emf 12 V and internal resistance of* $5 \times 10^{-2}\ \Omega$. *If the motor of self, for starting the car, draws a current of 90 A, determine the terminal voltage of the battery when the self is ON.*

b) *After long use, the internal resistance of the storage battery increases to 500 Ω. Determine the value of maximum current that can be drawn from the battery.*

c) *If the discharged battery is charged by an external emf source, determine whether terminal voltage of the battery during charging is greater or less than its emf 12 V?*

[NCERT]

Solution: $E = 12$ V, $r = 5 \times 10^{-2}\ \Omega$, $I = 90$ A, $r' = 500\ \Omega$

a) The terminal voltage of the battery, while the motor of the self is ON, may be given as:

$$V_t = E - I \times r = 12 - 90 \times 5 \times 10^{-2} = 7.5 \text{ V}$$

b) The value of maximum current (I_{max}), which can be drawn from the battery may be given by the expression:

$$V_t = E - I_{max} \times r' = 0$$

or, $I_{max} = \dfrac{E}{r'} = \dfrac{12}{500} = 0.024 \text{ A} = 24 \text{ mA}$

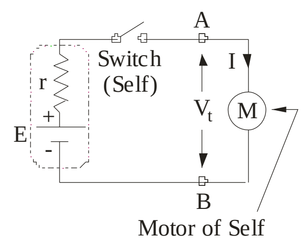

Fig. 3.80

The reader may observe and conclude that the storage battery, with this much internal resistance, is useless for starting purpose, and hence the storage battery must be replaced with the new storage battery with smaller internal resistance.

c) The external source forces the current inside the storage battery in opposite to that would flow due to storage battery, *i.e.* inside the storage battery from positive plate to negative plate of the battery, for its charging purpose.

So, now the value of current in the circuit may be written as:

$$I = \frac{V - E}{r} = \frac{V_t - E}{r}$$

or, $V_t = E + I \times r$ (So, terminal voltage must be greater than the emf of the battery.)

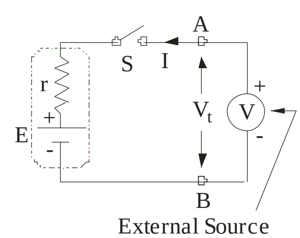

Fig. 3.81

Problem 3.157: *A battery of emf 12 V and internal resistance 0.5 Ω is to be charged by a battery charger which supplies 110 V d.c. Determine the value of resistance to be connected in series with the battery to limit the charging current to 5 A. Also determine the value of potential difference across the terminals of the battery during the charging.*

Solution: $E = 12$ V, $r = 0.5\ \Omega$, $V = 110$ V, $I = 5$ A

Let the series resistance required is R_{ext} to limit the charging current to 5 A.

So, $$I = \frac{V - E}{r + R_{ext}} = 5 \text{ A}$$

or, $$R_{ext} = \frac{V - E}{5} - r = \frac{110 - 12}{5} - 0.5 = 19.1\ \Omega$$

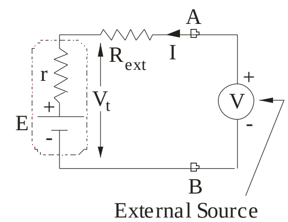

Fig. 3.82

The potential difference across the terminals of battery during charging may be given as:

$$V_t = V - I \times R_{ext} = 110 - 5 \times 19.1 = 14.5 \text{ V}$$

Problem 3.158: *A cell of emf 1.5 V and internal resistance 0.5 Ω is connected to a non-linear conductor whose V-I graph is shown in the Fig. 3.83 (a). Obtain graphically the current drawn from the cell and its terminal voltage.*

Solution: $E = 1.5$ V, $r = 0.5\ \Omega$

The terminal voltage of the cell may be given as:

$$V_t = E - I \times r$$

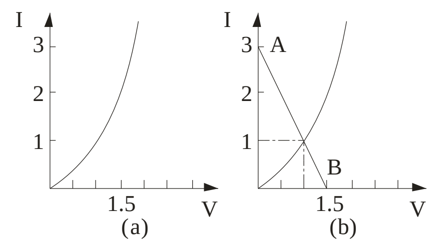

Fig. 3.83

The terminal voltage for various currents may be obtained as:

$I = 0$ A, $V_t = 1.5 - 0 \times 0.5 = 1.5$ V

$I = 1$ A, $V_t = 1.5 - 1 \times 0.5 = 1.0$ V

$I = 2$ A, $V_t = 1.5 - 2 \times 0.5 = 0.5$ V

$I = 3$ A, $V_t = 1.5 - 3 \times 0.5 = 0$ V

The $V_t - I$ graph for the cell is a straight line *AB* as drawn in the Fig. 3.83 (b). This straight line *AB* intersects the *V-I* graph of the non-linear conductor at:

Current = 1 A and, Voltage = 1 V

So, The current drawn from the cell = 1 A

and, The terminal voltage of the cell = 1 V

Problem 3.159: *The potential difference across the terminals of a cell was measured (in Volts) at different values of currents (in Amps) flowing through the cell. A graph was drawn which was a straight line ABC, as shown in the Fig. 3.84. Determine from the graph: i) emf of the cell, ii) the maximum current that can be obtained from the cell, iii) the internal resistance of the cell.*

Solution: The emf of the cell is the terminal voltage of the cell at zero current flowing through the cell. The emf may be obtained graphically with the help of the extrapolated graph and the slope of the line, to know the intercept of the line *ABC* at *y*-axis. The slope of the line may be given as:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0.4 - 0.8}{0.2 - 0.12} = -5 = \frac{0.4 - y_1}{0.2 - 0}$$

So, $y_1 = 0.4 + 0.2 \times 5 = 1.4$ V

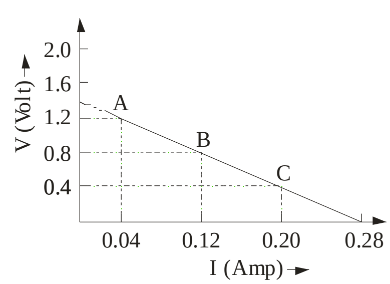

Fig. 3.84

The maximum current that can be obtained by the cell is the current when the terminal voltage of the cell becomes zero, *i.e.* the emf of the cell drops completely in the internal resistance of the cell. So, maximum current may be given as:

$I_{max} = 0.28$ A

The internal resistance of the cell may be given as:

$$r = \frac{E}{I_{SC}} = \frac{1.4}{0.28} = 5\ \Omega$$

Problem 3.160: *Determine the current drawn from a cell of emf 1 V and internal resistance of 2/3 Ω connected in the network shown in the Fig. 3.85.* [CBSE 2000-01]

Solution: $E = 1$ V, $r = \frac{2}{3}\ \Omega$

The equivalent resistance of the network across the points A and B as seen by the battery may be given as:

$$R_{eq} = 1 \parallel [(1 \parallel 1) + (1 \parallel 1)] \parallel 1$$

$$= 1 \parallel \left[\frac{1\times1}{1+1} + \frac{1\times1}{1+1}\right] \parallel 1 = 1 \parallel \left[\frac{1}{2} + \frac{1}{2}\right] \parallel 1$$

$$= \frac{1}{\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1}\right)} = \frac{1}{3}\ \Omega$$

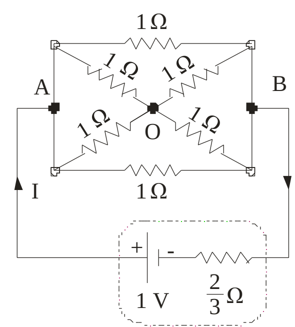

Fig. 3.85

So, the current drawn from the cell may be given as:

$$I = \frac{E}{r + R_{eq}} = \frac{1}{\left(\frac{2}{3} + \frac{1}{3}\right)} = 1 \text{ A}$$

Problem 3.161: *A uniform wire of resistance 12 Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A battery of emf 8 V and internal resistance 1 Ω is connected across the highest of the three resistors. Determine the current through each part of the circuit.* [Karnataka 1993-94]

Solution: $R_{Total} = 12\ \Omega$, Length Ratio $= l_1 : l_2 : l_3 = 1 : 2 : 3$, $E = 8$ V, $r = 1\ \Omega$

The resistances of three parts of the wire may respectively be given as:

$$R_1 = \frac{1}{1+2+3} \times 12 = 2\ \Omega$$

and, $$R_2 = \frac{2}{1+2+3} \times 12 = 4\ \Omega$$

and, $$R_3 = \frac{3}{1+2+3} \times 12 = 6\ \Omega$$

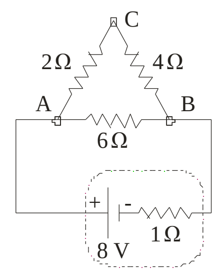

Fig. 3.86

The network formed by these wires is drawn in the Fig. 3.86. The equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (2 + 4) \parallel 6 = \frac{6\times6}{6+6} = 3\ \Omega$$

So, the current supplied by the battery may be given as:

$$I = \frac{E}{r + R_{eq}} = \frac{8}{1+3} = 2 \text{ A}$$

The current through the branch (2 + 4) Ω may be given as:

$$I_1 = \frac{6}{(2+4)+6} \times 2 = 1\text{ A}$$

The current through the branch 6 Ω may be given as:

$$I_2 = \frac{(2+4)}{(2+4)+6} \times 2 = 1\text{ A}$$

Problem 3.162: *The emf of a cell is 1.5 V. The terminal voltage of the cell falls to 1.4 V, when a resistor of 14 Ω is connected across the terminals of the cell. Determine the internal resistance of the cell.* [Haryana 2000-01]

Solution: $E = 1.5$ V, $V_t = 1.4$ V, $R_L = 14\ \Omega$

The current through the load resistance may be given as:

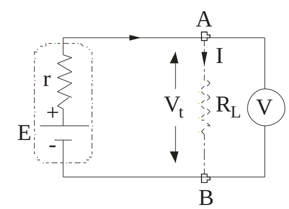

Fig. 3.87

$$I = \frac{V_t}{R_L} = \frac{1.4}{14} = 0.1\text{ A}$$

The terminal voltage of the cell may be given by the relationship:

$$V_t = E - I \times r$$

So, $$r = \frac{E - V_t}{I} = \frac{1.5 - 1.4}{0.1} = 1\ \Omega$$

Problem 3.163: *The potential difference across a cell is 1.8 V, when a current of 0.5 A is drawn from the cell. Again, the potential drop falls to 1.6 V, when a current of 1 A is drawn from the cell. Determine the emf and internal resistance of the cell.*

Solution: $V_{t1} = 1.8$ V, $I_1 = 0.5$ A, $V_{t2} = 1.6$ V, $I_2 = 1$ A

The terminal voltage of a cell may be given as:

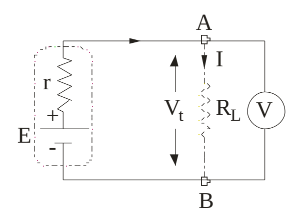

Fig. 3.88

$$V_t = E - I \times r$$

So, $V_{t1} = E - I_1 \times r$

or, $E - 0.5\,r = 1.8$ (3.95)

and, $V_{t2} = E - I_2 \times r$

or, $E - 1\,r = 1.6$ (3.96)

Equation (3.95) – (3.96):

$$0.5\,r = 0.2$$

or, $$r = \frac{0.2}{0.5} = 0.4\ \Omega$$

Putting this value in equation (3.96):

$$E = 1.6 + r = 1.6 + 0.4 = 2\text{ V}$$

Problem 3.164: *The potential difference across the terminals of a battery on open circuit is 6 V, which falls to 4 V when a current of 2 A is drawn from the battery. Determine the emf and internal resistance of the battery.*

Solution: $V_{OC} = 6$ V, $V_t = 4$ V, $I = 2$ A

The reader may easily conclude that the potential difference across the terminals of the battery on open circuit is known as the emf of the battery. So, emf of the battery may be given as:

$$E = V_{OC} = 6 \text{ V}$$

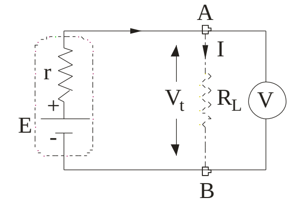

Fig. 3.89

Since, the terminal voltage of the battery on load may be given as:

$$V_t = E - I \times r$$

or, $$r = \frac{E - V_t}{I} = \frac{6-4}{2} = 1 \ \Omega$$

Problem 3.165: *The resistance of the ammeter is negligible and that of the voltmeter is infinitely high, in the circuit shown in the Fig. 3.90. The reading of voltmeter is 1.53 V, when the switch S is open. The reading of voltmeter falls to 1.03 V, when the reading of the ammeter is 1 A on closing the switch S. Determine: i) emf of the cell, ii) value of R_L, iii) internal resistance of the cell.*

Solution: $V_{OC} = 1.53$ V, $V_t = 1.03$ V, $I = 1$ A

The reader may easily conclude that the potential difference across the terminals of the battery on open circuit is known as the emf of the battery. So, emf of the battery may be given as:

$$E = V_{OC} = 1.53 \text{ V}$$

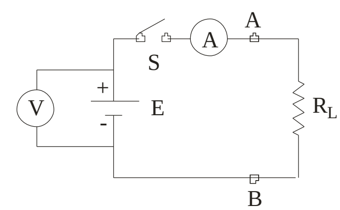

Fig. 3.90

The value of load resistance may be given as:

$$R_L = \frac{V_t}{I} = \frac{1.03}{1} = 1.03 \ \Omega$$

Since, the terminal voltage of the cell may be given as:

$$V_t = E - I \times r$$

or, $$r = \frac{E - V_t}{I} = \frac{1.53 - 1.03}{1} = 0.5 \ \Omega$$

Problem 3.166: *The potential difference across the terminals of a battery of emf 6 V and internal resistance 1 Ω drops to 5.8 V, when connected across an external resistor. Determine the value of external resistor.*

Solution: $E = 6$ V, $r = 1 \ \Omega$, $V_t = 5.8$ V

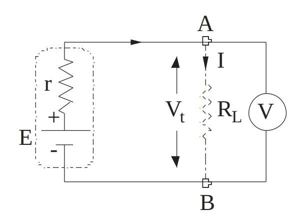

Fig. 3.91

Since, the terminal voltage of the battery may be given as:

$$V_t = E - I \times r$$

or, $$I = \frac{E - V_t}{r} = \frac{6 - 5.8}{1} = 0.2 \text{ A}$$

Now, $$R_L = \frac{V_t}{I} = \frac{5.8}{0.2} = 29 \ \Omega$$

Problem 3.167: *The potential difference across the terminals of a 6 V battery is 7.2 V, when it is being charged by a current of 2 A. Determine the internal resistance of the battery.*

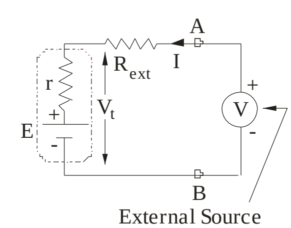

Fig. 3.92

Solution: $E = 6$ V, $V_t = 7.2$ V, $I = 2$ A

Since, the terminal voltage of the battery during its charging is greater than the emf of the cell, so it may be given as:

$$V_t = E + I \times r$$

or, $r = \dfrac{V_t - E}{I} = \dfrac{7.2 - 6}{2} = 0.6\ \Omega$

Problem 3.168: *A battery of emf 2 V and internal resistance 0.5 Ω is connected across a resistance of 9.5 Ω. How many electrons pass per second, through a cross section of the resistance?*

Solution: $E = 2$ V, $r = 0.5\ \Omega$, $R_L = 9.5\ \Omega$

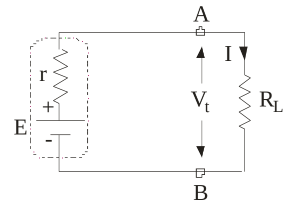

Fig. 3.93

The current through the circuit may be given as:

$$I = \frac{E}{r + R_L} = \frac{2}{0.5 + 9.5} = 0.2\ \text{A}$$

The charge passing through the resistor per second may be given as:

$$q = n\,e = I \times t = 0.2 \times 1 = 0.2\ \text{C}$$

So, $n = \dfrac{0.2}{e} = \dfrac{0.2}{1.6 \times 10^{-19}} = 1.25 \times 10^{18}$ electrons

Problem 3.169: *If a resistor of 2 Ω is connected across the terminals of a battery, the resultant current in the circuit is 0.5 A. When a resistance of 5 Ω is connected across the same battery, the resultant current in the circuit is 0.25 A. Determine the emf and internal resistance of the battery.*

Solution: $R_{L1} = 2\ \Omega$, $I_1 = 0.5$ A, $R_{L2} = 5\ \Omega$, $I_2 = 0.25\ \Omega$

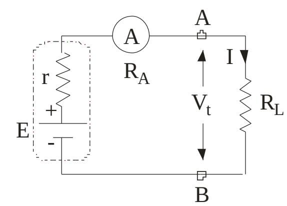

Fig. 3.94

R_{L1} = 2 Ω is connected across the terminals of the battery:

The expression for the circuit current may be given as:

$$I = \frac{E}{r + R_L} = \frac{E}{r + 2} = 0.5 \qquad (3.97)$$

R_{L2} = 5 Ω is connected across the terminals of the battery:

The expression for the circuit current may be given as:

$$I = \frac{E}{r + R_L} = \frac{E}{r + 5} = 0.25 \qquad (3.98)$$

Equation (3.97) / (3.98):

$$\frac{[E/(r+2)]}{[E/(r+5)]} = \frac{0.5}{0.25}$$

or, $\dfrac{r+5}{r+2} = 2$ or, r + 5 = 2 r + 4

or, $\text{r} = \dfrac{(5-4)}{(2-1)} = 1\ \Omega$

Putting this value in equation (3.97):

$$E = 0.5 \times (r + 2) = 0.5 \times (1 + 2) = 1.5\ \text{V}$$

Problem 3.170: *The emf of a battery is 4 V and its internal resistance is 1.5 Ω. Its potential difference is measured by a voltmeter of 1000 Ω. Determine the percentage error in the reading of the emf shown by the voltmeter.*

Solution: $E = 4$ V, $r = 1.5\ \Omega$, $R_V = 1000\ \Omega$

The potential difference across the terminals of the battery at the time of measurement may be given as:

$$V_t = I \times R_V = \frac{E}{r+R} \times R_V = \frac{4}{1.5+1000} \times 1000 = 3.994 \text{ V}$$

So, the percentage error in the reading of the voltmeter may be given as:

$$\% \text{ error} = \frac{E - V_t}{E} \times 100\% = \frac{4-3.994}{4} \times 100\% = 0.15\%$$

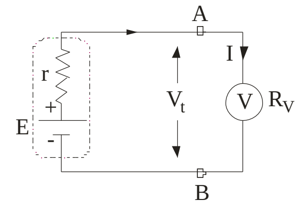

Fig. 3.95

Problem 3.171: *The emf of a battery is 6 V and its internal resistance is 0.6 Ω. A wire of resistance 2.4 Ω is connected to the two terminals of the battery. Determine: i) current in the circuit, ii) the potential difference between two terminals of the battery in closed circuit.*

Solution: $E = 6$ V, $r = 0.6\ \Omega$, $R_L = 2.4\ \Omega$

The current through the circuit may be given as:

$$I = \frac{E}{r+R} = \frac{6}{0.6+2.4} = 2 \text{ A}$$

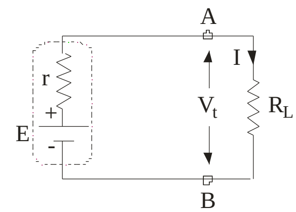

Fig. 3.96

The potential difference between two terminals of the battery may be given as:

$$V_t = E - I \times r = 6 - 2 \times 0.6 = 4.8 \text{ V}$$

Problem 3.172: *The potential difference across the terminals of a battery is 8.5 V, when a current of 3 A flows through it from its negative terminal to positive terminal. If a current of 2 A flows through it in the opposite direction, the potential difference between its two terminals is 11 V. Determine the emf and internal resistance of the battery.*

Solution: $V_{t1} = 8.5$ V, $I_1 = 3$ A

This is discharging of the battery, shown in the Fig. 3.97 (a), as current flows from negative to positive terminal through the battery.

$V_{t2} = 11$ V, $I_2 = 2$ A

This is charging of the battery, shown in the Fig. 3.97 (b), as current flows from positive to negative terminal through the battery.

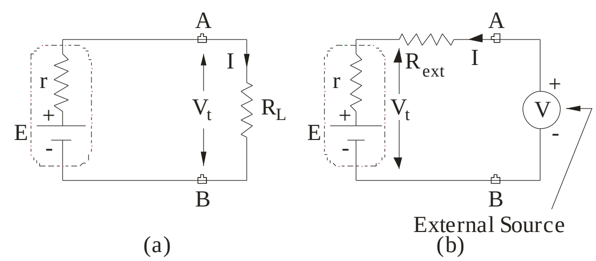

Fig. 3.97

The discharging current flowing through the battery:

The terminal voltage of the battery may be given as:

$$V_t = E - I \times r = E - 3 \times r = 8.5 \qquad (3.99)$$

The charging current flowing through the battery:

The terminal voltage of the battery may be given as:

$$V_t = E + I \times r = E + 2 \times r = 11 \qquad (3.100)$$

Equation (3.100) – (3.99):

$$5\,r = 2.5 \qquad \text{or,} \qquad r = \frac{2.5}{5} = 0.5\ \Omega$$

Putting this value in equation (3.99):

$$E = 8.5 + 3\,r = 8.5 + 3 \times 0.5 = 10\text{ V}$$

Problem 3.173: *The two terminals of a cell of emf 1.5 V are connected by a coil of resistance 10 Ω. If the current flowing through the coil is 0.1 A, determine the internal resistance of the cell.*

Solution: $E = 1.5$ V, $R_L = 10\ \Omega$, $I = 0.1$ A

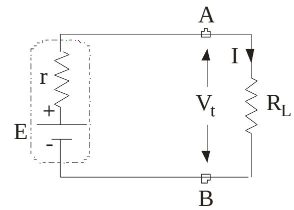

Fig. 3.98

Since, the current flowing through the circuit may be given as:

$$I = \frac{E}{r + R_L}$$

So, $r = \dfrac{E}{I} - R_L = \dfrac{1.5}{0.1} - 10 = 5\ \Omega$

Problem 3.174: *A potential difference of 3 V is required across the points A and B in the arrangement shown in the Fig. 3.99. If the value of R_2 is 8 Ω, determine the required value of R_1 for this purpose.*

Solution: $V_{AB} = 3$ V, $R_2 = 8\ \Omega$

The potential difference across the points *A* and *B* may be given as:

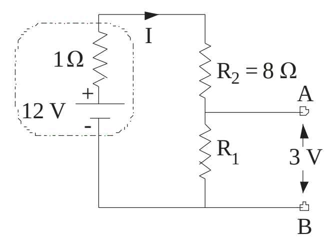

Fig. 3.99

$$V_{AB} = I \times R_1 = \frac{R_1}{r + R_2 + R_1} \times E = \frac{R_1}{1 + 8 + R_1} \times 12 = 3$$

or, $12\,R_1 = 3 \times (9 + R_1) = 27 + 3\,R_1$

So, $R_1 = \dfrac{27}{(12 - 3)} = 3\ \Omega$

3.20 Combination of Cells in Series and in Parallel: A single cell provides a very small *current* due to its smaller *emf*. Usually, in practice we need higher currents for specific purposes in our daily life. The higher values of *current* may be obtained by the combination of various *cells* in *series* or in *parallel* or a combination of *series and parallel* connected *cells*. A few of the examples of combination of cells to obtain higher currents for useful purposes are: torches for light, radio sets, automobiles *etc*.

Cells in Series: *When the cells are connected in a way that the positive terminal of one cell is connected to the negative terminal of next cell and so on, the cells are said to be connected in series.*

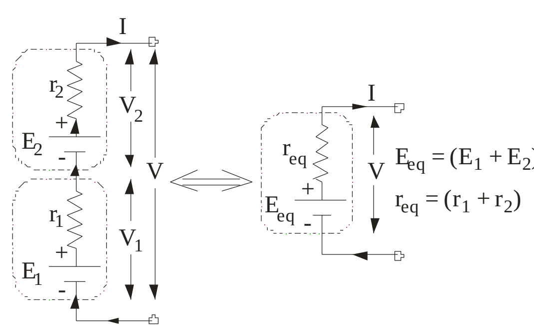

Fig. 3.100

The series combination of two cells and their equivalent battery is shown in the Fig. 3.100. Let E_1 and E_2 be the emf's of two cells and their internal resistances be r_1 and r_2 respectively. If a current I is flowing from the series combination of two cells, the potential differences (terminal voltages) across the individual cells may be given as:

$$V_1 = E_1 - I \times r_1$$

and, $V_2 = E_2 - I \times r_2$

So, the *potential difference* across the combination of the cells may be given as:

$$V_t = V_1 + V_2 = E_1 - I \times r_1 + E_2 - I \times r_2$$

$$= (E_1 + E_2) - I \times (r_1 + r_2) = E_{eq} - I \times r_{eq} \qquad (3.101)$$

The reader may conclude that:

$$E_{eq} = (E_1 + E_2) \tag{3.102}$$

and, $$r_{eq} = (r_1 + r_2) \tag{3.103}$$

The resultant emf (E_{eq}) of the resultant battery, containing two cells in series, will be (E_1 + E_2) and its equivalent internal resistance (r_{eq}) will be (r_1 + r_2).

In general for N cells to be connected in series, we may write that:

$$E_{eq} = (E_1 + E_2 + E_3 + \ldots\ldots.. + E_N) \tag{3.104}$$

and, $$r_{eq} = (r_1 + r_2 + r_3 + \ldots\ldots + r_N) \tag{3.105}$$

If any cell is connected in *series opposition* to the combination, *i.e.* positive terminal of one cell to the positive terminal of next cell, the value of emf is to be put with negative sign in the equation (3.104), as shown in the Fig. 3.101. However, the equivalent internal resistance (r_{eq}) will be (r_1 + r_2), *i.e.* same as earlier.

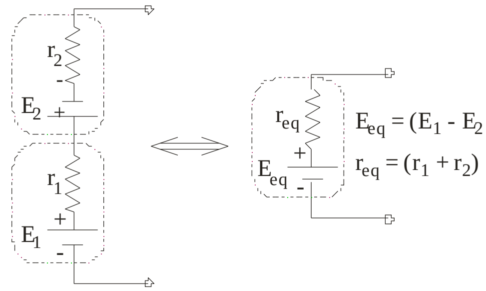

Fig. 3.101

Cells in Parallel: *When positive terminals of all the cells are connected together at a point* (A) *and negative terminals of all the cells are connected together at another point* (B), *the cells are said to be connected in parallel.*

The parallel combination of two cells and their equivalent battery is shown in the Fig. 3.102. Let E_1 and E_2 be the emf's of two cells and their internal resistances be r_1 and r_2 respectively. If the currents I_1 and I_2 are flowing from the two cells towards the junction A, the total current supplied by the parallel combination of the cells may be given as:

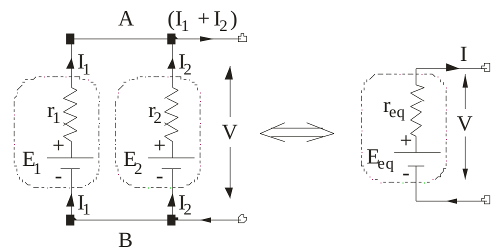

Fig. 3.102

$$I = I_1 + I_2 \tag{3.106}$$

As the two cells are connected across the same common points A and B, the potential difference across two cells will be same (V). The potential difference across two cells may be given as:

$$V = E_1 - I_1 \times r_1 = E_2 - I_2 \times r_2 \tag{3.107}$$

So, $$I_1 = \frac{E_1 - V}{r_1} \tag{3.108}$$

and, $$I_2 = \frac{E_2 - V}{r_2} \tag{3.109}$$

So, $$I = I_1 + I_2 = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right) - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

or, $$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right) - I$$

or, $$V = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) - I \times \left(\frac{r_1 r_2}{r_1 + r_2}\right) = E_{eq} - I \times r_{eq} \tag{3.111}$$

The reader may conclude that:

$$E_{eq} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) \tag{3.112}$$

and, $$r_{eq} = \left(\frac{r_1 r_2}{r_1 + r_2}\right) \tag{3.113}$$

Above results may conveniently be expressed in more simpler way as:

$$\frac{E_{eq}}{r_{eq}} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) \tag{3.114}$$

and, $$\frac{1}{r_{eq}} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{3.115}$$

In general for N cells to be connected in parallel, we may write that:

$$\frac{E_{eq}}{r_{eq}} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ... + \frac{E_N}{r_N}\right) \tag{3.116}$$

and, $$\frac{1}{r_{eq}} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_N}\right) \tag{3.117}$$

3.21 Choice between Series and Parallel Combination of the Cells:

Condition for Obtaining Maximum Current from a Series Combination of Cells: Let us assume that N similar cells of equal emf 'e' and internal resistance 'r' are connected in *series*. Let R_L be the external resistance connected across the series combination of cells, as shown in the Fig. 3.103.

The equivalent emf of N series connected cells may be given as:

$$E_{eq} = N \times e$$

The equivalent internal resistance of N series connected cells may be given as:

$$r_{eq} = r + r + r + \text{..........} N \text{ terms} = N \times r$$

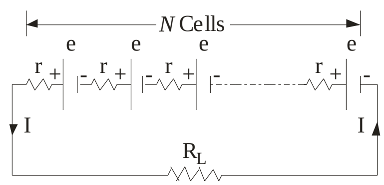

Fig. 3.103

So, total resistance in the circuit,

$$R_{Circuit} = R_L + r_{eq} = R_L + N \times r$$

The current flowing through the circuit may be given as:

$$I = \frac{E_{eq}}{R_{Circuit}} = \frac{N \times e}{R_L + N \times r} \tag{3.118}$$

If, $R_L \gg (N \times r)$ (*i.e.* external resistance in the circuit is much greater than the equivalent resistance of N series connected cells): The term $(N \times r)$ may be neglected in the denominator.

So, $$I = \frac{N \times e}{R_L} = N \times \text{current supplied by the single cell to external resistor} \left(\frac{e}{R_L}\right) \tag{3.119}$$

If, $R_L \ll (N \times r)$ (*i.e.* external resistance in the circuit is much smaller than the equivalent resistance of N series connected cells): The term R_L may be neglected in the denominator.

So, $$I = \frac{N \times e}{N \times r} = \frac{e}{r} \quad \text{(current supplied by the single cell on short circuiting a cell)} \tag{3.120}$$

Thus, *when the external resistance connected in the circuit is much greater than the equivalent internal resistance of the cells connected in the series, the cells must be connected in series to obtain the maximum current from the cells.*

Condition for Obtaining Maximum Current from a Parallel Combination of Cells: Let us assume that M similar cells of equal emf 'e' and internal resistance 'r' are connected in parallel. Let R_L be the external resistance connected across the parallel combination of cells, as shown in the Fig. 3.104.

The equivalent emf of N parallel connected cells may be given as:

$$E_{eq} = e$$

The equivalent internal resistance of N series connected cells may be given by the relationship:

$$\frac{1}{r_{eq}} = \left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}+......m \text{ terms}\right) = \frac{M}{r}$$

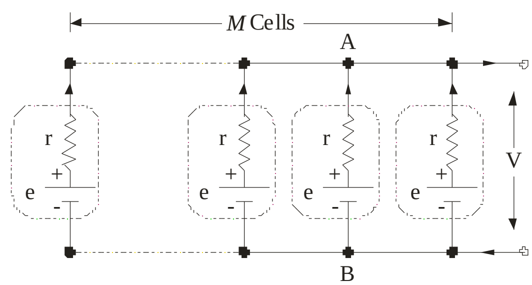

Fig. 3.104

or, $$r_{eq} = \frac{r}{M}$$

So, total resistance in the circuit,

$$R_{Circuit} = R_L + r_{eq} = R_L + \frac{r}{M}$$

The current flowing through the circuit may be given as:

$$I = \frac{E_{eq}}{R_{Circuit}} = \frac{e}{R_L + \frac{r}{M}} = \frac{M \times e}{M \times R_L + r} \qquad (3.121)$$

If, $R_L \ll \frac{r}{M}$ (*i.e.* external resistance in the circuit is much smaller than the equivalent resistance of N parallel connected cells): The term R_L may be neglected in the denominator.

So, $$I = \frac{M \times e}{r} = M \times \text{current supplied by the single cell on short circuit} \left(\frac{e}{r}\right) \qquad (3.122)$$

If, $R_L \gg \frac{r}{M}$ (*i.e.* external resistance in the circuit is much greater than the equivalent resistance of N parallel connected cells): The term $\frac{r}{M}$ may be neglected in the denominator.

So, $$I = \frac{e}{R_L} \qquad (3.123)$$

(the current supplied by the single cell to the external resistor)

Thus, *when the external resistance connected in the circuit is much smaller than the equivalent internal resistance of the cells connected in parallel, the cells must be connected in parallel to obtain the maximum current from the cells.*

Condition for Obtaining Maximum Current from Mixed (Series-Parallel) Combination of Cells: Let us assume that N similar cells of equal emf 'e' and internal resistance 'r' are connected in series in each row and M such rows are connected in parallel. Let R_L be the external resistance connected across the parallel combination of rows, as shown in the Fig. 3.105.

Total number of cells in the combination = $N \times M$

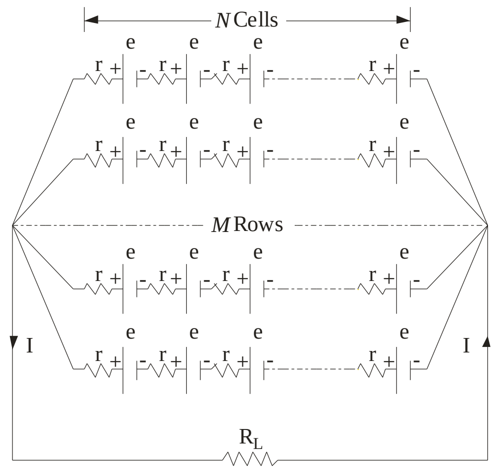

Fig. 3.105

Net emf across the external resistance, $E = N \times e$

Internal resistance of each row, $r_{eq\,(row)} = N \times r$

Net internal resistance of M rows in the combination may be given by the relationship:

$$\frac{1}{r_{eq}} = \left(\frac{1}{N\,r} + \frac{1}{N\,r} + \frac{1}{N\,r} + ...M \text{ terms}\right) = \frac{M}{N \times r}$$

So, $\quad r_{eq} = \dfrac{N \times r}{M}$

Now, the total resistance in the circuit may be given as:

$$R_{Circuit} = R_L + r_{eq} = R_L + \frac{N \times r}{M}$$

The current flowing through the circuit may be given as:

$$I = \frac{E_{eq}}{R_{Circuit}} = \frac{N \times e}{R_L + \dfrac{N \times r}{M}} = \frac{M\,N \times e}{M \times R_L + N \times r} = \frac{e}{\left(\dfrac{R_L}{N} + \dfrac{r}{M}\right)} \qquad (3.124)$$

Since, the cell emf '*e*' is constant the value of current drawn will be maximum, if denominator of above equation is minimum.

So,

$$\left(\sqrt{\frac{R_L}{N}}\right)^2 + \left(\sqrt{\frac{r}{M}}\right)^2 = \left(\sqrt{\frac{R_L}{N}}\right)^2 + \left(\sqrt{\frac{r}{M}}\right)^2 - 2\times\left(\sqrt{\frac{R_L}{N}}\right)\times\left(\sqrt{\frac{r}{M}}\right) + 2\times\left(\sqrt{\frac{R_L}{N}}\right)\times\left(\sqrt{\frac{r}{M}}\right)$$

$$= \left(\sqrt{\frac{R_L}{N}} - \sqrt{\frac{r}{M}}\right)^2 + 2\times\left(\sqrt{\frac{R_L}{N}}\right)\times\left(\sqrt{\frac{r}{M}}\right) \qquad (3.125)$$

As, square of any number cannot be negative, so above equation will have its minimum value when the square term will be zero.

So, $\quad \sqrt{\dfrac{R_L}{N}} - \sqrt{\dfrac{r}{M}} = 0$

or, $\quad \dfrac{R_L}{N} = \dfrac{r}{M}$

or, $\quad R_L = \dfrac{N \times r}{M} = r_{eq}$ (net internal resistance of the cells)

Thus, *when the external resistance connected in the circuit is equal to the net equivalent internal resistance of the cells, the current supplied by the mixed* (*series / parallel*) *combination of cells will be maximum.*

Problem 3.175: *a*) *Three cells of emf 2 V, 1.8 V and 1.5 V are connected in series. Their internal resistances are 0.05 Ω, 0.7 Ω and 1 Ω respectively. If an external resistor of 4 Ω is connected across the terminals of the battery so formed, through an extremely low resistance ammeter, determine the reading of the ammeter.*

b) *If three cells above are connected in parallel, would they be characterized by a definite emf and internal resistance (independent of external circuit)? If not, how can we obtain the currents in various branches?* [NCERT]

Solution: $E_1 = 2$ V, $r_1 = 0.05\ \Omega$, $E_2 = 1.8$ V, $r_2 = 0.7\ \Omega$, $E_3 = 1.5$ V, $r_3 = 1\ \Omega$

$R_L = 4\ \Omega$

The current flowing through the external resistance and ammeter may be given as:

$$I = \frac{E_{eq}}{R_L + r_{eq}} = \frac{(E_1 + E_2 + E_3)}{R_L + (r_1 + r_2 + r_3)} = \frac{(2 + 1.8 + 1.5)}{4 + (0.05 + 0.7 + 1)} = 0.9217 \text{ A}$$

No, there is no formula to characterize such a combination as a definite emf and internal resistance (independent of external resistance), by the knowledge gathered by us till now. The electrical branch of engineering may have some methods for it. We can use *Kirchhoff's laws* to evaluate the currents in various branches.

Problem 3.176: *A cell of emf 1.1 V and internal resistance 0.5 Ω is connected to a wire of resistance 0.5 Ω. If a second cell of the same emf is now connected in the series with earlier cell to feed the external resistance but the current in the circuit remains same, determine the internal resistance of second cell.* [CBSE 1995-96]

Solution: $E_1 = E_2 = 1.1$ V, $r_1 = 0.5\ \Omega$, $R_L = 0.5\ \Omega$

The current supplied by the single cell to external resistance may be given as:

$$I_1 = \frac{E_1}{R_L + r_1} = \frac{1.1}{0.5 + 0.5} = 1.1 \text{ A}$$

The current supplied by the two cells connected in series to external resistance may be given as:

$$I_2 = \frac{E_1 + E_2}{R_L + (r_1 + r_2)} = \frac{1.1 + 1.1}{0.5 + (0.5 + r_2)} = 1.1$$

or, $$1 + r_2 = \frac{2.2}{1.1} = 2$$

So, $r_2 = 2 - 1 = 1\ \Omega$

Problem 3.177: *Two identical cells of emf 1.5 V, connected in parallel, supplies two external resistances of 17 Ω each connected in parallel. A very high resistance voltmeter is used to measure the terminal voltage of the parallel combination of cells which reads 1.4 V. Determine the internal resistance of each cell.* [CBSE 1994-95]

Solution: $E_1 = E_2 = 1.5$ V, $M = 2$, $R_L = (17 \parallel 17)\ \Omega$, $V_t = 1.4$ V

The equivalent external resistance, $R_L = \dfrac{17 \times 17}{17 + 17} = \dfrac{17}{2}\ \Omega$

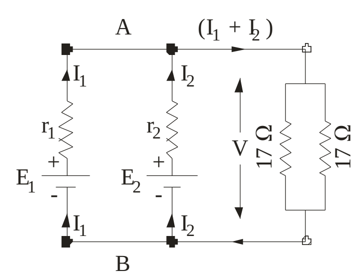

Fig. 3.106

The current through the external resistance may be given as:

$$I = \frac{V_t}{R_L} = \frac{1.4}{(17 \backslash 2)} = \frac{2.8}{17} = \frac{E}{R_L + (r/M)}$$

or, $$\frac{1.5}{(17/2) + (r/2)} = \frac{2.8}{17}$$

or, $1.5 \times \frac{17}{2.8} = \frac{1}{2}(17 + r)$

So, $r = \left(1.5 \times 2 \times \frac{17}{2.8} - 17\right) = 17 \times \left(\frac{3}{2.8} - 1\right) = 17 \times \left(\frac{3 - 2.8}{2.8}\right) = 1.214\ \Omega$

Problem 3.178: *Four identical cells of emf 2 V connected in parallel supplies two external resistances of 15 Ω each connected in parallel. A very high resistance voltmeter is used to measure the terminal voltage of the parallel combination of cells which reads 1.6 V. Determine the internal resistance of each cell.* [CBSE 2001-02]

Solution: $E_1 = E_2 = E_3 = E_4 = 2$ V, $M = 4$, $R_L = (15 \parallel 15)\ \Omega$, $V_t = 1.6$ V

The equivalent external resistance, $R_L = \frac{15 \times 15}{15 + 15} = \frac{15}{2}\ \Omega$

The current through the external resistance may be given as:

$$I = \frac{V_t}{R_L} = \frac{1.6}{(15\backslash 2)} = \frac{3.2}{15} = \frac{E}{R_L + (r/M)}$$

or, $\frac{2}{(15/2) + (r/4)} = \frac{3.2}{15}$

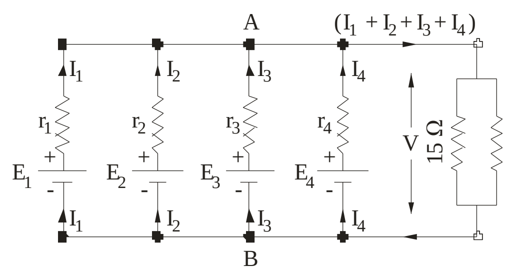

Fig. 3.107

or, $2 \times \frac{15}{3.2} = \frac{15}{2} + \frac{r}{4}$

So, $r = 4 \times \left(\frac{15}{1.6} - \frac{15}{2}\right)$

$= 15 \times \left(\frac{4}{1.6} - 2\right) = 7.5\ \Omega$

Problem 3.179: *Two cells E_1 and E_2 of emf's 4 V and 8 V having internal resistances of 0.5 Ω and 1 Ω respectively are connected in phase opposition to each other. This combination is connected in series with resistances of 4.5 Ω and 3 Ω. Another resistance of 6 Ω is connected in parallel across the 3 Ω resistor.*

a) *Draw the circuit diagram.*

b) *Determine the total current flowing through the circuit and through each resistor.*

[CBSE 2001-02]

Solution: $E_1 = 4$ V, $E_2 = 8$ V, $r_1 = 0.5\ \Omega$,

$r_2 = 1\ \Omega$, $R_L = [4.5 + (3 \parallel 6)]\ \Omega$

The circuit diagram is drawn in the Fig. 3.108.

The equivalent external resistance may be given as:

$$R_{eq} = 4.5 + (6 \parallel 3) = 4.5 + \frac{6 \times 3}{6 + 3}$$

$$= 4.5 + 2 = 6.5\ \Omega$$

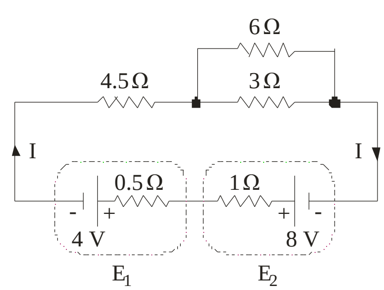

Fig. 3.108

The current flowing through the circuit may be given:

$$I = \frac{(E_1 + E_2)}{R_L + (r_1 + r_2)} = \frac{(8-4)}{6.5 + (0.5+1)} = 0.5 \text{ A}$$

(Clockwise, as shown in the figure)

The current through the two parallel branches may respectively be given as:

$$I_{6\,\Omega} = \frac{3}{3+6} \times 0.5 = 0.167 \text{ A} \qquad \text{and,} \qquad I_{3\,\Omega} = \frac{6}{3+6} \times 0.5 = 0.333 \text{ A}$$

Problem 3.180: *Two cells E_1 and E_2 of emf's 2 V and 4 V having internal resistances of 1 Ω and 2 Ω respectively are connected in phase opposition to each other, as shown in the Fig. 3.109. The value of external resistance (R_L) is 5 Ω. Determine the current flowing through the circuit, the potential difference across the points AB and the potential difference across the points AC.*

Solution: $E_1 = 2$ V, $E_2 = 4$ V, $r_1 = 1\ \Omega$, $r_2 = 2\ \Omega$, $R_L = 5\ \Omega$

The current flowing through the circuit may be given as:

$$I = \frac{(E_1 + E_2)}{R_L + (r_1 + r_2)} = \frac{(4-2)}{5 + (1+2)} = 0.25 \text{ A}$$

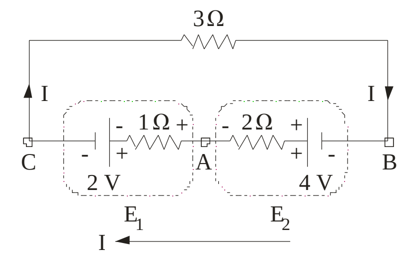

Fig. 3.109

(Clockwise, as shown in the figure)

The potential difference across the points *A* and *B* may be given as:

$$V_{AB} = E_2 - I \times r_2 = 4 - 0.25 \times 2 = 3.5 \text{ V}$$

The potential difference across the points *A* and *C* may be given as:

$$V_{AC} = E_1 + I \times r_1 = 2 - 0.25 \times 1 = 2.25 \text{ V}$$

Problem 3.181: *A network of resistors is connected to a battery of emf 16 V with an internal resistance of 1 Ω, as shown in the Fig. 3.110.*

a) *Determine the equivalent resistance of the network.*

b) *Determine the current through each resistor.*

c) *Determine the voltage drops V_{AB}, V_{BC}, V_{CD}.* [NCERT]

Solution: $E = 16$ V, $r = 1\ \Omega$

The equivalent resistance of the network may be given as:

$$R_{eq} = (4 \parallel 4) + 1 + (12 \parallel 6)$$

$$= \frac{4\times4}{4+4} + 1 + \frac{12\times6}{12+6} = 2 + 1 + 4 = 7\ \Omega$$

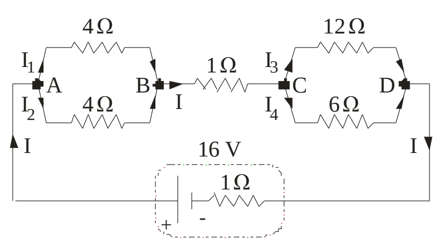

Fig. 3.110

The current flowing through the circuit (I) and the central resistance of 1 Ω may be given as:

$$I = \frac{E}{R_{eq} + r} = \frac{16}{7+1} = 2 \text{ A}$$

The current through the two parallel branches (4 ‖ 4) Ω may respectively be given as:

$$I_1 = \frac{4}{4+4} \times 2 = 1 \text{ A} \qquad \text{and,} \qquad I_2 = \frac{4}{4+4} \times 2 = 1 \text{ A}$$

The current through the two parallel branches (12 ‖ 6) Ω may respectively be given as:

$$I_3 = \frac{6}{12+6} \times 2 = 0.667 \text{ A} \qquad \text{and,} \qquad I_4 = \frac{12}{12+6} \times 2 = 1.333 \text{ A}$$

The potential drops V_{AB}, V_{BC}, V_{CD} may respectively be given as:

$$V_{AB} = I_1 \times 4 = I_2 \times 4 = 1 \times 4 = 4 \text{ V}$$

and, $V_{BC} = I \times 1 = 2 \times 1 = 2$ V

and, $V_{CD} = I_3 \times 12 = 0.667 \times 12 = 8$ V

$= I_4 \times 6 = 1.333 \times 6 = 8$ V

Problem 3.182: *A 20 V battery of internal resistance 1 Ω is connected across three coils of 12 Ω, 6 Ω and 4 Ω connected in parallel, a resistor of 5 Ω and a reversed battery (emf = 8 V and internal resistance = 2 Ω), as shown in the Fig. 3.111. Determine the current in each resistor and the terminal potential difference across each battery.*

Solution: $E_1 = 20$ V, $r_1 = 1$ Ω, $E_2 = 8$ V, $r_2 = 2$ Ω, $R_L = 5 + (12 \parallel 6 \parallel 4)$ Ω

The equivalent resistance of the network may be given as:

$$R_{eq} = 5 + (12 \parallel 6 \parallel 4) = 5 + \frac{1}{\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right)}$$

$$= 5 + \frac{12}{(1+2+3)} = 5 + 2 = 7 \ \Omega$$

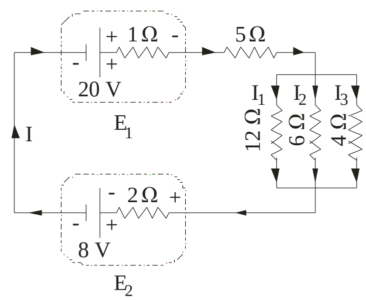

Fig. 3.111

The current flowing through the circuit and through the 5 Ω resistor may be given as:

$$I = \frac{E_1 + E_2}{R_{eq} + (r_1 + r_2)} = \frac{20-8}{7+(1+2)} = 1.2 \text{ A}$$

The current flowing through the three parallel connected resistors may respectively be given as:

$$I_1 = \frac{\frac{1}{12}}{\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right)} \times 1.2 = \frac{1}{(1+2+3)} \times 1.2 = 0.2 \text{ A}$$

and, $$I_2 = \frac{\frac{1}{6}}{\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right)} \times 1.2 = \frac{2}{(1+2+3)} \times 1.2 = 0.4 \text{ A}$$

and, $$I_3 = \frac{\frac{1}{4}}{\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right)} \times 1.2 = \frac{3}{(1+2+3)} \times 1.2 = 0.6 \text{ A}$$

The terminal voltage across the battery of 20 V may be given as:

$$V_{t\,(20\,V)} = E_1 - I \times r_1 = 20 - 1.2 \times 1 = 18.8\text{ V}$$

The terminal voltage across the battery of 8 V may be given as:

$$V_{t\,(8\,V)} = E_2 + I \times r_2 = 8 + 1.2 \times 2 = 10.4\text{ V}$$

Problem 3.183: *36 cells each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external resistor of 2 Ω. Determine the best mode of grouping them and the current through the external circuit.*

Solution: n = 36 cells, r = 0.5 Ω, e = 1.5 V, R_L = 2 Ω

Let N cells are connected in series and M such rows are connected in parallel.

So, $$N \times M = n = 36 \tag{3.126}$$

The optimum condition for obtaining the maximum current for the combination may be given as:

$$R_L = \frac{N \times r}{M} = \frac{0.5N}{M} = 2 \tag{3.127}$$

Equation (3.126) × (3.127):

$$0.5 \times N^2 = 72$$

So, $$N = \sqrt{\frac{72}{0.5}} = 12$$

and, $$M = \frac{36}{N} = \frac{36}{12} = 3$$

So, 12 cells are to be connected in series and 3 such rows are to be connected in parallel to obtain the maximum current from the combination. The maximum current so obtained may be given as:

$$I = \frac{e}{\left(\frac{R_L}{N} + \frac{r}{M}\right)} = \frac{1.5}{\left(\frac{2}{12} + \frac{0.5}{3}\right)} = \frac{1.5 \times 12}{(2+2)} = 4.5\text{ A}$$

Problem 3.184: *12 cells each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external resistor of 1.5 Ω. Determine the best mode of grouping them and the current through the external circuit.* [CBSE 2007-08]

Solution: n = 12 cells, r = 0.5 Ω, e = 1.5 V, R_L = 1.5 Ω

Let N cells are connected in series and M such rows are connected in parallel.

So, $$N \times M = n = 12 \tag{3.128}$$

The optimum condition for obtaining the maximum current for the combination may be given as:

$$R_L = \frac{N \times r}{M} = \frac{0.5N}{M} = 1.5 \tag{3.129}$$

Equation (3.128) × (3.129):

$$0.5 \times N^2 = 18$$

So, $N = \sqrt{\dfrac{18}{0.5}} = 6$ and, $M = \dfrac{12}{N} = \dfrac{12}{6} = 2$

So, 6 cells are to be connected in series and 2 such rows are to be connected in parallel to obtain the maximum current from the combination. The maximum current so obtained may be given as:

$$I = \frac{e}{\left(\dfrac{R_L}{N} + \dfrac{r}{M}\right)} = \frac{1.5}{\left(\dfrac{1.5}{6} + \dfrac{0.5}{2}\right)} = \frac{1.5 \times 6}{(1.5 + 1.5)} = 3 \text{ A}$$

Problem 3.185: *Two cells E_1 and E_2 having emfs of 5 V and 9 V and internal resistances of 0.3 Ω and 1.2 Ω respectively are connected in series, as shown in the circuit in Fig. 3.112. Determine the current flowing through the resistor of value 3 Ω.* [CBSE 2005-06]

Solution: $E_1 = 5$ V, $r_1 = 0.3$ Ω, $E_2 = 9$ V, $r_2 = 1.2$ Ω

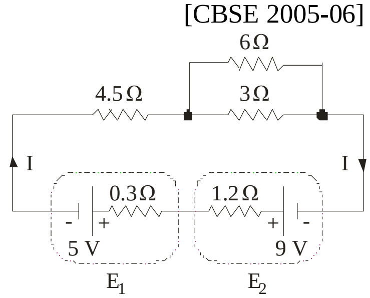

Fig. 3.112

The equivalent external resistance may be given as:

$$R_{eq} = 4.5 + (6 \parallel 3) = 4.5 + \frac{6 \times 3}{6 + 3}$$

$$= 4.5 + 2 = 6.5 \text{ Ω}$$

The current flowing through the circuit may be given:

$$I = \frac{(E_1 + E_2)}{R_L + (r_1 + r_2)} = \frac{(9 - 5)}{6.5 + (0.3 + 1.2)} = 0.5 \text{ A (Clockwise, as shown in the figure)}$$

The current through the 3 Ω resistor may be given as:

$$I_{3\,\Omega} = \frac{6}{3 + 6} \times 0.5 = 0.333 \text{ A}$$

Problem 3.186: *Three identical cells, each of emf 2 V and internal resistance 0.2 Ω are connected in series across an external resistance of 7.4 Ω. Determine the current flowing through the circuit.* [CBSE 1992-93]

Solution: $E_1 = E_2 = E_3 = 2$ V, $r_1 = r_2 = r_3 = 0.2$ Ω, $R_L = 7.4$ Ω

The current flowing through the circuit may be given as:

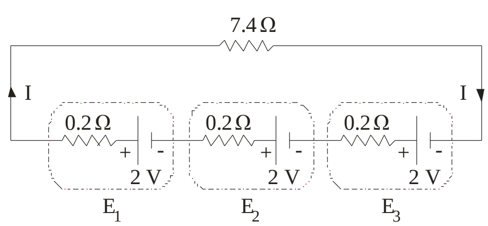

Fig. 3.113

$$I = \frac{N \times E}{R_L + N \times r}$$

$$= \frac{3 \times 2}{7.4 + 3 \times 0.2} = \frac{3}{4} = 0.75 \text{ A}$$

Problem 3.187: *Three identical cells, each of emf 2 V and unknown internal resistance are connected in parallel across an external resistance of 5 Ω. If the terminal voltage across the cells is 1.5 V, determine the internal resistance of each cell.* [CBSE 1998-99]

Solution: $E_1 = E_2 = E_3 = 2$ V, $r_1 = r_2 = r_3 = r$, $M = 3$, $R_L = 5$ Ω, $V_t = 1.5$ V

The current flowing through the external resistor may be given as:

$$I = \frac{V_t}{R_L} = \frac{1.5}{5} = 0.3 \text{ A}$$

$$I = \frac{E}{R_L + (r/M)} = \frac{2}{5 + r/M} = 0.3$$

or, $$r = M \times \left(\frac{2}{0.3} - 5\right) = 3 \times \left(\frac{2}{0.3} - 5\right) = 5\ \Omega$$

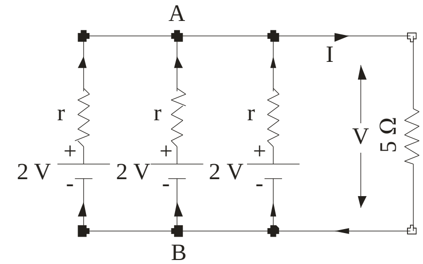

Fig. 3.114

Problem 3.188: *Two cells connected in series have emf of 1.5 V each and internal resistances of 0.5 Ω and 0.25 Ω respectively. This combination is connected to an external resistor of 2.25 Ω. Determine the current flowing through the circuit and the potential difference across the terminals of each cell.*

Solution: $E_1 = E_2 = 1.5$ V, $r_1 = 0.5\ \Omega$, $r_2 = 0.25\ \Omega$, $R_L = 2.25\ \Omega$

The current flowing through the circuit may be given as:

$$I = \frac{E_1 + E_2}{R_L + (r_1 + r_2)} = \frac{1.5 + 1.5}{2.25 + (0.5 + 0.25)} = 1 \text{ A}$$

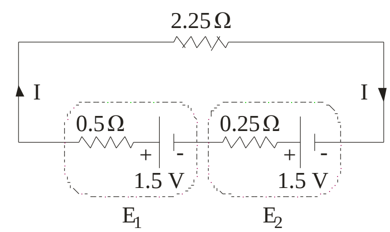

Fig. 3.115

The potential difference across the cells may respectively be given as:

$$V_{t1} = E_1 - I \times r_1 = 1.5 - 1 \times 0.5 = 1 \text{ V}$$

$$V_{t2} = E_2 - I \times r_2 = 1.5 - 1 \times 0.25 = 1.25 \text{ V}$$

Problem 3.189: *If 10 identical cells in series are connected across the ends of an external resistor of 59 Ω, the resultant current is found to be 0.25 A, but when the same cells after being connected in parallel are connected across the points of an external resistor of 0.05 Ω, the resultant current is found to be 25 A. Determine the internal resistance and emf of each cell.*

Solution: $N = 10$ cells, $R_{L1} = 59\ \Omega$, $I_1 = 0.25$ A, $R_{L2} = 0.05\ \Omega$, $I_2 = 25$ A

The current flowing in the circuit due to the series combination of cells may be given as:

$$I_1 = \frac{N \times e}{R_{L1} + N \times r} = \frac{10 \times e}{59 + 10 \times r} = 0.25$$

or, $$10\,r + 59 = \frac{10 \times e}{0.25} = 40\,e$$

or, $40\,e - 10\,r = 59$ or, $4\,e - r = 5.9$ (3.130)

The current flowing in the circuit due to the parallel combination of cells may be given as:

$$I_2 = \frac{e}{R_{L2} + (r/N)} = \frac{e}{0.05 + (r/10)} = 25$$

or, $e = 1.25 + 2.5\,r$ or, $e - 2.5\,r = 1.25$ (3.131)

Equation (3.130) × 2.5 – (3.131):

$$10\,e - e = 5.9 \times 2.5 - 1.25 = 13.5 \quad \text{or,} \quad e = \frac{13.5}{9} = 1.5 \text{ V}$$

Putting this value in equation (3.130):

$$r = 4\,\text{e} - 5.9 = 4 \times 1.5 - 5.9 = 0.1\ \Omega$$

Problem 3.190: *Determine the minimum number of cells required to produce an electric current of 1.5 A through an external resistor of 30 Ω. Given that the emf of each cell is 1.5 V and internal resistance is 1 Ω.*

Solution: $I = 1.5$ A, $R_L = 30\ \Omega$, $e = 1.5$ V, $r = 1\ \Omega$

Let N cells are connected in series and M such rows are connected in parallel for the required current through the external resistor of 30 Ω.

The optimum condition for obtaining the maximum current for the combination may be given as:

$$R_L = \frac{N \times r}{M} = \frac{N \times 1}{M} = 30 \qquad \text{or,} \qquad N = 30\,M \qquad (3.132)$$

The current flowing through the circuit may be given as:

$$I = \frac{e}{\left(\frac{R_L}{N} + \frac{r}{M}\right)} = \frac{1.5}{\left(\frac{30}{N} + \frac{1}{M}\right)} = 1.5$$

or, $$\left(\frac{30}{N} + \frac{1}{M}\right) = 1$$

or, $$\left(\frac{30}{30M} + \frac{1}{M}\right) = \left(\frac{1}{M} + \frac{1}{M}\right) = 1 \qquad \text{[from equation (3.132)]}$$

or, $M = 2$

Putting this value in equation (3.132):

$$N = 30 \times M = 30 \times 2 = 60$$

So, the number of total cells = $N \times M = 60 \times 2 = 120$ cells

Thus, total 120 cells are required for the purpose, 60 cells are connected in series and two such rows are connected in parallel.

Problem 3.191: *How would you arrange 64 similar cells each having an emf of 2 V and internal resistance of 2 Ω, so as to obtain a maximum current through an external resistor of 8 Ω. Also determine the value of maximum current through 8 Ω resistor.*

Solution: $N = 64$ cells, $e = 2$ V, $r = 2\ \Omega$, $R_L = 8\ \Omega$

Let N cells are connected in series and M such rows are connected in parallel.

So, $N \times M = n = 64 \qquad (3.133)$

The optimum condition for obtaining the maximum current for the combination may be given as:

$$R_L = \frac{N \times r}{M} = \frac{2N}{M} = 8 \qquad (3.134)$$

Equation (3.133) × (3.134):

$$2 \times N^2 = 64 \times 8 \qquad \text{or,} \qquad N = \sqrt{\frac{64 \times 8}{2}} = 8 \times 2 = 16$$

and, $$M = \frac{64}{N} = \frac{64}{16} = 4$$

So, 16 cells are to be connected in series and 4 such rows are to be connected in parallel to obtain the maximum current through the 8 Ω resistor. The maximum current so obtained may be given as:

$$I = \frac{e}{\left(\frac{R_L}{N} + \frac{r}{M}\right)} = \frac{2}{\left(\frac{8}{16} + \frac{2}{4}\right)} = \frac{2 \times 16}{(8+8)} = 2 \text{ A}$$

Problem 3.192: *Two identical cells, whether connected together in series or in parallel give the same current, when connected across an external resistance of 1 Ω. Determine the internal resistance of each cell.*

Solution: $N = 2$ cells, $R_L = 1\ \Omega$

Two identical cells connected in series:

The current flowing through the circuit may be given as:

$$I = \frac{N \times e}{R_L + N \times r} = \frac{2 \times e}{1 + 2 \times r}$$

Two identical cells connected in parallel:

The current flowing through the circuit may be given as:

$$I = \frac{e}{R_L + (r/N)} = \frac{e}{1 + (r/2)} = \frac{2e}{2 + r}$$

Since, two currents are equal,

So, $$I = \frac{2 \times e}{1 + 2 \times r} = \frac{2e}{2 + r}$$

or, $2 + r = 1 + 2\,r$ or, $r = 2 - 1 = 1\ \Omega$

Problem 3.193: *A set of 4 cells, each of emf 2 V and internal resistance 1.5 Ω are connected across an external load of 10 Ω with 2 rows, 2 cells in each row. Determine the current through each row and potential difference across the 10 Ω load.*

Solution: $n = 4$ cells, $e = 2$ V, $r = 1.5\ \Omega$, $R_L = 10\ \Omega$, $N = 2$, $M = 2$

The current flowing through the load resistance / drawn from the combination of cells may be given as:

$$I = \frac{e}{\left(\frac{R_L}{N} + \frac{r}{M}\right)} = \frac{2}{\left(\frac{10}{2} + \frac{1.5}{2}\right)} = \frac{2 \times 2}{(10 + 1.5)} = \frac{4}{11.5} \text{ A}$$

The current supplied by each row may be given as:

$$I_{row} = \frac{I}{2} = \frac{1}{2} \times \frac{4}{11.5} = \frac{2}{11.5} = 0.174 \text{ A}$$

The potential difference across the load resistance may be given as:

$$V_t = I \times R_L = \frac{4}{11.5} \times 10 = 3.478 \text{ V}$$

3.22 Heating Effect of Current: When a *potential difference* is applied across the terminals of a conductor an *electric field* is setup inside the conductor and free electrons of the conductor accelerates in the opposite direction of the *electric field* under the influence of it. But the speed of electrons does not increase beyond a constant *drift speed*. The reason is small *relaxation time* between repeated collisions of electrons with the metal ions. The *kinetic energy* gained by the electrons during *relaxation time* is lost in the collisions and the energy of metal ions increases due to which the amplitude of vibration of metal ions about their mean position also increases and hence the average kinetic energy of metal ions and the temperature of the metal increases due to the flow of current (electrons) through the conductor. Obviously, the *electrical energy* supplied by the source of emf is being converted into *heat* inside the conductor.

So, "*the phenomenon of the production of heat in a resistor due to the flow of an electric current through a conductor is known as the **heating effect of current** or **Joule heating***".

3.23 Heat Produced by Electric Current: Consider a conductor *AB* of resistance *R*, as shown in the Fig. 3.116. A source of emf maintains a *potential difference V* across the ends of the conductor *AB* and sends a *steady current I* from the end *A* to end *B*. Obviously the *potential* at the point *A* is greater than the *potential* at the point *B*, *i.e.* $V_A > V_B$.

So, $$V = V_A - V_B > 0 \quad (3.135)$$

The amount of charge that flows from the point *A* to point *B* in time *t* may be given as:

$$q = I \times t \quad (3.136)$$

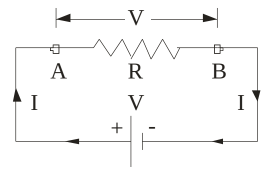

Fig. 3.116

As, the charge moves through a decrease in potential ($V_A > V_B$) of magnitude *V*, the decrease in *potential energy* of the charge *q* may be given as:

U = Potential Energy of the charge q at point B – Potential Energy of the charge q at point A

$$= q \times V_B - q \times V_A = -q \times (V_A - V_B) = -q \times V < 0 \quad (3.137)$$

This decrease (change) in potential energy of the charge *q* appears as heat in the conductor due to collisions of electrons with metal ions and the increased kinetic energy of the electrons (at the cost of potential energy) being transferred to the metal ions completely, because the electrons moves inside the metal at a constant drift velocity without any acceleration.

So, the heat produced in the metal due to *heating effect of the current* may be given as:

$$H = -U = -(-q \times V) = q \times V = I \times t \times V = V I \times t > 0$$

or, $$H = V I \times t = (I R) \times I \times t = I^2 R \times t$$

or, $$H = V \times \frac{V}{R} \times t = \frac{V^2}{R} \times t$$

So, $$H = V I \times t \text{ Joules} = I^2 R \times t \text{ Joules} = \frac{V^2}{R} \times t \text{ Joules} \quad (3.138.1)$$

or, $$H = \frac{V I \times t}{4.18} \text{ cal} = \frac{I^2 R \times t}{4.18} \text{ Cal} = \frac{V^2 \times t}{4.18 \times R} \text{ Cal} \quad (3.138.2)$$

The above two equations are known as ***Joule's Law of Heating***. The postulates of this law are given as follows:

i) The *heat* produced in given resistor (constant *R*) is directly proportional to the *square of the current* (I^2) *flowing through the resistor*.

ii) The *heat* produced by the resistor, for a *constant current* (*I*), is directly proportional to the *resistance* (*R*) *of the resistor*.

iii) The *heat* produced by the resistor, for a *constant potential difference* (V) *across it*, is inversely proportional to the resistance (R) of the resistor.

iv) The *heat* produced by the resistor is directly proportional to the *time of flow of current*.

3.24 Electric Power: "*The rate of work done by the source of emf to maintain the potential difference across the terminals of the conductor to maintain the flow of current is known as the* ***Electrical Power*** *of the circuit*".

Or in other words, "*the rate of conversion of electrical power to any other useful form by an appliance is known as the* ***Electric Power*** *of the circuit*".

If a current I flows through a circuit for time t at a constant potential difference V across it, then the work done by the source of emf or energy consumed by the circuit may be given as:

$$W = V\,I \times t \text{ Joule}$$

So, the *electric power* may be given as:

$$P = \frac{W}{t} = \frac{V\,I \times t}{t} = V\,I = (I\,R) \times I = I^2 R$$

$$= V \times \frac{V}{R} = \frac{V^2}{R}$$

or, $$P = V\,I = I^2 R = \frac{V^2}{R} \tag{3.139}$$

SI Unit of Electric Power: The SI unit of electric power is *Watt* (W). *The power of an electric appliance is 1 Watt, if it consumes power at the rate of 1 Joule/second*.

Or in other words, *the power of an electric circuit is 1 Watt, if a current of 1 Amp flows through the circuit on applying a potential difference of 1 Volt across it.*

So, $$1 \text{ Watt} = \frac{1 \text{ Joule}}{1 \text{ Second}} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}} \times \frac{1 \text{ Coulomb}}{1 \text{ Second}}$$

or, 1 Watt = 1 Volt × 1 Ampere

The larger units of power are kilo-Watt (kW) and Mega-Watt (MW).

1 kW = 10^3 Watt

1 MW = 10^3 kilo-Watt = 10^6 Watt

One another popular unit of power is Horse Power (HP).

1 HP = 746 Watt = 0.746 kW

3.25 Electric Energy: "*The total work done* (*or the energy expanded*) *by the source of emf in maintaining an electric current through a circuit for a given time is known as* ***Electric Energy*** *consumed by the circuit*". The electric energy consumed by a circuit / appliance depends on the power of the circuit / appliance and the time for which this power is being consumed. The electric energy consumed may be given as:

$$E = W = P \times t = V\,I \times t \text{ Joules} = I^2 R \times t \text{ Joules} = \frac{V^2}{R} \times t \text{ Joules}$$

The SI unit of electric energy is Joule (J), like all other energies.

1 Joule = 1 Watt × 1 second = 1 Volt × 1 Ampere × 1 second

Commercial Unit of Electric Energy: *The electricity consumption is a continuous process over the hours, days, weeks, months, years, decades and centuries*. So, a larger unit is required for the convenient measurement of *electric energy*. The commercial unit of electric energy is **kilo-Watt-hour** (**kWh**).

"*One kilo-Watt-hour of energy may be defined as the power consumed by an appliance having a powerof 1 kilo-Watt, if it remains ON continuously for one hour*".

So, 1 kWh = 1 kilo-Watt × 1 hour = 1000 × 60 × 60 = 36,00,000 Joules = 3.6×10^6 Joules

The reader may observe that the unit 1 kWh is quite big unit in terms of Joule, although when we see it in context of a storage water heater of 2 kW (installed in our bathrooms) or total load in a home of about 5 kW, the unit 1 kWh is quite smaller unit. So, the reader may understand the requirement of *commercial unit of electric energy*.

The *electric energy meters* in our houses and industries measure the power in kilo-Watt-hour (kWh).

3.26 Power Rating:

"*The power rating of a circuit / electrical appliance may be defined as the power consumed per second when it is connected across the marked voltage* (*rated voltage*) *of the mains*".

If a voltage V is applied across a circuit element of resistance R and it sets up a current I Amp through it, the power rating of the element may be given as:

$$P = \frac{V^2}{R} = I^2 R = V I \qquad (3.140)$$

(3.140)

Measurement of Electric Power: The electric power of a circuit element (say a lamp) may be measured by measuring the voltage applied across it and the current flowing through it due to this voltage. Let us consider the lamp connected across a battery of emf E. The current (I) flowing through the lamp is measured by an ammeter connected in series with the lamp and the potential difference (V) across the lamp is measured by the voltmeter in parallel with the lamp. Now the power supplied by the battery to the lamp / rated power of the lamp may be given by:

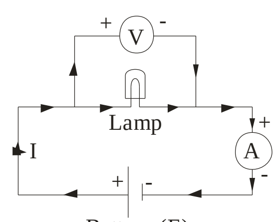

Fig. 3.117

$$P = V \times I \text{ Watt} \qquad (3.141)$$

3.27 Power Consumed in a Combination of Appliances:

Power Consumed in Series Combination of Appliances: Consider a series combination of three lamps / appliances of powers P_1, P_2 and P_3 with same rated voltage V. The resistances of three lamps may respectively be given as:

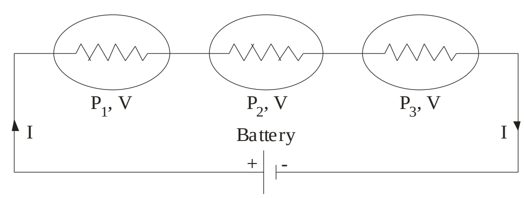

Fig. 3.118

$$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}, \qquad R_3 = \frac{V^2}{P_3} \qquad (3.142)$$

Note here, that the resistance of the individual lamp is inversely proportional to the power of lamp,

i.e. $$R \propto \frac{1}{P_{individual}} \qquad (3.143)$$

Since, the lamps are connected in series, the equivalent resistance of the combination may be given as:

$$R_{eq} = R_1 + R_2 + R_3 = V^2 \times \left(\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}\right) = \frac{V^2}{P_{eq}}$$

So, $$\frac{1}{P_{eq}} = \left(\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}\right) \qquad (3.144)$$

So, *for series combination of appliances the reciprocal of total power is equal to the sum of reciprocal of individual powers of the appliances, so it is seldom used in practice*.

Obviously, if N appliances of same power are connected in series across the same supply source, the total power may be given by the relationship:

$$\frac{1}{P_{eq}} = \left(\frac{1}{P} + \frac{1}{P} + \frac{1}{P} + \text{........}n\ terms\right) = \frac{N}{P}$$

or, $$P_{\text{eq}} = \frac{P}{N} \qquad (3.145)$$

This confirms that the use of appliances in series is not useful, so it is not used in practice except in some special cases.

The current flowing through the series connected appliances may be given as:

$$I = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2 + R_3} \qquad (3.146)$$

The brightness of the lamps may be given as:

$$P_1 = I^2 R_1, \qquad P_2 = I^2 R_2, \qquad P_3 = I^2 R_3 \qquad (3.147)$$

Since, the current is same through all the lamps, so the brightness of the lamp with greatest resistance will be maximum.

i.e. $$\text{Brightness } (P) \propto R \qquad (3.148)$$

If we combine the equations (3.143) and (3.148), we will get an interesting result:

$$\text{Brightness } (P) \propto R \propto \frac{1}{P_{individual}} \qquad (3.149)$$

So, ***the lamp with the minimum power will glow with the most brightness***. So, *the lamp with minimum power rating will fused first if the current through the circuit exceeds the safe limit*.

Power Consumed in Parallel Combination of Appliances: Consider a parallel combination of three lamps / appliances of powers P_1, P_2 and P_3 with same rated voltage V. The resistances of three lamps may respectively be given as:

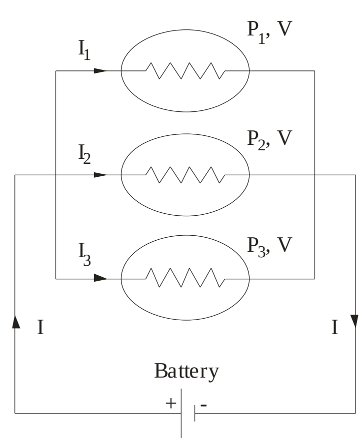

Fig. 3.119

$$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}, \qquad R_3 = \frac{V^2}{P_3} \qquad (3.150)$$

Note here, that the resistance of the individual lamp is inversely proportional to the power of lamp,

i.e. $$R \propto \frac{1}{P_{individual}} \qquad (3.151)$$

Since, the lamps are connected in parallel, the equivalent resistance of the combination may be given as:

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = \frac{1}{V^2} \times (P_1 + P_2 + P_3) = \frac{P_{eq}}{V^2}$$

So, $P_{eq} = (P_1 + P_2 + P_3)$ (3.152)

So, *for parallel combination of appliances the total power is equal to the sum of individual powers of the appliances, so it is used frequently in practice*.

Obviously, if N appliances of same power are connected in parallel across the same supply source, the total power may be given as:

$$P_{eq} = (P + P + P + \ldots\ldots.. + N \text{ terms}) = N \times P$$

or, $P_{eq} = N \times P$ (3.153)

This confirms that the use of appliances in parallel is very useful, so it is frequently used in practice. All the loads in our home are connected in parallel across the supply voltage.

The current flowing through the parallel connected appliances may be given as:

$$I = \frac{V}{R_{eq}} = V \times \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) \qquad (3.154)$$

The brightness of the lamps may be given as:

$$P_1 = \frac{V^2}{R_1}, \qquad P_2 = \frac{V^2}{R_2}, \qquad P_3 = \frac{V^2}{R_3} \qquad (3.155)$$

Since, the voltage is same across all the lamps, so the brightness of the lamp with smallest resistance will be maximum.

i.e. Brightness $(P) \propto \frac{1}{R}$ (3.156)

If we combine the equations (3.151) and (3.156), we will get an interesting result:

$$\text{Brightness } (P) \propto \frac{1}{R} \propto P_{\text{individual}} \qquad (3.157)$$

So, ***the lamp with the maximum power will glow with the most brightness***.

3.28 Efficiency of a Source of EMF: "*The efficiency of a power source of emf may be defined as the ratio of power supplied by the source to that of the power consumed by the circuit elements / appliances*".

Let us assume that a source of emf E, having internal resistance, r is supplying an external resistance R_L. The efficiency of the system may be given as:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{V_t \times I}{E \times I} = \frac{V_t}{E} = \frac{I \times R_L}{I \times (R_L + r)} = \frac{R_L}{(R_L + r)} \qquad (3.158)$$

Maximum Power Transfer Theorem (Condition for maximum power transfer to the external resistance R_L): This theorem states that, "*The power supplied by a source of emf to an external resistance will be maximum, when the external resistance (R_L) is equal to the internal resistance (r) of the source of emf*".

Let us assume that a source of emf E, having internal resistance, r is supplying an external resistance R_L. The current flowing through the circuit may be given as:

$$I = \frac{E}{R_L + r}$$

The power supplied / dissipated in the external resistance (R_L) may be given as:

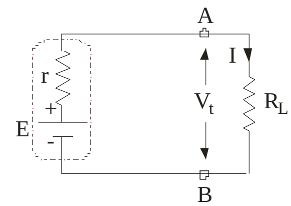

Fig. 3.120

$$P = I^2 R_L = \left(\frac{E}{R_L + r}\right)^2 \times R_L = \frac{E^2}{(R_L + r)^2} \times R_L \qquad (3.159)$$

$$= \frac{E^2}{(R_L - r)^2 + 4 r R_L} \times R_L$$

The power supplied / dissipated in the external resistance (R_L) will obviously be maximum, if the square term in the denominator will be zero.

i.e. $\quad R_L - r = 0 \quad$ or, $\quad R_L = r \qquad (3.160)$

Alternatively: Consider the equation (3.159):

$$P = \frac{E^2}{(R_L + r)^2} \times R_L = \frac{E^2}{R_L^2 + r^2 + 2 r R_L} \times R_L = \frac{E^2}{R_L + \frac{r^2}{R_L} + 2r}$$

Differentiating the numerator of above equation *w.r.t.* external resistance (R_L is variable), for minimizing the denominator, so that the power will be maximized:

$$\frac{d}{d R_L}\left[R_L + \frac{r^2}{R_L} + 2r\right] = 1 - \frac{r^2}{R_L^2} + 0 = 0$$

or, $\quad \frac{r^2}{R_L^2} = 1 \quad$ or, $\quad R_L = r$

So, ***the power supplied by a source of emf to an external resistance will be maximum, when the external resistance is equal to the internal resistance of the source of emf***.

The value of maximum power supplied to the external resistance may be given as:

$$P_{\max} = \left(\frac{E}{R_L + r}\right)^2 \times R_L = \left(\frac{E}{r + r}\right)^2 \times r = \frac{E^2}{4r} \qquad (3.161)$$

The power supplied by the source at this instant may be given as:

$$P_{\text{Source}} = E \times I = E \times \frac{E}{R_L + r} = \frac{E^2}{2r} \qquad (3.162)$$

So, the maximum efficiency of the system may be given as:

$$\eta_{\max} = \frac{\text{Power supplied to external resistance}}{\text{Power supplied by the source}} \times 100\% = \frac{(E^2/4r)}{(E^2/2r)} \times 100\%$$

$$= 50\%$$

Special Case: If the d.c. source (battery) is short circuited, *i.e.* $R_L = 0$

The power output, $P = 0$

So, the efficiency, $\eta = \frac{\text{Output Power}}{\text{Supplied Power}} \times 100\% = \frac{0}{\text{Supplied Power}} \times 100\% = 0\%$

The power supplied by the source is dissipated completely in the internal resistance of the d.c. source (battery). The power dissipated inside the battery in its internal resistance on short circuiting the terminals of the battery may be given as:

$$P_{\text{Short circuit}} = I^2 \times r = \left(\frac{E}{r}\right)^2 \times r = \frac{E^2}{r} \qquad (3.163)$$

3.28 Efficiency of an Electrical Device / Appliance: "*The efficiency of any electrical device / appliance may be defined as the ratio of power output of the appliance to that of the power input to the appliance*".

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \qquad (3.164)$$

An electric motor is a device to convert electrical energy into mechanical energy. So, "*the efficiency of an electric motor may be defined as the ratio of mechanical energy output of the motor to the electrical energy input to the motor*".

$$\eta_{\text{motor}} = \frac{\text{Output mechanical power}}{\text{Input electrical power}} \qquad (3.165.1)$$

$$= \frac{\text{Output mechanical power}}{\text{Output mechanical power} + \text{Losses}} \qquad (3.165.2)$$

$$= \frac{\text{Input electrical power} - \text{Losses}}{\text{Input electrical power}} \qquad (3.165.3)$$

Condition for Maximum Power Output from a Source of EMF Supplying an Electric Motor: Let us assume that a source of emf E, with its internal resistance r, is supplying a motor (the electrical machine to convert input electrical energy to mechanical energy for some useful work). The motor is drawing a current of I Ampere from the source.

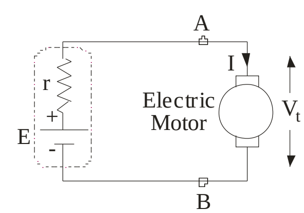

Fig. 3.121

The power supplied by the emf source may be given as:

$$P_{\text{Source output}} = E \times I - I^2 \times r \qquad (3.166)$$

Since the motor current is variable (E and r are constant), we can differentiate above equation *w.r.t.* the motor current I to maximize the power delivered by the source to the motor.

i.e. $$\frac{d}{dI}(E \times I - I^2 \times r) = E - 2\,I \times r = 0$$

or, $$I = \frac{E}{2r} \qquad (3.167)$$

So, the power output of the emf source will be maximum, if the motor draws a current of $\frac{E}{2r}$ from the supply source.

Condition for Maximum Power Output of an Electric Motor: Let us assume that a back emf E_b is being induced in the motor and the resistance of the motor winding is zero. The current drawn by the motor from the emf source may be given as:

$$I = \frac{E - E_b}{r}$$

Since, the resistance of the motor windings are assumed as zero, so the copper losses ($I^2 R$) of the motor will be zero. The power input to the motor will completely be converted into mechanical power and available at the shaft of motor as output mechanical power. Now, the output mechanical power of the motor may be given as:

$$P_{\text{mechanical output}} = P_{\text{electrical input}} = P_{\text{source output}} = E \times I - I^2 \times r$$

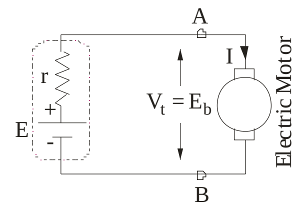

Fig. 3.122

This power is maximum for the condition given by the equation (3.167),

i.e. $I = \dfrac{E}{2r}$

So, $I = \dfrac{E}{2r} = \dfrac{E - E_b}{r}$

or, $E = 2\,(E - E_b)$ or, $E_b = \dfrac{E}{2}$ (3.168)

So, "*the mechanical output of the motor will be maximum, when the back emf induced in the motor will be half of the emf of supply source*".

3.29 Applications of Heating Effect of Current: Some of the important applications of the heating effect of current are described below for the awareness of the reader.

Household Heating Appliances: We need heating of various objects or places in our daily life for the comfortable life of human beings, *e.g.* room heater, storage water heater, electric toaster, electric iron, electric oven, electric kettle etc. The designing and manufacturing of these appliances needs the selection of a resistor of proper resistance and other physical properties to withstand the wear and tear due to heating and the environment. The selected resistor must have a sufficiently high value of resistance, so as to limit the current flowing through it within a safe limit and to convert most of the electric power to heat. The material used for the heating element in most of the appliances is Nichrome, because of the following properties of the Nichrome:

i) *Its melting point is high*.

ii) *Its resistivity is large*.

iii) *Its tensile strength is high*, so it can easily be drawn in the form of thin wires to make its resistance very high.

iv) *It is not easily oxidized by the oxygen in the air in its heated state*.

Incandescent Electric Lamp: It is an important application of *Joule's heating effect* in the production of light. It consists of a fine metallic wire (*tungsten*) enclosed in a glass bulb filled with chemically inactive gas like nitrogen and argon, so that the tungsten wire may not be burnt. The filament material must have a high resistivity and high melting point. So, the tungsten wire (melting point ≈ 3380°) is used for the purpose. The tungsten filament gets heated to a very high temperature due to the flow of current through it and starts to emit light due to its very-very high temperature. Most of the electrical energy consumed by the filament of lamp is converted into heat energy and a very little part of the energy is converted to light. *A lamp will produce approximately 1 candela of light energy for the consumption of every watt of electric energy*.

Electric Fuse: A safety device to protect electrical appliances, a household electrical connection mains, any electrical machine (transformer, motor, generator *etc*.) or a power system from accidental high / large currents is known as "*Electrical Fuse*". A fuse wire has a high resistivity but low melting point. It is usually made up of tin and lead (63% : 37%) alloy. The fuse wire is put in series with the appliance or connection to carry the full current through it. If the current taken by the appliance or system exceeds the safe limit, the fuse wire melts due to its temperature rise because of *heating effect* and the electrical supply to the appliance or system breaks to interrupt any further malfunction or fire. The fuse wire of suitable ratings (0.1 A, 0.2 A, 0.5 A, 1 A, 5 A, 10 A *etc.*) are available in market and must be used in the circuit depending on the load current in the circuit.

For example if we are using a storage water heater of ratings 2 kW, 230 V; the required current for the satisfactory working of storage water heater is $\frac{2\times10^3}{230}$ = 8.696 A. So, a fuse of rating 10 A must be used in series with the storage water heater for the safety of heating element to prevent it from burning due to over-currents.

Electric Arc: The electric arc may be used for lighting or for melting the metals due to high temperature in the electric arc. It may be created by two carbon rods with sharp ends and separated by a small gap between their pointed ends. If a high voltage (potential difference ≈ 40 – 60 V) is applied across the ends of two rods, the air in-between the pointed ends get ionized due to very high electric field in the gap and corona discharge at the sharp ends, and a sharp glowing discharge starts to flow from one pointed end to another pointed end of other rod through the air. A very intense light is emitted in the air gap between two pointed ends of two rods. This intense light may be used for illumination of the objects as well as for melting the metals due to high temperature of electric arc.

Many other devices like electric arc welding, thermionic valves, thermocouples, hotwire ammeters, voltmeters, RTD's for temperature measurements, small handheld refrigerators (*for preserving medicines while outdoor like polio vaccine*) *etc.* may be constructed by the use of heating effect of electric currents.

Reason of Power Transmission at High Voltages: The electrical power we use in our homes or in industries for useful works is at the level of 230 V (single phase) or 400 V (three phase). But when the power is being transmitted from one place to another place over long distances (from the generation power house to the power sub-station in our city), it is being transmitted at very high voltages (220 kV, 132 kV, 33 kV, 11 kV). We know about the *heating effect of the current*, and the knowledge that every conductor has a resistivity and its resistance depending on the resistivity, area of cross section and length of the conductor. So, the power wasted in the long transmission lines will be more, if such a high current will flow in a longer conductor as ($I^2 R$) loss.

Let us assume that the power P is being delivered by a transmission line to the load. The power delivered may be given as:

$$P = VI \qquad \text{or,} \qquad I = \frac{P}{V} \tag{3.169}$$

If this current I is flowing through the conductor of transmission line having resistance R_t, the losses in transmission line may be given as:

$$P_{\text{Line losses}} = I^2 R_t = \left(\frac{P}{V}\right)^2 \times R_t = \frac{P^2 R_t}{V^2} \tag{3.170}$$

So, $\quad P_{\text{Line losses}} \propto \frac{1}{V^2}$

Thus, the line losses will be small, if the electrical power is being transmitted at high voltages. We transmit the power over long distances at high voltages but low currents. These high voltages are step down by transformers before supplying to homes and industries.

Problem 3.194: *An electric current of 4 A flows through a resistor of 12 Ω. Determine the rate at which heat is being produced by the resistor.* [NCERT]

Solution: I = 4 A, $\quad R$ = 12 Ω

The rate at which the heat is being produced by the resistor will be equal to the power wasted in the resistor due to electric current flowing through it, and may be given as:

$$P = I^2 R = (4)^2 \times 12 = 192 \text{ W}$$

Problem 3.195: *How many electrons flow through the filament of 120 V, 60 W electric lamp per second? Given: e = 1.6 × 10^{−19} C.*

Solution: *Lamp Ratings* = 120 V, 60 W, $e = 1.6 \times 10^{-19}$ C

The rated current flowing through the lamp may be given as:

$$I = \frac{P}{V} = \frac{60}{120} = 0.5 \text{ A} = \text{charge } (q) \text{ flowing through the lamp per second}$$

or, $q = n\,e = 0.5$

So, $$n = \frac{0.5}{e} = \frac{0.5}{1.6 \times 10^{-19}} = 3.125 \times 10^{18} \text{ electrons/second}$$

Problem 3.196: *A 210 V, 630 W heating element is connected across a 210 V d.c. mains. Determine the resistance of the heating element and the current flowing through the heating element.* [NCERT]

Solution: *Heating Element Ratings* = 210 V, 630 W, $E = 210$ V

The resistance of the heating element may be given as:

$$R = \frac{V^2}{P} = \frac{(210)^2}{630} = 70\ \Omega$$

The current flowing through the heating element may be given as:

$$I = \frac{P}{V} = \frac{630}{210} = 3 \text{ A}$$

Problem 3.197: *A 10 V storage battery of negligible internal resistance and infinite capacity is connected across a 50 Ω resistor made up of an alloy. How much heat energy is produced in the resistor in 1 hour? What is the source of this energy?* [NCERT]

Solution: $E = 10$ V, $R_L = 50\ \Omega$, $t = 1$ hour $= 60 \times 60 = 3600$ sec

The heat energy produced by the resistor may be given as:

$$H = P \times t = \frac{V^2}{R_L} \times t = \frac{(10)^2}{50} \times 3600 = 7200 \text{ J}$$

The source of this energy is the chemical energy stored in the battery, which is being dissipated as heat in the resistor due to the heating effect of the electric current.

Problem 3.198: *An electric motor operates on 50 V supply and draws a current of 12 A. If the motor is producing a mechanical power of 450 W, determine the percentage efficiency of the electric motor.* [NCERT]

Solution: $E = 50$ V, $I = 12$ A, $P_{mech} = 450$ W

The efficiency of the electric motor may be given as:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100\% = \frac{P_{mech}}{E \times I} \times 100\% = \frac{450}{50 \times 12} \times 100\% = 75\%$$

Problem 3.199: *An electric motor operates on 50 V supply and draws a current of 12 A. If the efficiency of the motor is 76%, determine the winding resistance of the motor assuming the total losses in the motor due to heating effect of the electric current.* [NCERT]

Solution: $E = 50$ V, $I = 12$ A, $\eta = 76\%$

Total losses in the motor may be given as:

$$P_{\text{Losses}} = (1 - \eta) \times P_{\text{Input}} = (1 - 0.76) \times 50 \times 12 = 144 \text{ W} = I^2 R$$

So, $$R = \frac{144}{I^2} = \frac{144}{(12)^2} = 1\ \Omega$$

Problem 3.200: *a*) *A nichrome heating element connected across 230 V supply consumes 1.5 kW of power and heats up to temperature of 750 °C. A tungsten lamp across the same supply operates at a much higher temperature of 1600 °C in order to be able to emit light. Does it mean that the tungsten lamp necessarily consumes greater power?*

b) *Which of the two has greater resistance: a 1 kW heater or a 100 W tungsten lamp, both marked for 230 V?* [NCERT]

Solution: *a*) $V = 230$ V, $P = 1.5$ kW, $T_{\text{element}} = 750$ °C, $T_{\text{Lamp}} = 1600$ °C

No, the steady state heated temperature of an element does not depend only on the power consumed but also on the physical properties like resistivity, surface area, emissivity *etc.* which determines its power loss due to radiation.

b) $V = 230$ V, $P_{\text{heater}} = 1$ kW, $P_{\text{Lamp}} = 100$ W

The resistance of both the objects may respectively be given as:

$$R_{\text{heater}} = \frac{V^2}{P_{heater}} = \frac{(230)^2}{1\times 10^3} = 52.9\ \Omega$$

$$R_{\text{Lamp}} = \frac{V^2}{P_{Lamp}} = \frac{(230)^2}{100} = 529\ \Omega$$

So, a 100 W lamp has greater resistance than that of a 1 kW heater.

Problem 3.201: *An electric power station* (*100 MW*) *transmits power to a remote distant load through long and thin cables. Which of two mode of transmission results in lesser power wastage: power transmission of: a*) *20 kV, or ii*) *200 V?* [NCERT]

Solution: $P = 100$ MW, $V_1 = 20$ kV, $V_2 = 200$ V

Let us assume that the resistance of the cables be $R\ \Omega$.

The ratio of line losses in two cases may be given as:

$$\frac{P_{Losses\,2}}{P_{Losses\,1}} = \frac{I_2^2 R}{I_1^2 R} = \frac{(P/V_2)^2}{(P/V_1)^2} = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{20\times 10^3}{200}\right)^2 = 10^4$$

or, $P_{\text{Losses 2}} = 10^4\, P_{\text{Losses 1}}$

So, $P_{\text{Losses 1}} \ll P_{\text{Losses 2}}$

There will be much lesser losses in first case, when transmitting the power at 20 kV.

Problem 3.202: *Two ribbons are given with following specifications:*

Ribbon	***A***	***B***
Alloy	Constantan	Nichrome
Length (m)	8.456	4.235
Width (mm)	1.0	2.0
Thickness (mm)	0.03	0.06
Temperature coefficient of resistivity (°C^{-1})	Negligible	Negligible
Resistivity	4.9	11

Which of the two ribbons corresponds to greater rate of heat production, for a fixed voltage supply? [NCERT]

Solution: The ratio of heats produced by two ribbons (A and B) may be given as:

$$\frac{H_B}{H_A} = \frac{(V^2/R_B)}{(V^2/R_A)} = \frac{R_A}{R_B} = \frac{(\rho_A l_A / A_A)}{(\rho_B l_B / A_B)} = \frac{\rho_A l_A}{w_A \times t_A} \times \frac{w_B \times t_B}{\rho_B l_B}$$

$$= \frac{4.9 \times 8.456}{1 \times 0.03} \times \frac{2 \times 0.06}{11 \times 4.235} = 3.558$$

or, $H_B = 3.558\, H_A$

So, the ribbon B (*Nichrome Ribbon*) will correspond to greater rate of heat production.

Problem 3.203: *A heater coil is rated 100 W, 200 V. It is cut into two identical parts and both parts are connected in parallel across the supply source of 200 V. Determine the energy librated per second in this new combination.* [CBSE 1999-2000]

Solution: $P = 100$ W, $V = 200$ V

The resistance of two halves of the heater coil may be given as:

$$R_{1/2} = \frac{1}{2} \times R = \frac{1}{2} \times \frac{V^2}{P} = \frac{1}{2} \times \frac{(200)^2}{100} = 200\ \Omega$$

The equivalent resistance of parallel combination may be given as:

$$R_{\text{eq}} = \frac{200 \times 200}{200 + 200} = 100\ \Omega$$

So, the energy liberated per second (Power) of the combination may be given as:

$$P_{\text{new}} = \frac{V^2}{R} = \frac{(200)^2}{100} = 400\ \text{W}$$

Problem 3.204: *An electric lamp is marked 100 W, 230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the lamp in 20 minutes? Also determine the current flowing through the lamp.* [NCERT, CBSE 1993-94]

Solution: $P = 100$ W, $V = 230$ V, $V' = 115$ V, $t = 20 \text{ minutes} = \frac{20}{60} = \frac{1}{3}$ hr.

The resistance of the lamp may be given as:

$$R = \frac{V^2}{P} = \frac{(230)^2}{100} = 529\ \Omega$$

The total heat and the light energy produced by the lamp in 20 minutes with 115 V supply may now be given as:

$$H = P \times t = \frac{V'^2}{R} \times t = \frac{(115)^2}{529} \times \frac{1}{3} = 8.333 \text{ Wh}$$

The current flowing through the lamp may be given as:

$$I = \frac{V}{R} = \frac{115}{529} = 0.217 \text{ A}$$

Problem 3.205: *A 500 W, 100 V electric lamp is connected across a 200 V supply. Determine the resistance R to be connected in series with the lamp so that the lamp delivers 500 W.* [IIT 87]

Solution: *Lamp Ratings* = 500 W, 100 V, $V = 200$ V, $R_{ext} = R$

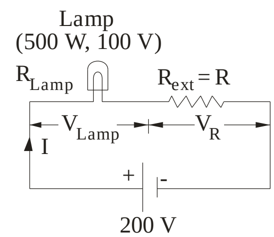

Fig. 3.123

The required circuit is shown in the Fig. 3.123. The reader may observe that the current flowing through the combination is same (I) as both are in connected in series. The lamp will deliver 500 W for its rated current only, which may be given as:

$$I = \frac{P}{V} = \frac{500}{100} = 5 \text{ A}$$

The voltage across the lamp must also be its rated voltage for delivering its rated output (*i.e.* 500 W). So, the voltage across the external resistance may be given as:

$$V_R = V - V_{Lamp} = 200 - 100 = 100 \text{ V}$$

So, $$R = \frac{V_R}{I} = \frac{100}{5} = 20\ \Omega$$

Problem 3.206: *An electric heater and an electric lamp are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V d.c. mains. Determine the power consumed by : i) the heater, ii) the electric lamp.* [CBSE 1996-97]

Solution: *Heater ratings* = 500 W, 220 V, *Lamp Ratings* = 100 W, 220 V, $V = 220$ V

The individual resistance of the heater and the lamp may respectively be given as:

$$R_{heater} = \frac{V^2}{P_{heater}} = \frac{(220)^2}{500} = 96.8\ \Omega$$

and, $$R_{Lamp} = \frac{V^2}{P_{Lamp}} = \frac{(220)^2}{100} = 484\ \Omega$$

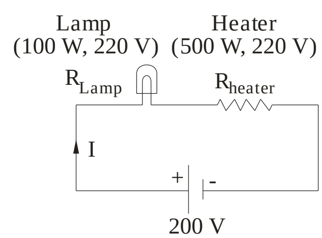

Fig. 3.124

The equivalent resistance of the series combination of the heater and the lamp may be given as:

$$R_{eq} = R_{heater} + R_{Lamp} = 96.8 + 484 = 580.8\ \Omega$$

The current flowing through the series combination may be given as:

$$I = \frac{V}{R_{eq}} = \frac{220}{580.8} = 0.379 \text{ A}$$

The power consumed by the heater and the lamp may respectively be given as:

$$P_{heater} = I^2 R_{heater} = (0.379)^2 \times 96.8 = 13.9 \text{ Watt}$$

and, $P_{\text{Lamp}} = I^2 R_{\text{Lamp}} = (0.379)^2 \times 484 = 69.52$ Watt

Problem 3.207: *The maximum power rating of a 20 Ω resistor is 2 kW. Would you connect this resistor directly across a 300 V d.c. source of negligible internal resistance? Explain your answer.* [NCERT, Haryana 1996-97]

Solution: $R = 20\ \Omega$, $P = 2$ kW, $V = 300$ V

The supply voltage required for this 20 Ω resistor to dissipate its rated power may be given as:

$$V_{\text{rated}} = \sqrt{P \times R} = \sqrt{2 \times 10^3 \times 20} = 200 \text{ V}$$

As, $V_{\text{rated}} < 300$ V

So, this resistor may not be connected directly across a 300 V supply, if it is done the resistor will get permanently damaged due to excess heat generated in the resistor.

Problem 3.208: *Two heaters are rated as 300 W, 200 V and 600 W, 200 V respectively. If two heaters are combined in series and the combination is connected to a 200 V d.c. supply, which heater will produce more heat?* [CBSE 1996-97]

Solution: *Heater*$_1$ *ratings* = 300 W, 200 V, *Heater*$_2$ *Ratings* = 600 W, 200 V, $V = 200$ V

The individual resistance of two heaters may respectively be given as:

$$R_{\text{heater.1}} = \frac{V^2}{P_{heater.1}} = \frac{(200)^2}{300} = 133.333\ \Omega$$

and, $$R_{\text{heater.2}} = \frac{V^2}{P_{heater.2}} = \frac{(200)^2}{600} = 66.667\ \Omega$$

The equivalent resistance of the series combination of two heaters may be given as:

$$R_{\text{eq}} = R_{\text{heater.1}} + R_{\text{heater.2}} = 133.333 + 66.667 = 200\ \Omega$$

The current flowing through the series combination may be given as:

$$I = \frac{V}{R_{eq}} = \frac{200}{200} = 1 \text{ A}$$

The heat produced / power consumed by two heaters may respectively be given as:

$$P_{\text{heater.1}} = I^2 R_{\text{heater.1}} = (1)^2 \times 133.333 = 133.333 \text{ Watt}$$

and, $$P_{\text{heater.2}} = I^2 R_{\text{heater.2}} = (1)^2 \times 66.667 = 66.667 \text{ Watt}$$

So, the heater of ratings 300 W, 200 V will produce more heat.

Problem 3.209: *The rate of heat dissipation in the 4 Ω resistor in the Fig. 3.125 is 100 J/sec, determine the heat dissipated in the 3 Ω resistor in 10 seconds.*

Solution: $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, $P_{4\,\Omega} = 100$ J/sec = 100 W, $t = 10$ sec

The current flowing through the 4 Ω resistor may be given as:

$$I_{4\,\Omega} = \sqrt{\frac{P_{4\,\Omega}}{4}} = \sqrt{\frac{100}{4}} = 5 \text{ A}$$

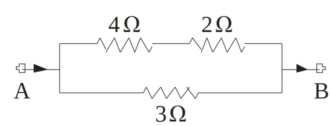

Fig. 3.125

So, the potential difference across the 3 Ω resistor may be given as:

$$V_{3\,\Omega} = V_{(4+2)\,\Omega} = I_{4\,\Omega} \times (4 + 2) = 5 \times 6 = 30 \text{ V}$$

Now, the heat produced / power dissipated in the 3 Ω resistor in 10 seconds may be given as:

$$H_{3\,\Omega} = P_{3\,\Omega} \times t = \frac{V_{3\,\Omega}^2}{3} \times t = \frac{(30)^2}{3} \times 10 = 3000 \text{ J} = 3 \text{ kJ}$$

Problem 3.210: *A series parallel combination of three resistances, 4 Ω each, is connected across a d.c. supply source of emf E, as shown in the Fig. 3.126. The ammeter shows a reading of 1 A. Determine the power dissipated in the circuit.*

Solution: The equivalent resistance of the circuit as seen by the d.c. supply source may be given as:

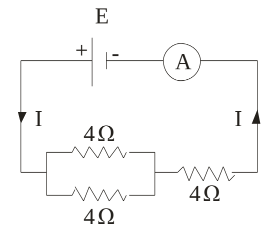

Fig. 3.126

$$R_{eq} = (4 \parallel 4) + 4 = \frac{4\times 4}{4+4} + 4 = 2 + 4 = 6\ \Omega$$

So, the power dissipated in the circuit may be given as:

$$P = I^2 \times R_{eq} = (1)^2 \times 6 = 6 \text{ Watt}$$

Problem 3.211: *A house is fitted with 20 lamps of 60 W each, 10 fans consuming 0.5 A each and an electric kettle of resistance 110 Ω. If the energy is supplied at 220 V and costs 4 rupees per unit, determine the monthly bill for running appliances for 6 hours per day.*

(*Take 1 month = 30 days*)

Solution: Lamps = 20 × (60W), Fans = 10 × (0.5 A), Electric Kettle = 1 × (110 Ω)

V = 220 V, Electricity tariff = 4 Rupees/ Unit Running per day = 6 hours

The energy consumed by these appliances in a month may be given as:

$$E = (P_{\text{Lamps}} + P_{\text{Fans}} + P_{\text{Kettle}}) \times t \times 30$$

$$= \left[60\times 20 + 10\times(220\times 0.5) + 1\times\frac{(220)^2}{110}\right] \times 6 \times 30$$

$$= (1200 + 1100 + 440) \times 180 = 493200 \text{ Wh} = 493.2 \text{ kWh}$$

So, the total cost of energy for the month may be given as:

Cost = 493.2 × 4 =1972.80 Rs.

Problem 3.212: *Two electric lamps are rated for 60 W, 110 V and 100 W, 110 V. They are connected in series with a 220 V d.c. supply. Will any lamp gets fuse? What will happen if they are connected in parallel with the same supply?*

Solution: Lamp_1 = 60 W, 110 V, Lamp_2 = 100 W, 110 V, V = 220 V

Series Combination of two lamps across 220 V supply:

The rated currents of both the lamps may respectively be given as:

$$I_1 = \frac{P_1}{V_1} = \frac{60}{110} = 0.545 \text{ A}$$

and, $$I_2 = \frac{P_2}{V_2} = \frac{100}{110} = 0.909 \text{ A}$$

The ratio of potential differences across two lamps may be given as:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{V^2/P_1}{V^2/P_2} = \frac{P_2}{P_1}$$

So, the voltages and currents across the individual lamps may respectively be given as:

$$V_1 = \frac{P_2}{P_1 + P_2} \times V = \frac{100}{60+100} \times 220 = 137.5 \text{ V},$$

So, $$I = \frac{V_1}{R_1} = \frac{V_1}{(V_{rated}^2 / P_1)} = \frac{137.5 \times 60}{(110)^2} = 0.682 \text{ A}$$

and, $$V_2 = \frac{P_1}{P_1 + P_2} \times V = \frac{60}{60+100} \times 220 = 82.5 \text{ V},$$

So, $$I = \frac{V_2}{R_2} = \frac{V_2}{(V_{rated}^2 / P_2)} = \frac{82.5 \times 100}{(110)^2} = 0.682 \text{ A}$$

The voltage across the 60 W lamp will exceed the rated limit (137.5 V > 110 V). This will result in excess current (0.682 A > 0.545 A) in 60 W lamp, so it will get fuse and both the lamps will turn OFF simultaneously, as the current through both the lamps will interrupt on damage of filament of 60 W lamp. Although the 100 W lamp is alright because the current through it was still below its rated current (0.682 A < 0.909 A) before the circuit is interrupted due to malfunction of 60 W lamp.

Parallel Combination of two lamps across 220 V supply: Both the lamps will get fuse immediately as the voltage across both the lamps is double of the rated value of the voltage, so the current double of their rated values will flow through each of them.

Problem 3.213: *The resistance of a hot 200 W, 240 V filament of electric lamp is 10 times the resistance when it is cold. If the working temperature of the lamp is 2000 °C, determine its resistance at room temperature and temperature coefficient of resistivity of the lamp's filament.*

Solution: Lamp Ratings = 200 W, 240 V, $R_{\text{hot}} = 10\, R_{\text{cold}}$, $T = 2000$ °C

The resistance of the filament at working temperature may be given as:

$$R_{\text{hot}} = \frac{V^2}{P} = \frac{(240)^2}{200} = 288\ \Omega$$

The cold temperature of the filament of lamp may be given as:

$$R_{\text{cold}} = \frac{R_{hot}}{10} = \frac{288}{10} = 28.8\ \Omega$$

The relationship between hot and cold temperature may be given as:

$$R_{\text{hot}} = R_{\text{cold}} \times (1 + \alpha\, t)$$

So, $$\alpha = \frac{1}{T} \times \left(\frac{R_{hot}}{R_{cold}} - 1 \right) = \frac{1}{2000} \times \left(\frac{288}{28.8} - 1 \right) = 4.5 \times 10^{-3}\ °\text{C}^{-1}$$

Problem 3.214: *A thin metallic wire of resistance 100 Ω is immersed in a calorimeter containing 250 gm of water at 10 °C and a current of 0.5 A is passed through it for half an hour. If the water equivalent of calorimeter is 10 gm, determine the rise of temperature.* [Haryana 1994-95]

Solution: $R = 100\ \Omega$, $m_{\text{water}} = 250$ gm, $T_1 = 10$ °C, $I = 0.5$ A, $t = 30$ minutes

$w_{\text{Cal.}} = 10$ gm

The heat produced by the thin metallic wire may be given as:

$$H = I^2 R \times t = (0.5)^2 \times 100 \times 30 \times 60 = 45 \text{ kJ}$$

This heat is absorbed by the water and calorimeter completely as the metallic wire is immersed in the water.

So, $H_{\text{absorbed}} = (m_{\text{water}} + w_{\text{Cal.}}) \times c \times \Delta t \times 4.18 = 45$ kJ

or, $$\Delta t = \frac{45\times10^3}{(m_{water} + w_{Cal.})\times c\times 4.18} = \frac{45\times10^3}{(250+10)\times1\times 4.18} = 41.41\ °\text{C}$$

Problem 3.215: *A copper electric kettle weighing 1000 gm contains 900 gm of water at 20 °C. It takes 12 minutes to raise the temperature to boiling point. If electric energy is supplied at 210 V, determine the strength of the current assuming that 10% of heat produced is waste. Specific heat of copper is 0.1.*

Solution: $m_{\text{kettle(cu)}} = 1000$ gm, $m_{\text{water}} = 900$ gm, $T_1 = 20$ °C, $t = 12$ minutes, $V = 210$ V

Heat Lost = 10 %, $s_{\text{cu}} = 0.1$

Water equivalent of kettle, $w_{\text{Cal.}} = m_{\text{kettle}} \times s = 1000 \times 0.1 = 100$ gm

The temperature rise, $\Delta t = T_2 - T_1 = 100$ °C – 20 °C = 80 °C

The heat absorbed by the system may be given as:

$$H_{\text{absorbed}} = (m_{\text{water}} + w_{\text{Cal.}}) \times c \times \Delta t \times 4.18 = (900 + 100) \times 1 \times 80 \times 4.18 = 334.4 \text{ kJ}$$

The heat produced by the electrical system may be given as:

$$H_{\text{developed}} = V I \times t = 210 \times I \times 12 \times 60 = (151.2\, I) \text{ kJ}$$

The useful heat utilized in temperature rise of the system may be given as:

$$H_{\text{useful}} = (1 - 0.1) \times 151.2\, I = (136.08\, I) \text{ kJ} = H_{\text{absorbed}}$$

or, $136.08\, I = 334.4$ or, $$I = \frac{334.4}{136.08} = 2.457 \text{ A}$$

Problem 3.216: *A coil of enameled copper wire of resistance 50 Ω is embedded in a block of ice and a potential difference of 210 V is applied across it. Determine the rate at which the ice will melt. Latent heat of ice is 80 cal per gram.*

Solution: $R = 50\ \Omega$, $V = 210$ V, $L = 80$ cal/gm

The heat produced by the coil per second may be given as:

$$H = \frac{V^2}{R} \times t \times \frac{1}{4.18} = \frac{(210)^2}{5} \times 1 \times \frac{1}{4.18} = 211 \text{ cal/sec}$$

If m gm of ice melts per second,

then, $m L = H$ or, $$m = \frac{H}{L} = \frac{211}{80} = 2.6375 \text{ gm/sec}$$

Problem 3.217: *An electric kettle has two heating coils, when one of the coil is switched ON the water in kettle begins to boil in 6 minutes and when the other is switched ON, the boiling begins in 8 minutes. Determine the time taken to boil the water, if both the coils are switched ON simultaneously connected in: i) series, ii) parallel.* [IIT]

Solution: $t_1 = 6$ minutes, $t_2 = 8$ minutes

Let us assume that the resistances of two coils be R_1 and R_2.

The heat produced by two coils for boiling the same quantity of water may respectively be given as:

$$H_1 = \frac{P_1 \times t_1}{4.18} = \frac{V^2 \times t_1 \times 60}{4.18 \times R_1} = \frac{V^2 \times 6 \times 60}{4.18 \times R_1}$$

and, $$H_2 = \frac{P_2 \times t_2}{4.18} = \frac{V^2 \times t_2 \times 60}{4.18 \times R_2} = \frac{V^2 \times 8 \times 60}{4.18 \times R_2}$$

Since, $H_1 = H_2$ (Heat required to boil same amount of water)

or, $$\frac{V^2 \times 6 \times 60}{4.18 \times R_1} = \frac{V^2 \times 8 \times 60}{4.18 \times R_2}$$

or, $$\frac{R_2}{R_1} = \frac{8}{6} = \frac{4}{3}$$

Both coils are connected in series:

The equivalent resistance of two coils may be given as, $R_{eq} = R_1 + R_2$

The heat produced by the combination may be given as:

$$H_{series} = \frac{P \times t_{series} \times 60}{4.18} = \frac{V^2 \times t_{series} \times 60}{4.18 \times (R_1 + R_2)}$$

Since, this is the heat required to boil the water,

So, $$H_{series} = \frac{V^2 \times t_{series} \times 60}{4.18 \times (R_1 + R_2)} = H_1 = \frac{V^2 \times 6 \times 60}{4.18 \times R_1}$$

or, $$t_{series} = 6 \times \left(\frac{R_1 + R_2}{R_1}\right) = 6 \times \left(1 + \frac{R_2}{R_1}\right) = 6 \times \left(1 + \frac{4}{3}\right) = 6 \times \frac{7}{3} = 14 \text{ min}$$

Both coils are connected in parallel:

The equivalent resistance of two coils may be given as, $R_{eq} = \dfrac{R_1 \times R_2}{R_1 + R_2}$

The heat produced by the combination may be given as:

$$H_{parallel} = \frac{P \times t_{parallel} \times 60}{4.18} = \frac{V^2 \times t_{parallel} \times 60 \times (R_1 + R_2)}{4.18 \times (R_1 R_2)}$$

Since, this is the heat required to boil the water,

So, $$H_{parallel} = \frac{V^2 \times t_{parallel} \times 60 \times (R_1 + R_2)}{4.18 \times (R_1 R_2)} = H_1 = \frac{V^2 \times 6 \times 60}{4.18 \times R_1}$$

or, $$t_{parallel} = 6 \times \frac{R_2}{R_1 + R_2} = 6 \times \frac{1}{1 + (R_1 / R_2)} = 6 \times \frac{1}{1 + (3/4)} = 6 \times \frac{4}{7} = 3.43 \text{ min}$$

Problem 3.218: *The heater coil of an electric kettle is rated at 2 kW, 200 V. How much time will it take in raising the temperature of 1 liter of water from 20 °C to 100 ° C, assuming that 80% of the total heat energy produced by the heater coil is used in raising the temperature of water? Density of water = 1 gm/cm^3, specific heat of water = 1 cal gm^{-1} °C^{-1}.*

Solution: *heating coil* = 2 kW, 200 V, $Vol._{water}$ = 1 liter = 1000 cm^3, T_1 = 20 °C,

T_2 = 100 °C, $H_{useful} = 0.8\ H_{produced}$, ρ_{water} = 1 gm/cm^3, s = 1 cal gm^{-1} °C^{-1}

Mass of one liter of water, m = 1000 × 1 = 1000 gm

Heat absorbed by the water may be given as:

$$H_{absorbed} = m\ s\ \Delta t \times 4.18 = 1000 \times 1 \times (100 - 20) \times 4.18 = 334.4 \text{ kJ}$$

The heat produced by the heating element in time t may be given as:

$$H_{Produced} = P \times t = 2000 \times t \text{ J} = (2 \times t) \text{ kJ}$$

The useful heat to raise the temperature of the water may be given as:

$$H_{useful} = 0.8 \times H_{produced} = 0.8 \times (2 \times t) = (1.6\ t) \text{ kJ} = H_{absorbed}$$

or, $1.6\ t = 334.4$ or, $t = \dfrac{334.4}{1.6}$ = 209 seconds

Problem 3.219: *A 1 kW electric heater is to be used with 220 V d.c. supply. Determine: i) the current through the heater, ii) its resistance, iii) power dissipated in the heater, iv) heat produced in calories, v) how many gms of water at 100 °C will be converted per minute into steam at 100 °C, with the heater. Latent heat of steam = 540 cal per gram.* [IIT]

Solution: P = 1 kW, 220 V, T = 100 °C, L_{steam} = 540 cal/gm

The current through the heater may be given as:

$$I = \frac{P}{V} = \frac{1\times10^3}{220} = 4.545 \text{ A}$$

The resistance of the heater may be given as:

$$R = \frac{V^2}{P} = \frac{(220)^2}{1\times10^3} = 48.4\ \Omega$$

The power dissipated in the heater may be given as:

$$P = 1 \text{ kW}$$

If the mass of water converted to steam per minute is m,

Then, $m \times L = \dfrac{P}{4.18} \times 60$ or, $m = \dfrac{P\times60}{4.18\times L} = \dfrac{1000\times60}{4.18\times540}$ = 26.58 gm/minute

Problem 3.220: *A 10 V battery of negligible internal resistance is charged by a 200 V d.c. supply. If the resistance in the charging circuit is 38 Ω, determine the value of charging current.* [NCERT]

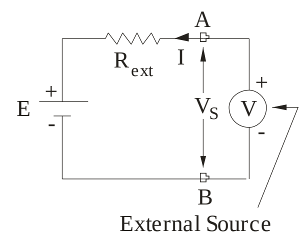

Fig. 3.127

Solution: E = 10 V, V_S = 200 V, R_{ext} = 38 Ω

The charging current through the circuit may be given as:

$$I = \frac{V_S - E}{R_{ext}} = \frac{200-10}{38} = 5 \text{ A}$$

Problem 3.221: *A series battery of 10 lead accumulators, each of emf 2 V and internal resistance of 0.25 Ω is charged by a 220 V d.c. mains. A 47.5 Ω resistor is use in series with the charging circuit in order to limit the charging current. Determine: i) the power supplied by the mains, ii) power dissipated as heat. Account for the difference in power in (i) and (ii).*

Solution: $E = 10 \times 2 = 20$ V, $r = 10 \times 0.25 = 2.5\ \Omega$

$V_S = 220$ V, $R_{ext} = 47.5\ \Omega$

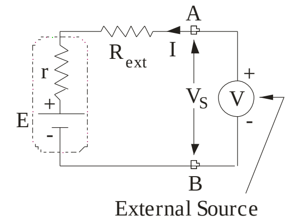

Fig. 3.128

The current through the charging circuit may be given as:

$$I = \frac{V_S - E}{R_{ext} + r} = \frac{220 - 20}{47.5 + 2.5} = 4\text{ A}$$

The power supplied by the mains may be given as:

$$P_S = V_S \times \text{I} = 220 \times 4 = 880\text{ W}$$

The power dissipated as heat may be given as:

$$P_{Loss} = I^2 \times (R_{ext} + r) = (4)^2 \times (47.5 + 2.5) = 800\text{ W}$$

The difference of two powers (880 – 800 = 80 W) is stored in the cells (battery) as chemical energy due to charging of the cells (battery).

Problem 3.222: *A dry cell of emf 1.6 V and internal resistance 0.1 Ω is connected across a resistor of resistance R Ω. If the current drawn from the cell is 2 A, determine: i) voltage across the resistance R, ii) the energy dissipated in the resistor.* [CBSE 1992-93]

Solution: $E = 1.6$ V, $r = 0.1\ \Omega$, $I = 2$ A

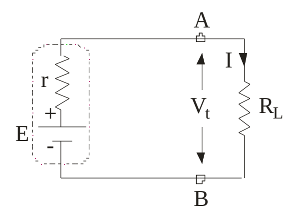

Fig. 3.129

The voltage across the resistance R may be given as:

$$V_t = E - I \times r = 1.6 - 2 \times 0.1 = 1.4\text{ V}$$

The energy dissipated in the resistor may be given as:

$$P = V_t \times I = 1.4 \times 2 = 2.8\text{ W}$$

Problem 3.223: *A series battery of 6 lead accumulators, each of emf 2 V and internal resistance of 0.5 Ω is charged by a 100 V d.c. mains. Determine the value of series resistance to limit charging current to 8 A. Using the required resistor determine: i) the power supplied by the d.c. source, ii) the energy stored in the battery in 15 minutes due to power supplied by the d.c. source.* [NCERT]

Solution: $E = 6 \times 2 = 12$ V, $r = 6 \times 0.5 = 3\ \Omega$

$V_S = 100$ V, $I = 8$ A

Since the current through the circuit may be given by the relationship:

$$I = \frac{V_S - E}{R_{ext} + r}$$

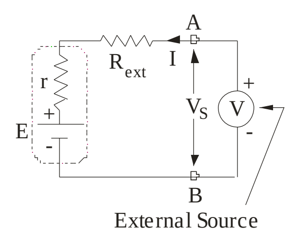

Fig. 3.130

or, $$R_{ext} = \frac{V_S - E}{I} - r = \frac{100 - 12}{8} - 3 = 8\ \Omega$$

The power supplied by the mains may be given as:

$$P_S = V_S \times I = 100 \times 8 = 800\text{ W}$$

The power dissipated as heat may be given as:

$$P_{Loss} = I^2 \times (R_{ext} + r) = (8)^2 \times (8 + 3) = 704\text{ W}$$

The power stored in the battery may be given as:

$$P_{stored} = P_S - P_{Loss} = 800 - 704 = 96\text{ W}$$

So, the energy stored in the battery in 15 minutes may be given as:

$$U = P_{stored} \times t = 96 \times 15 \times 60 = 86400 \text{ J} = 96 \times \frac{15}{60} = 24 \text{ Wh}$$

Problem 3.224: *Power from a 64 V d.c. supply gets to charge a battery of 8 lead accumulators each of emf 2 V and internal resistance 1/8 Ω. The charging current also runs an electric motor placed in series with the charging circuit. If the resistance of the winding of the motor is 7 Ω and the steady supply current is 3.5 A, determine: i) the mechanical energy supplied by the motor, ii) the chemical energy stored in the battery during charging in 1 hour.* [NCERT]

Solution: $V_S = 64$ V, $E = 8 \times 2 = 16$ V, $r = 8 \times \frac{1}{8} = 1\ \Omega$, $R_m = 7\ \Omega$, $I = 3.5$ A

Let the back emf induced in the electric motor be E_b.

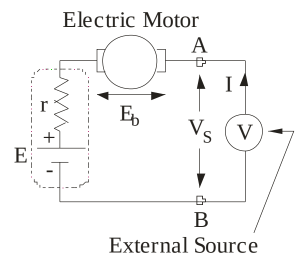

Fig. 3.131

So the current flowing through the charging circuit may be given as:

$$I = \frac{V_S - (E_b + E)}{R_m + r} = \frac{64 - (E_b + 16)}{7 + 1} = 3.5 \text{ A}$$

or, $E_b = 64 - 16 - 3.5 \times 8 = 20$ V

So, the mechanical energy supplied by the motor may be given as:

$$W_{mech.} = P_{mech.} \times t = E_b \times I \times t = 20 \times 3.5 \times 1 \times 60 \times 60 = 252 \text{ kJ}$$

and, the chemical energy stored in the battery may be given as:

$$W_{chem.} = P_{chem.} \times t = E \times I \times t = 16 \times 3.5 \times 1 \times 60 \times 60 = 201.6 \text{ kJ}$$

Problem 3.225: *A 24 V battery of internal resistance of 4 Ω is connected to a variable resistor. Determine the value of current for which the rate of heat produced in the variable resistor will be maximum.* [NCERT]

Solution: $V_S = 24$ V, $r = 4\ \Omega$

The maximum rate of heat production indicates the maximum power dissipation in the external resistance. The condition of maximum power dissipation in the external resistance may be given according to "*Maximum Power Transfer Theorem*", which is given below:

$$R_{ext} = r$$

So, the current corresponding to the maximum power dissipation in the external resistance may be given as:

$$I = \frac{V_S}{R_{ext} + r} = \frac{V_S}{2r} = \frac{24}{2 \times 4} = 3 \text{ A}$$

Problem 3.226: *Four cells, of identical emf e and internal resistance r, are connected in series across a variable resistor. The graph shown in the Fig. 3.132 is drawn for the variation of terminal voltage (V_t) w.r.t. the output current (I). Determine: i) the emf of each cell. ii) the value of current for maximum power dissipation in the circuit, iii) the internal resistance of each cell.* [NCERT]

Solution: $E = 4\,e$, $r_{int.} = 4\,r$

Emf of the battery (E) may be read from the graph (V_t –vs– I) corresponding to $I = 0$ A.

i.e. $E = 4\,e = 5.6$ V

So, emf of each cell may be given as:

$$e = \frac{5.6}{4} = 1.4 \text{ V}$$

The terminal voltage V_t is 2.8 V corresponding to the current I = 1 A.

Since, the terminal voltage at any current may be given as:

$$V_t = E - I \times r_{int}$$

or, $$r_{int} = \frac{E - V_t}{I} = \frac{5.6 - 2.8}{1} = 2.8\ \Omega$$

So, $$r = \frac{r_{int}}{4} = \frac{2.8}{4} = 0.7\ \Omega$$

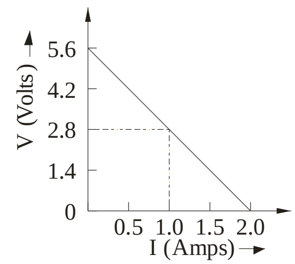

Fig. 3.132

The condition of maximum power dissipation in the external resistance may be given according to "*Maximum Power Transfer Theorem*", which is given below:

$$R_{ext} = r_{int} = 2.8\ \Omega$$

So, the current corresponding to the maximum power dissipation in the external resistance may be given as:

$$I = \frac{E}{R_{ext} + r_{int}} = \frac{E}{2r_{int}} = \frac{5.6}{2 \times 2.8} = 1 \text{ A}$$

Problem 3.227: *Two batteries, each of emf E and internal resistance r, are connected in parallel. If an external resistance (R) is connected across the combination of batteries, determine the value of R corresponding to maximum power dissipation in it. Also, determine the value of maximum power dissipated in the resistance R.*

Solution: The condition of maximum power dissipation in the external resistance may be given according to "*Maximum Power Transfer Theorem*", which is given below:

$$R = r_{int} = \frac{r \times r}{r + r} = \frac{r}{2}$$

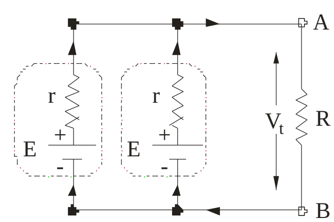

Fig. 3.133

So, the current corresponding to the maximum power dissipation in the external resistance may be given as:

$$I = \frac{E}{R + r_{int}} = \frac{E}{2r_{int}} = \frac{E}{2 \times (r/2)} = \frac{E}{r} \text{ A}$$

So, the maximum power dissipated in the external resistance may be given as:

$$P_{max} = I^2 \times R = \left(\frac{E}{r}\right)^2 \times r_{int} = \left(\frac{E}{r}\right)^2 \times \frac{r}{2} = \frac{E^2}{2r}$$

Alternatively: The maximum power dissipated in the external resistance may be given as:

$$P_{max} = \frac{E^2}{4r_{int}} = \frac{E^2}{4 \times (r/2)} = \frac{E^2}{2r}$$

Problem 3.228: *Two wires made up of tinned copper having identical cross section (= 10^{-6} m^2) and lengths 10 cm and 15 cm respectively are to be used as fuses. Show that the fuses will melt at the same value of current in each case.* [NCERT]

Solution: $A_1 = A_2 = 10^{-6}$ m^2, l_1 = 10 cm, l_2 = 15 cm

The melting of a fuse wire is dependent on the steady state temperature attained by the wire due to heat produced by the flow of current. The steady state temperature depends on the heat produced due to flow of current and the heat lost to the surroundings of the wire by radiations.

The heat produced by the current may be given as:

$$H = I^2 R = I^2 \times \frac{\rho l}{A} \text{ Joules} = \frac{I^2 \rho l}{\pi r^2 \times 4.18} \text{ Cal.}$$

If the heat lost per second per unit surface area for this metal is h, the amount of heat lost from the fuse wire may be given as:

$$H = h \times 2\,\pi\, r\, l$$

For the steady state temperature of the wire:

$$H = h \times 2\,\pi\, r\, l = \frac{I^2 \rho l}{\pi r^2 \times 4.18}$$

or, $$h = \frac{I^2 \rho}{2\pi^2 r^3 \times 4.18}$$

or, $$h \propto \frac{I^2 \rho}{r^3}$$

The reader may observe that h is independent of the length (l) of the conductor. It depends only on the current flowing through the wire, resistivity of the wire and the radius of the conductor. Since, resistivity of the wire and area of cross section of both the wires are same in present case, so both the wires will melt at the same value of current.

Problem 3.229: *A fuse with circular cross sectional area with radius 0.15 mm blows at 15 A. Determine the radius of the fuse made up of same material which will blow at 30 A.* [NCERT]

Solution: $r_1 = 0.15$ mm, $I_1 = 15$ A, $I_2 = 30$ A

The melting of a fuse wire is dependent on the steady state temperature attained by the wire due to heat produced by the flow of current. The steady state temperature depends on the heat produced due to flow of current and the heat lost to the surroundings of the wire by radiations.

The heat produced by the current may be given as:

$$H = I^2 R = I^2 \times \frac{\rho l}{A} \text{ Joules} = \frac{I^2 \rho l}{\pi r^2 \times 4.18} \text{ Cal.}$$

If the heat lost per second per unit surface area for this metal is h, the amount of heat lost from the fuse wire may be given as:

$$H = h \times 2\,\pi\, r\, l$$

For the steady state temperature of the wire:

$$H = h \times 2\,\pi\, r\, l = \frac{I^2 \rho l}{\pi r^2 \times 4.18}$$

or, $$h = \frac{I^2 \rho}{2\pi^2 r^3 \times 4.18}$$

or, $h \propto \dfrac{I^2 \rho}{r^3}$

For a given material of the values of h and ρ are constant, so we can say that, $r^3 \propto I^2$

So, $\left(\dfrac{r_2}{r_1}\right)^3 = \left(\dfrac{I_2}{I_1}\right)^2 = \left(\dfrac{30}{15}\right)^2 = 4$

or, $r_2 = \sqrt[3]{4} \times r_1 = 1.5874 \times 0.15 = 0.2381 \text{ mm} \approx 0.24 \text{ mm}$

Problem 3.230: *Calculate the value of current flowing through a heater rated at 2 kW, when connected to a 300 V d.c. supply.* [CBSE 1993-94]

Solution: $P = 2$ kW, $V = 300$ V

The current flowing through the heater may be given as:

$$I = \frac{P}{V} = \frac{2 \times 10^3}{300} = 6.667 \text{ A}$$

Problem 3.231: *Calculate the amount of heat produced per second (in calories), when an electric lamp rated for 100 W, 220 V glows. Assume that only 20% of electric energy is converted into light. Given: J = 4.2 J cal^{-1}.* [Haryana 1993-94]

Solution: *Lamp ratings* = 100 W, 220 V, *Light energy* = 20%, J = 4.2 J cal^{-1}

The electric energy consumed per second may be given as:

$$E = P \times t = 100 \times 1 = 100 \text{ J}$$

The heat produced per second may be given as:

$$H = (1 - 0.2) \times 100 = 80 \text{ J} = \frac{80}{4.2} = 19.05 \text{ cal}$$

Problem 3.232: *An electric heating element of 480 W, 240 V is to be made from a nichrome ribbon which is 1 mm wide and 0.05 mm thick. Determine the length of the ribbon required, if the resistivity of the nichrome is 1.1 × 10^{-6} Ω-m.*

Solution: *Heater ratings* = 480 W, 240 V, $w = 1$ mm, $t = 0.05$ mm, $\rho = 1.1 \times 10^{-6}$ Ω-m

The required resistance of the heating element may be given as:

$$R = \frac{V^2}{P} = \frac{(240)^2}{480} = 120\ \Omega = \frac{\rho l}{A}$$

So, $$l = 120 \times \frac{A}{\rho} = 120 \times \frac{w \times t}{\rho} = 120 \times \frac{1 \times 10^{-3} \times 0.05 \times 10^{-3}}{1.1 \times 10^{-6}} = 5.455 \text{ m}$$

Problem 3.233: *A 100 W, 220 V lamp is connected to 110 V mains. Determine the power consumed by the lamp.* [Roorkee 1985-86]

Solution: *Lamp ratings* = 100 W, 220 V, $V_S = 110$ V

Since, $P = \dfrac{V^2}{R}$ so, $P \propto V^2$ (for R constant)

The ratio of power at rated voltage and that on 110 V supply may be given as:

$$\frac{P_{110\,V}}{P_{220\,V}} = \left(\frac{V_s}{V_{rated}}\right)^2$$

or, $$P_{110\,\text{V}} = \left(\frac{V_s}{V_{rated}}\right)^2 \times P_{220\,\text{V}} = \left(\frac{110}{220}\right)^2 \times 100 = 25\text{ W}$$

Problem 3.234: *How many electrons flow per second through an electric lamp of 100 W, 220 V.*

[BIT Ranchi 1998]

Solution: *Lamp ratings* = 100 W, 220 V

The charge flowing through the lamp per second may be given as:

$$q = n \times e = I \times t = \frac{P}{V} \times t = \frac{100}{220} \times 1 = 0.455$$

So, $$n = \frac{0.455}{e} = \frac{0.455}{1.6\times10^{-19}} = 2.844 \times 10^{18}\text{ electrons/sec}$$

Problem 3.235: *An ammeter reads a current of 30 A on connecting across the terminals of a cell of emf 1.5 V. Neglecting the ammeter resistance, determine the heat produced in the battery in 10 seconds.*

Solution: I = 30 A, E = 1.5 V, t = 10 seconds

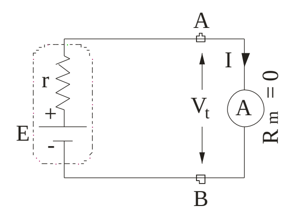

Fig. 3.134

Since, the current flowing through the circuit may be given as:

$$I = \frac{E}{R_m + r} = \frac{E}{r}$$

So, $$r = \frac{E}{I} = \frac{1.5}{30} = 0.05\ \Omega$$

So, heat produced in the battery in 10 seconds may be given as:

$$H = I^2 r \times t \times \frac{1}{4.18} = (30)^2 \times 0.05 \times 10 \times \frac{1}{4.18} = 107.66\text{ cal}$$

Alternatively:

The heat produced in the battery in 10 seconds may be given as:

$$H = E \times I \times t = 1.5 \times 30 \times 10 \times \frac{1}{4.18} = 107.66\text{ cal}$$

Problem 3.236: *A coil of resistance 100 Ω is connected across a battery of emf 6 V. The heat developed in the coil is used to raise its temperature. If the thermal capacity of the coil is 4 JK^{-1}, how long would it take to raise the temperature of the coil by 15 °C?*

Solution: R = 100 Ω, E = 6 V, k = 4 JK^{-1}, ΔT = 15 °C

Let the time taken by the coil to raise in temperature be t. The heat produced by the coil in time t may be given as:

$$H = \frac{E^2}{R} \times t = \frac{(6)^2}{100} \times t = 0.36 \times t\text{ Joules}$$

The heat required by the coil (H) = thermal capacity (k) × Rise in temperature (ΔT)

or, $$0.36 \times t = 4 \times 15$$

So, $t = \dfrac{4 \times 15}{0.36} = 166.667 \text{ sec} = \dfrac{166.667}{60} = 2 \text{ min } 46.667 \text{ sec}$

Problem 3.237: *A generator is supplying power to a factory by cables of resistance 20 Ω. If the generator is generating 50 kW power at 5 kV, determine the power received by the factory.*

Solution: $r = 20\ \Omega$, $P = 50$ kW, $V_S = 5$ kV

The current supplied to the factory may be given as:

$$I = \frac{P}{V} = \frac{50 \times 10^3}{5 \times 10^3} = 10 \text{ A}$$

The voltage received at the factory end may be given as:

$$V_t = E - I \times r = 5000 - 10 \times 20 = 4800 \text{ V}$$

So, the power received by the factory may be given as:

$$P_{\text{Factory}} = V_t \times I = 4800 \times 10 = 48 \text{ kW}$$

(the rest of the power, *i.e.* 50 – 48 = 2 kW, is being wasted in the cable due to the *Joules heating effect* in the cable)

Problem 3.238: *Two electric lamps are marked 100 W, 220 V and 50 W, 220 V respectively. They are connected in series to a 220 V supply mains. Determine the ratio of heats generated in them.*

Solution: $Lamp_1 = 100$ W, 220 V, $Lamp_2 = 50$ W, 220 V, $V_S = 220$ V

The ratio of potential differences across two lamps may be given as:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{V_S^2 / P_{rated.1}}{V_S^2 / P_{rated.2}} = \frac{P_{rated.2}}{P_{rated.1}}$$

Since, the current flowing through both the lamps is same in series combination, the ratio of heats generated in two lamps may be given as:

$$\frac{H_1}{H_2} = \frac{P_{produced.1}}{P_{produced.2}} = \frac{V_1 \times I}{V_2 \times I} = \frac{V_1}{V_2} = \frac{P_{rated.2}}{P_{rated.1}} = \frac{50}{100} = \frac{1}{2}$$

So, $H_1 : H_2 = 1 : 2$

Problem 3.239: *Following appliances are working in a house at constant voltage 220 V supply: i) a 60 W lamp, ii) a 1000 W heater, iii) a 40 W radio. Determine the current drawn by the heater and the current passing through the fuse on main line.*

Solution: *Lamp* = 60 W, 220 V, *Heater* = 1 kW, 220 V, *Radio* = 40 W, 220 V

The current drawn by the heater may be given as:

$$I_{\text{heater}} = \frac{P_{heater}}{V} = \frac{1000}{220} = 4.545 \text{ A}$$

The current flowing through the fuse on main line may be given as:

$$I_{\text{fuse}} = \frac{1}{V} \times (P_{\text{heater}} + P_{\text{lamp}} + P_{\text{radio}}) = \frac{1}{220} \times (1000 + 60 + 40) = 5 \text{ A}$$

Problem 3.239: *Three equal resistances connected in series across a source of emf consumes 20 W. If the same resistances are connected in parallel across the same source of emf, determine the value of power dissipated by the new combination.* [Punjab 1998-99]

Solution: $R_1 = R_2 = R_3 = R$

The equivalent resistance of these three resistors connected in series and then three resistances connected in parallel may respectively be given as:

$$R_{\text{eq(series)}} = (R + R + R) = 3\,R$$

and, $$R_{\text{eq(parallel)}} = \frac{1}{\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)} = \frac{R}{(1+1+1)} = \frac{R}{3}$$

The power consumed by the series connected combination of these resistors may be given as:

$$P_{\text{series}} = \frac{V^2}{R_{eq(series)}} = \frac{V^2}{3R} = 20\text{ W} \qquad \text{or,} \qquad \frac{V^2}{R} = 3 \times 20 = 60\text{ W}$$

The power consumed by the parallel connected combination of these resistors may be given as:

$$P_{\text{parallel}} = \frac{V^2}{R_{eq(parallel)}} = \frac{V^2}{(R/3)} = \frac{3V^2}{R} = 3 \times \frac{V^2}{R} = 3 \times 60 = 180\text{ W}$$

Problem 3.240: *An electric heater consists of 20 m length of manganin wire of 0.23 mm^2 cross sectional area. Determine the wattage of heater when a potential difference of 200 V is applied across it. The resistivity of menganin is given as 4.6 × 10^{-7} Ω-m.*

Solution: $L = 20$ m, $A = 0.23$ mm^2, $V = 200$ V, $\rho = 4.6 \times 10^{-7}$ Ω-m

The resistance of the heating element may be given as:

$$R = \frac{\rho l}{A} = \frac{4.6\times10^{-7}\times 20}{0.23\times10^{-6}} = 40\ \Omega$$

The wattage of heater on connecting a supply of 200 V across it may be given as:

$$P = \frac{V^2}{R} = \frac{(200)^2}{40} = 1000\text{ Watt} = 1\text{ kW}$$

Problem 3.241: *An electric line having total resistance of 0.2 Ω delivers 10 kW at 220 V to a small factory. Determine the efficiency of the transmission.*

Solution: $R_{\text{Line}} = 0.2\ \Omega$, $P_{\text{receiving end}} = 10$ kW, $V = 220$ V

The current drawn by the factory may be given as:

$$I = \frac{P_{receiving\ end}}{V} = \frac{10\times10^3}{220} = 45.455\text{ A}$$

The line losses may be given as:

$$P_{\text{losses}} = I^2 \times R_{\text{Line}} = (45.455)^2 \times 0.2 = 413.23\text{ W} = 0.41323\text{ kW}$$

The efficiency of transmission may be given as:

$$\eta_{\text{transmission}} = \frac{P_{receiving\ end}}{P_{receiving\ end} + P_{losses}} \times 100\% = \frac{10}{10+0.41323}\times100\% = 96.031\%$$

Alternatively:

The voltage at sending end may be given as:

$$V_{\text{sending end}} = V_{\text{recieving end}} + I \times R_{\text{Line}} = 220 + 45.455 \times 0.2 = 229.091 \text{ V}$$

So, the power at sending end may be given as:

$$P_{\text{sending end}} = V_{\text{sending end}} \times I = 229.091 \times 45.455 = 10.41333 \text{ kW}$$

The efficiency of transmission may be given as:

$$\eta_{\text{transmission}} = \frac{P_{receiving\ end}}{P_{sending\ end}} \times 100\% = \frac{10}{10.41333} \times 100\% = 96.031\%$$

Problem 3.242: *An electric motor operating on 120 V draws a current of 2 A. If heat is developed in the motor at the rate of 9 cal sec^{-1}, determine the efficiency of the motor.*

Solution: $V = 120$ V, $I = 2$ A, $H = 9$ cal/sec

The electrical input to motor may be given as:

$$P_{\text{input}} = V \times I = 120 \times 2 = 240 \text{ W}$$

The losses of the machines are given as:

$$P_{\text{losses}} = 9 \text{ cal/sec} = 9 \times 4.18 \text{ J/sec} = 37.62 \text{ Watt}$$

So, the efficiency of the motor may be given as:

$$\eta = \frac{P_{output}}{P_{input}} \times 100\% = \frac{P_{input} - P_{losses}}{P_{input}} \times 100\% = \frac{240 - 37.62}{240} \times 100\% = 84.325\%$$

Problem 3.243: *A 500 W electric heater is designed to work with a supply voltage of 200 V. If the supply voltage drops to 160 V, determine the percentage loss of the heat developed.*

Solution: $P_{\text{rated}} = 500$ W, $V_{\text{rated}} = 200$ V, $V = 160$ V

The resistance of the heater may be given as:

$$R = \frac{V^2}{P_{rated}} = \frac{(200)^2}{500} = 80\ \Omega$$

The power dissipated with 160 V across the heater may be given as:

$$P = \frac{V^2}{R} = \frac{(160)^2}{80} = 320 \text{ W}$$

So, the percentage loss in the production of heat may be given as:

$$\%\text{ loss} = \frac{P_{rated} - P}{P_{rated}} \times 100\% = \frac{500 - 320}{500} \times 100\% = 36\%$$

Problem 3.244: *A 50 W electric lamp is connected across 200 V supply. Determine the resistance of the lamp and the current flowing through the lamp. If 10% of the total electric power is converted into light, determine the production of heat. Given J = 4.2 J cal^{-1}.*

Solution: $P = 50$ W, $V = 200$ V, *Light energy* = 10% of total electric energy, $J = 4.2$ J cal^{-1}

The resistance of the lamp may be given as:

$$R = \frac{V^2}{P} = \frac{(200)^2}{50} = 800\ \Omega$$

The current flowing through the lamp may be given as:

$$I = \frac{P}{V} = \frac{50}{200} = 0.25 \text{ A} \qquad \text{Also,} \qquad I = \frac{V}{R} = \frac{200}{800} = 0.25 \text{ A}$$

The rate of production of heat may be given as:

$$P = (1 - 0.1) \times 50 \text{ W} = 45 \text{ J/sec} = \frac{45}{4.2} \text{ cal/sec} = 10.714 \text{ cal/sec}$$

Problem 3.245: *Two electric lamps rated at 25 W, 220 V and 100 W, 220 V are connected in series to a 440 V supply. i) Investigate with proper calculations, which lamp will fuse. ii) What will happen if two lamps are connected in parallel to the same supply?*

Solution: $Lamp_1$ = 25 W, 220 V, $Lamp_2$ = 100 W, 220 V, V = 440 V

Series Combination of two lamps across 440 V supply:

The rated currents of both the lamps may respectively be given as:

$$I_1 = \frac{P_1}{V_1} = \frac{25}{220} = 0.114 \text{ A} \qquad \text{and,} \qquad I_2 = \frac{P_2}{V_2} = \frac{100}{220} = 0.455 \text{ A}$$

The ratio of potential differences across two lamps may be given as:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{V^2/P_1}{V^2/P_2} = \frac{P_2}{P_1}$$

So, the voltages and currents across the individual lamps may respectively be given as:

$$V_1 = \frac{P_2}{P_1 + P_2} \times V = \frac{100}{25+100} \times 440 = 352 \text{ V},$$

So, $$I = \frac{V_1}{R_1} = \frac{V_1}{(V_{rated}^2 / P_1)} = \frac{352 \times 25}{(220)^2} = 0.182 \text{ A}$$

and, $$V_2 = \frac{P_1}{P_1 + P_2} \times V = \frac{25}{25+100} \times 440 = 88 \text{ V},$$

So, $$I = \frac{V_2}{R_2} = \frac{V_2}{(V_{rated}^2 / P_2)} = \frac{88 \times 100}{(220)^2} = 0.182 \text{ A}$$

The voltage across the 25 W lamp will exceed the rated limit (352 V > 220 V). This will result in excess current (0.182 A > 0.114 A) in 25 W lamp, so it will get fuse and both the lamps will turn OFF simultaneously, as the current through both the lamps will interrupt on damage of filament of 25 W lamp. Although the 100 W lamp is alright because the current through it is still below its rated current (0.182 A < 0.455 A) before the circuit is interrupted due to malfunction of 25 W lamp.

Parallel Combination of two lamps across 440 V supply: Both the lamps will get fuse immediately as the voltage across both the lamps is double of the rated value of the voltage, so the current double of their rated values will flow through each of them.

Problem 3.246: *A servo voltage stabilizer restricts the output voltage to 220 V ± 1%. If an electric lamp rated at 100 W, 220 V is connected to it, determine the minimum and maximum power consumed by the lamp?*

Solution: $V = 220 \pm 1\%$, $Lamp$ = 100 W, 220 V

The resistance of the lamp may be given as:

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

So, the minimum and maximum power consumed by the lamp may respectively be given as:

$$P_{\text{minimum}} = \frac{(V_{minimum})^2}{R} = \frac{[(1-0.01)\times 220]^2}{484} = 98.01\text{ W}$$

and, $$P_{\text{maximum}} = \frac{(V_{maximum})^2}{R} = \frac{[(1+0.01)\times 220]^2}{484} = 102.01\text{ W}$$

Problem 3.247: *A room is lighted by 200 W, 124 V incandescent lamp fed by a generator, whose output voltage is 130 V. The connecting wires from the generator to the user are made up of aluminum wire of total length 150 m and cross sectional area of 15 mm^2. How many such lamps can be installed? What is the total power consumed by the user? Specific resistance of the aluminum is given as 2.9 × 10^{-8} Ω-m.*

Solution: *Lamp* = 200 W, 124 V, V_S = 130 V, l = 150 m, A = 15 mm^2, $\rho = 2.9 \times 10^{-8}$ Ω-m

The resistance of connecting wires may be given as:

$$R_{\text{wires}} = \frac{\rho l}{A} = \frac{2.9\times 10^{-8}\times 150}{15\times 10^{-6}} = 0.29\ \Omega$$

The current from the generator to supply the lamps of 124 V may be given as:

$$I = \frac{V_S - V_{rated}}{R} = \frac{130-124}{0.29} = 20.69\text{ A}$$

The rated current of one lamp may be given as:

$$I_{\text{rated}} = \frac{P_{rated}}{V_{rated}} = \frac{200}{124} = 1.613\text{ A}$$

So, the number of lamps that may be installed,

$$N = \frac{I}{I_{rated}} = \frac{20.69}{1.613} = 12.827 \approx 12 \text{ or } 13 \text{ Lamps}$$

The total power consumed by the lamps may be given as:

If 12 lamps are installed:

$$V_{\text{Lamps}} = V_S - 12 \times I_{\text{rated}} \times R_{\text{wires}} = 130 - 12 \times 1.613 \times 0.29 = 124.387\text{ V}$$

So, $$P = 12 \times \frac{V_{Lamp}^2}{R_{Lamp}} = 12 \times V_{Lamps}^2 \times \frac{P_{Lamp}}{V_{rated}^2} = 12 \times (124.387)^2 \times \frac{200}{(124)^2} = 2.415\text{ kW}$$

If 13 lamps are installed:

$$V_{\text{Lamps}} = V_S - 13 \times I_{\text{rated}} \times R_{\text{wires}} = 130 - 13 \times 1.613 \times 0.29 = 123.92\text{ V}$$

So, $$P = 13 \times \frac{V_{Lamp}^2}{R_{Lamp}} = 13 \times V_{Lamps}^2 \times \frac{P_{Lamp}}{V_{rated}^2} = 13 \times (123.92)^2 \times \frac{200}{(124)^2} = 2.597\text{ kW}$$

Problem 3.248: *Two wires A and B of same material and mass have their lengths in the ratio 1 : 2. On connecting them one at a time to the same source of emf, the rate of heat dissipation in B is found to be 5 W. Determine the rate of heat dissipation in the wire A.*

Solution: $m_A = m_B$, $l_A : l_B = 1 : 2$, $P_B = 5$ W

Since mass of the wires and the material is same. So their volumes are equal,

$$Volume_A = A_A\, l_A = Volume_B = A_B\, l_B$$

So, $$\frac{A_A}{A_B} = \frac{l_B}{l_A} = 2$$

The ratio of the rate of heat dissipation (power) in two wires may be given as:

$$\frac{P_A}{P_B} = \frac{(V^2/R_A)}{(V^2/R_B)} = \frac{R_B}{R_A} = \frac{\rho\, l_B / A_B}{\rho\, l_A / A_A} = \frac{l_B}{l_A} \times \frac{A_A}{A_B} = 2 \times 2$$

So, $P_A = 4 \times P_B = 4 \times 5 = 20$ W

Problem 3.249: *Two electric lamps rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V d.c. mains. Determine: i) current through them, ii) potential difference across them, iii) actual power consumed by each lamp.*

Solution: $Lamp_1 = 25$ W, 220 V, $Lamp_2 = 100$ W, 220 V, $V = 220$ V

Series Combination of two lamps across 220 V supply:

The resistances of two lamps may respectively be given as:

$$R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{25} = 1936\ \Omega$$

and, $$R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{100} = 484\ \Omega$$

The current through the series combination of two lamps may be given as:

$$I = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2} = \frac{220}{1936 + 484} = 0.091\text{ A}$$

The ratio of potential differences across two lamps may be given as:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{V^2/P_1}{V^2/P_2} = \frac{P_2}{P_1}$$

So, the voltages and currents across the individual lamps may respectively be given as:

$$V_1 = \frac{P_2}{P_1 + P_2} \times V = \frac{100}{25 + 100} \times 220 = 176\text{ V}$$

and, $$V_2 = \frac{P_1}{P_1 + P_2} \times V = \frac{25}{25 + 100} \times 220 = 44\text{ V}$$

The actual power consumed by each lamp may be given as:

$$P_{actual.1} = V_1 \times I = 176 \times 0.091 = 16.016\text{ W}$$

and, $$P_{actual.2} = V_2 \times I = 44 \times 0.091 = 4.004\text{ W}$$

Problem 3.250: *The heater coil of an electric kettle is rated at 2000 W, 200 V. How much time will it take to heat one liter of water from 20 °C to boiling point, assuming that entire electric energy* *librated from the heater coil is utilized for heating the water? Also calculate the resistance of the coil. Density of water is 1 gm/cm³.*

Solution: *Heater* = 2000 W, 200 V, T_1 = 20 °C, T_2 = 100 °C, ρ_d = 1 gm/cm^3

Mass of one liter of water, $m = 1000 \times 1 = 1000$ gm

Heat absorbed by the water may be given as:

$$H_{\text{absorbed}} = m\, s\, \Delta t \times 4.18 = 1000 \times 1 \times (100 - 20) \times 4.18 = 334.4 \text{ kJ}$$

The heat produced by the heating element in time t may be given as:

$$H_{\text{Produced}} = P \times t = 2000 \times t \text{ J} = (2 \times t) \text{ kJ} = H_{\text{absorbed}}$$

or, $2\,t = 334.4$

So, $$t = \frac{334.4}{2} = 167.2 \text{ seconds} = 2 \text{ minute } 47.2 \text{ sec}$$

The resistance of the heater coil may be given as:

$$R = \frac{V^2}{P} = \frac{(200)^2}{2000} = 20\ \Omega$$

Problem 3.251: *An electric kettle was rated at 500 W, 230 V and was found to raise 1 kg of water at 15 °C to the boiling point in 15 minutes. Determine the heat efficiency of the kettle.*

Solution: *Heater* = 500 W, 230 V, T_1 = 15 °C, T_2 = 100 °C, t = 15 minutes

Heat absorbed by the water may be given as:

$$H_{\text{absorbed}} = m\, s\, \Delta t \times 4.18 = 1000 \times 1 \times (100 - 15) \times 4.18 = 355.3 \text{ kJ}$$

The heat produced by the heating element in 15 minutes may be given as:

$$H_{\text{Produced}} = P \times t = 500 \times 15 \times 60 = 450 \text{ kJ}$$

The heat efficiency of the kettle may be given as:

$$\eta = \frac{H_{abosrbed}}{H_{produced}} \times 100\% = \frac{355.3}{450} \times 100\% = 78.96\%$$

Problem 3.252: *A copper kettle weighing 1000 gm holds 1900 gm of water at 19 °C. It takes 12 minutes to raise the temperature to 100 °C. If energy is supplied at 210 V, determine the strength of current, assuming that 10% of heat is wasted. Specific heat of copper = 0.1 cal gm^{-1} °C^{-1}, 1 cal = 4.2 J.*

Solution: m_{kettle} = 1000 gm, m_{water} = 1900 gm, T_1 = 19 °C, T_2 = 100 °C,

t = 12 minutes, V = 210 V, wastage = 10%, s_{cu} = 0.1 cal gm^{-1} °C^{-1}

Let the heat is produced by the current I, so the heat produced in 12 minutes may be given as:

$$H_{\text{produced}} = V\, I \times t = 210 \times I \times 12 \times 60 = (151.2 \times I) \text{ kJ}$$

The useful heat absorbed by the water may be given as:

$$H_{\text{useful}} = (1 - 0.1) \times (151.2 \times I) = (136.08 \times I) \text{ kJ}$$

The heat absorbed by the water to raise its temperature to boiling point may be given as:

$$H_{\text{absorbed}} = (m_{\text{kettle}}\, s_{\text{cu}} + m_{\text{water}}\, s)\, \Delta t \times 4.2$$

$$= (1000 \times 0.1 + 1900 \times 1) \times (100 - 19) \times 4.2 = 680.4 \text{ kJ}$$

Since, $H_{\text{useful}} = H_{\text{absorbed}}$

or, $(136.08 \times I) = 680.4$ or, $I = \dfrac{680.4}{136.08} = 5$ A

Problem 3.253: *A 30 V storage battery is being charged by 120 V d.c. supply. A resistor has been connected in series with the battery to limit the charging current to 15 A. Determine the rate at which energy is being dissipated in the resistor. If the total heat produced could be made available for heating of water, how long would it take to bring 1 kg of water from 15 °C to the boiling point? Specific heat of water = 1 cal gm^{-1} °C^{-1}, 1 cal = 4.2 J.* [MNREC 1984]

Solution: $E = 30$ V, $V_S = 120$ V, $I = 15$ A, $m_{water} = 1$ kg, $T_1 = 15$ °C,

$T_2 = 100$ °C, $s = 1$ cal gm^{-1} °C^{-1}, 1 cal = 4.2 J

The rate of energy dissipation in the external resistor may be given as:

$$P_{ext} = V_{ext} \times I = (V_S - E) \times I = (120 - 30) \times 15$$

$$= 1350 \text{ W} = 1350 \text{ J/sec } 1.35 \text{ kJ/sec}$$

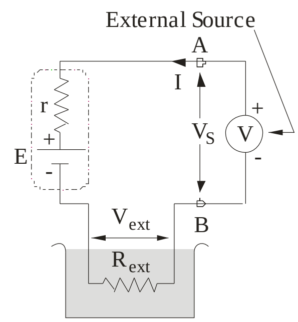

Fig. 3.135

The heat produced by the heater in time t may be given as:

$$H_{produced} = (1.35 \times t) \text{ kJ}$$

The heat required for 1 kg of water to raise it temperature from 15 °C to boiling point may be given as:

$$H_{absorbed} = m_{water}\, s\, \Delta t = 1000 \times 1 \times (100 - 15) \times 4.2 = 357 \text{ kJ}$$

For boiling of water:

$$H_{produced} = H_{absorbed}$$

or, $(1.35 \times t) = 357$ or, $t = \dfrac{357}{1.35} = 264.44$ sec $= 4.407$ minute

Problem 3.254: *Each of the three resistors of 4 Ω, shown in the Fig. 3.136, can have a maximum power of 20 W* (*without melting*). *What may be the maximum power taken by the whole circuit?*

Solution: $R_1 = R_2 = R_3 = 4\ \Omega$, $P_{max} = 20$ W

The current corresponding to maximum power in the circuit may be given as:

$$I_{max} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{20}{4}} = \sqrt{5} \text{ A}$$

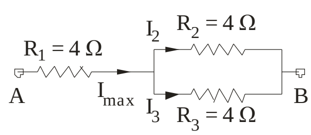

Fig. 3.136

The equivalent resistance of the whole circuit may be given as:

$$R_{eq} = 4 + (4 \parallel 4) = 4 + \frac{4 \times 4}{4 + 4} = 6\ \Omega$$

So, the total power dissipated in the whole circuit may be given as:

$$P_{Total} = I_{max}^2 \times R_{eq} = (\sqrt{5})^2 \times 6 = 30 \text{ W}$$

Alternatively:

The current passing through two parallel branches may be given as:

$$I_2 = I_3 = \frac{4}{4+4} \times \sqrt{5} = \frac{\sqrt{5}}{2} \text{ A}$$

The sum of power dissipated in two parallel resistors may be given as:

$$P_{2\&3} = 2 \times I_2^2 \times R_2 = 2 \times \left(\frac{\sqrt{5}}{2}\right)^2 \times 4 = 10 \text{ W}$$

So, the total power dissipated in the whole circuit may be given as:

$$P_{Total} = P_{max} + P_{2\&3} = 20 + 10 = 30 \text{ W}$$

Problem 3.255: *Determine the heat produced per minute in each of the resistor shown in the Fig. 3.137.*

Solution: $R_1 = 1\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 3\ \Omega$, $P_{max} = 20$ W

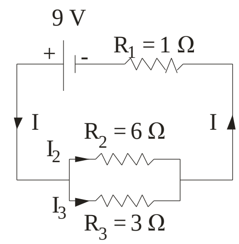

Fig. 3.137

The equivalent resistance of the circuit as seen by the battery may be given as:

$$R_{eq} = 1 + (6 \parallel 3) = 1 + \frac{6\times 3}{6+3} = 1 + 2 = 3\ \Omega$$

The current flowing through the circuit may be given as:

$$I_1 = \frac{E}{R_{eq}} = \frac{9}{3} = 3 \text{ A}$$

The current flowing through two parallel branches may respectively be given as:

$$I_2 = \frac{R_3}{R_2 + R_3} \times I = \frac{3}{6+3} \times 3 = 1 \text{ A}$$

and, $$I_3 = \frac{R_2}{R_2 + R_3} \times I = \frac{6}{6+3} \times 3 = 2 \text{ A}$$

So, the heat produced per minute in three resistors may respectively be given as:

$$H_1 = P_1 \times t = I_1^2 \times R_1 \times t = (3)^2 \times 1 \times 60 = 540 \text{ J/minute}$$

and, $$H_2 = P_2 \times t = I_2^2 \times R_2 \times t = (1)^2 \times 6 \times 60 = 360 \text{ J/minute}$$

and, $$H_3 = P_3 \times t = I_3^2 \times R_3 \times t = (2)^2 \times 3 \times 60 = 720 \text{ J/minute}$$

Problem 3.256: *Determine the current drawn from the battery of emf 15 V and internal resistance 0.5 Ω in the circuit shown in the Fig. 3.138. Also determine the power dissipated in the 6 Ω resistor.* [IIT]

Solution: $E = 15$ V, $r = 0.5\ \Omega$

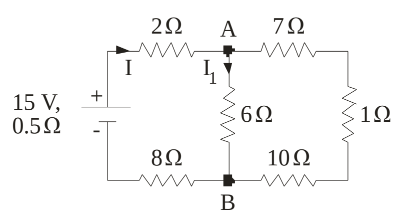

Fig. 3.138

The equivalent resistance of the (7 Ω, 1 Ω and 10 Ω) branch across points *A* and *B* may be given as:

$$R_{7\,\Omega,\,1\,\Omega,\,10\,\Omega} = 7 + 1 + 10 = 18\ \Omega$$

The equivalent resistance across the points *A* and *B* may be given as:

$$R_{AB} = \frac{6\times 18}{6+18} = 4.5\ \Omega$$

The equivalent resistance of the circuit as seen by the battery may be given as:

$$R_{eq} = 2 + R_{AB} + 8 = 2 + 4.5 + 8 = 14.5\ \Omega$$

So, the current drawn from the battery may be given as:

$$I = \frac{E}{R_{eq} + r} = \frac{15}{14.5 + 0.5} = 1 \text{ A}$$

The current flowing through the 6 Ω resistor may be given as:

$$I_{6\,\Omega} = \frac{18}{6 + 18} \times 1 = 0.75 \text{ A}$$

So, the power dissipated in the 6 Ω resistor may be given as:

$$P_{6\,\Omega} = I_{6\,\Omega}^2 \times 6 = (0.75)^2 \times 6 = 3.375 \text{ W}$$

Problem 3.257: *The rate of heat production in 4 Ω resistor is 40 cal/sec due to current flowing through it, in the circuit shown in the Fig. 3.139. Determine the rate at which heat is being produced in the 2 Ω resistor.*

Solution: $P_1 = 40$ cal/sec

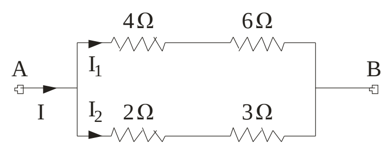

Fig. 3.139

Let I be the total current in the circuit, then the ratio of currents I_2 and I_1 may be given as:

$$\frac{I_2}{I_1} = \frac{4 + 6}{2 + 3} = \frac{10}{5} = 2$$

The ratio of heat produced in the two resistances (2 Ω and 4 Ω) may be given as:

$$\frac{P_2}{P_1} = \frac{I_2^2 \times 2}{I_1^2 \times 4} = \left(\frac{I_2}{I_1}\right)^2 \times \frac{1}{2} = (2)^2 \times \frac{1}{2} = 2$$

So, $P_2 = 2 \times P_1 = 2 \times 40 = 80$ cal/sec

Problem 3.258: *The 2 Ω resistor shown in the Fig. 3.140 is immersed into a calorimeter containing water. The heat capacity of the calorimeter together with water is 2000 JK^{-1}. i) If the circuit is active for 30 minutes, what would be the rise in the temperature of the water? ii) Suppose the 6 Ω resistor gets burnt. What would be the rise in temperature of the water in the next 30 minutes?*

Solution: The equivalent resistance of the circuit as seen by the battery may be given as:

$$R_{eq} = 1 + (2 \parallel 6) = 1 + \frac{2 \times 6}{2 + 6} = 1 + \frac{3}{2} = \frac{5}{2}\ \Omega$$

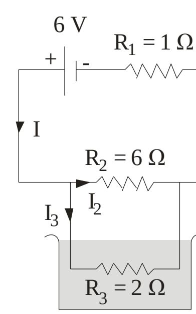

Fig. 3.140

The current drawn from the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{6}{(5/2)} = 2.4 \text{ A}$$

The current flowing through 2 Ω resistor may be given as:

$$I_{2\,\Omega} = \frac{6}{2 + 6} \times 2.4 = 1.8 \text{ A}$$

The heat produced by the resistor in 30 minutes may be given as:

$$H_{\text{produced}} = P_{2\,\Omega} \times t = (1.8)^2 \times 2 \times 30 \times 60 = 11.664 \text{ kJ}$$

The temperature rise of the calorimeter may be given as:

$$\Delta t = \frac{11.664 \times 10^3}{2000} = 5.832\ °\text{K} = 5.832\ °\text{C}$$

If the resistor 6 Ω is burnt, the equivalent resistance of the circuit as seen by the battery may be given as:

$$R_{eq} = 1 + 2 = 3\ \Omega$$

The current flowing through the 2 Ω resistor may now be given as:

$$I = \frac{E}{R_{eq}} = \frac{6}{3} = 2\ \text{A}$$

The heat produced by the resistor in 30 minutes may be given as:

$$H_{\text{produced}} = P_{2\,\Omega} \times t = (2)^2 \times 2 \times 30 \times 60 = 14.4\ \text{kJ}$$

The temperature rise of the calorimeter may be given as:

$$\Delta t = \frac{14.4 \times 10^3}{2000} = 7.2\ °\text{K} = 7.2\ °\text{C}$$

Problem 3.259: *Three resistors R_1, R_2 and R_3, each of 240 Ω, are connected across a 120 V supply, as shown in the Fig. 3.141. Determine: i) the potential difference across each resistor, ii) the total heat developed across the three resistors in 1 minute.*

Solution: $R_1 = R_2 = R_3 = 240\ \Omega$, $E = 120$ V

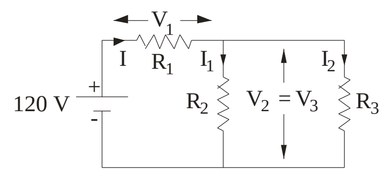

Fig. 3.141

The equivalent resistance of the circuit as seen by the battery may be given as:

$$R_{eq} = R_1 + (R_2 \parallel R_3) = 240 + \frac{240 \times 240}{240 + 240}$$

$$= 240 + 120 = 360\ \Omega$$

The current drawn from the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{120}{360} = \frac{1}{3}\ \text{A}$$

The current flowing through the R_2 and R_3 may be given as:

$$I_2 = I_3 = \frac{240}{240 + 240} \times \frac{1}{3} = \frac{1}{6}\ \text{A}$$

The potential difference across the resistor R_1, R_2 and R_3 may respectively be given as:

$$V_1 = I \times R_1 = \frac{1}{3} \times 240 = 80\ \text{V}$$

and, $$V_2 = V_3 = I_2 \times R_2 = \frac{1}{6} \times 240 = 40\ \text{V}$$

The total heat developed by three resistors in one minute may be given as:

$$H = (P_1 + P_2 + P_3) \times t = (I^2 R_1 + I_2^2 R_2 + I_3^2 R_3) \times t$$

$$= \left[\left(\frac{1}{3}\right)^2 \times 240 + \left(\frac{1}{6}\right)^2 \times 240 + \left(\frac{1}{6}\right)^2 \times 240\right] \times 60 = 2400\ \text{Joules}$$

Problem 3.260: *A heating coil is connected in series with a resistor R. The coil is immersed in a liquid of mass 2 kg and specific heat 0.5 cal gm^{-1} °C^{-1}. A potential difference of 200 V is applied and the* *temperature of the liquid is found to increase by 60 °C in 20 minutes. If R is removed, the same rise in temperature is reached in 15 minutes. Determine the value of R.*

Solution: m_{liquid} = 2 kg, s = 0.5 cal gm^{-1} °C^{-1}, V = 200 V

ΔT = 60 °C, t_1 = 20 minutes, t_2 = 15 minutes

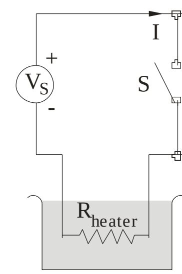

Fig. 3.142

The potential difference across the heater coil, while switch (S) is open, may be given as:

$$V_{\text{heater.1}} = \frac{R_{heater}}{R + R_{heater}} \times 200 \text{ V}$$

The potential difference across the heater coil, while switch (S) is closed, may be given as:

$$V_{\text{heater.2}} = 200 \text{ V}$$

The heat balance equation for the second case may be written as:

$$H_{\text{produced}} = H_{\text{absorbed}}$$

or, $$\frac{V_{heater.2}^2}{R_{heater}} \times t_2 = m_{\text{liquid}} \times s \times \Delta t \times 4.2$$

So, $$R_{\text{heater}} = \frac{V_{heater.2}^2}{m_{liquid} \times s \times \Delta t \times 4.2} \times t_2 = \frac{(200)^2}{2000 \times 0.5 \times 60 \times 4.2} \times 15 \times 60 = 142.857 \ \Omega$$

The heat produced in two cases is equal, as the temperature rise of the same amount of liquid is same in two cases.

So, $$\frac{H_1}{H_2} = \frac{(V_{heater.1}^2 / R_{heater}) \times t_1}{(V_{heater.2}^2 / R_{heater}) \times t_2} = \frac{V_{heater.1}^2 \times t_1}{V_{heater.2}^2 \times t_2} = 1$$

or, $$V_{heater.1}^2 \times t_1 = V_{heater.2}^2 \times t_2$$

or, $$\left(\frac{R_{heater}}{R + R_{heater}} \times 200\right)^2 \times 20 = (200)^2 \times 15$$

or, $$\frac{R_{heater}}{R + R_{heater}} = \sqrt{\frac{15}{20}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

or, $$\frac{R + R_{heater}}{R_{heater}} = \frac{2}{\sqrt{3}}$$

or, $$\frac{R}{R_{heater}} = \frac{2}{\sqrt{3}} - 1 = 0.1547$$

or, $$\text{R} = 0.1547 \times R_{\text{heater}} = 0.1547 \times 142.857 = 22.1 \ \Omega$$

Problem 3.261: *A house is fitted with two electric lamps, each of 100 W; one heater of resistance 110 Ω; and two fans each consuming 0.25 A. If electric energy is supplied at 200 V and each appliance works for 5 hours per day, determine the monthly bill for an electric tariff of Rs. 4.00 per unit.*
[Punjab 1997-98]

Solution: *Electric Lamps* = 2 × (100 W), *Heater* = 1 × (110 Ω), *Fans* = 2 × (0.25 A)

V = 200 V, Working = 5 hours/day, Electricity tariff = Rs. 4.00 per unit

The energy consumed by these appliances in a month may be given as:

$$E = (P_{\text{Lamps}} + P_{\text{Heater}} + P_{\text{Fans}}) \times t \times 30$$

$$= \left[2\times100+1\times\frac{(200)^2}{110}+2\times(200\times0.25)\right] \times 5 \times 30$$

$$= (200 + 363.64 + 100) \times 150 = 99546 \text{ Wh} = 99.546 \text{ kWh}$$

So, the total cost of energy for the month may be given as:

Cost = 99.546 × 4 = Rs. 398.18 ≈ Rs. 398.20

Problem 3.262: *An electric kettle has two coils. When one coil is switched ON, it takes 5 minutes to boil the water and when second coil is switched ON, it takes 10 minutes to boil the water. How long will it take to boil the water, when both the coils are used in series?*

Solution: t_1 = 5 minutes, t_2 = 10 minutes

Let us assume that the resistances of two coils be R_1 and R_2.

The heat produced by two coils for boiling the same quantity of water may respectively be given as:

$$H_1 = \frac{P_1\times t_1}{4.2} = \frac{(V)^2\times t_1}{4.2\times R_1} = \frac{(V)^2\times5\times60}{4.2\times R_1}$$

and, $$H_2 = \frac{P_2\times t_2}{4.2} = \frac{(V)^2\times t_2}{4.2\times R_2} = \frac{(V)^2\times10\times60}{4.2\times R_2}$$

Since, $H_1 = H_2$

or, $$\frac{(V)^2\times5\times60}{4.2\times R_1} = \frac{(V)^2\times10\times60}{4.2\times R_2}$$

or, $$\frac{R_2}{R_1} = \frac{10}{5} = 2$$

Both coils are connected in series:

The equivalent resistance of two coils may be given as, $R_{eq} = R_1 + R_2$

The heat produced by the combination may be given as:

$$H_{\text{series}} = \frac{P\times t_{series}\times60}{4.2} = \frac{(V)^2\times t_{series}\times60}{4.2\times(R_1+R_2)}$$

Since, this is the heat required to boil the water,

So, $H_{\text{series}} = H_1$

or, $$\frac{(V)^2\times t_{series}\times60}{4.2\times(R_1+R_2)} = \frac{(V)^2\times5\times60}{4.2\times R_1}$$

or, $$t_{\text{series}} = 5\times\left(\frac{R_1+R_2}{R_1}\right) = 5\times\left(1+\frac{R_2}{R_1}\right) = 5\times(1+2) = 15 \text{ min}$$

Problem 3.263: *A series battery of 6 lead accumulators, each of emf 2 V and internal resistance of 0.25 Ω are charged by a 230 V d.c. mains. A resistance of 53 Ω is connected in series with the battery to* *limit charging current. Determine: i) the power supplied by the d.c. mains, ii) the power dissipated as heat. Account for the difference in two cases.* [NCERT]

Solution: $E = 6 \times 2 = 12$ V, $r = 6 \times 0.25 = 1.5\ \Omega$, $V_S = 230$ V, $R_{ext} = 53\ \Omega$

The current through the circuit may be given as:

$$I = \frac{V_S - E}{R_{ext} + r} = \frac{230 - 12}{53 + 1.5} = 4 \text{ A}$$

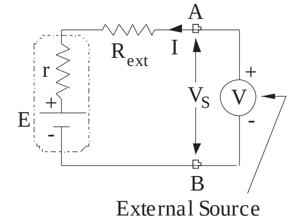

Fig. 3.143

The power supplied by the mains may be given as:

$$P_S = V_S \times I = 230 \times 4 = 920 \text{ W}$$

The power dissipated as heat may be given as:

$$P_{Loss} = I^2 \times (R_{ext} + r) = (4)^2 \times (53 + 1.5) = 872 \text{ W}$$

The difference of two powers (920 – 872 = 48 W) is stored in the cells (battery) as chemical energy due to charging of the cells (battery).

Problem 3.264: *A storage battery of emf 8 V and internal resistance 1 Ω is being charged by a 120 V d.c. mains using a 15 Ω resistor in series with the charging circuit. Determine: i) the current in the circuit, ii) terminal voltage across the battery during the charging, iii) chemical energy stored in the battery during 5 minutes.* [CBSE 2000-01, 2007-08]

Solution: $E = 8$ V, $r = 1\ \Omega$, $V_S = 120$ V, $R_{ext} = 15\ \Omega$, $t = 5$ minutes

The current through the circuit may be given as:

$$I = \frac{V - E}{R_{ext} + r} = \frac{120 - 8}{15 + 1} = 7 \text{ A}$$

The terminal voltage of the battery during the charging may be given as:

$$V_t = V_S - I \times R_{ext} = 120 - 7 \times 15 = 15 \text{ V}$$
$$= E + I \times r = 8 + 7 \times 1 = 15 \text{ V}$$

The power supplied by the mains may be given as:

$$P_S = V_S \times I = 120 \times 7 = 840 \text{ W}$$

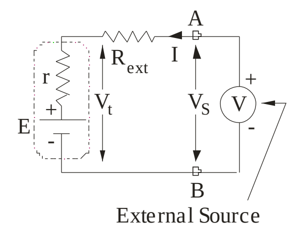

Fig. 3.144

The power dissipated as heat may be given as:

$$P_{Loss} = I^2 \times (R_{ext} + r) = (7)^2 \times (15 + 1) = 735 \text{ W}$$

The power stored in the battery may be given as:

$$P_{stored} = P_S - P_{Loss} = 840 - 735 = 105 \text{ W}$$

So, the energy stored in the battery in 5 minutes may be given as:

$$U = P_{stored} \times t = 105 \times 5 \times 60 = 31500 \text{ J} = 105 \times \frac{5}{60} = 8.75 \text{ Wh}$$

Problem 3.265: *Three cells, of identical emf e and internal resistance r, are connected in series across a variable resistor. The graph shown in the Fig. 3.145 is drawn for the variation of terminal voltage (V_t) w.r.t. the output current (i). Determine: a) the emf of each cell. ii) the value of current for maximum power dissipation in the circuit, iii) the internal resistance of each cell.*

Solution: $E = 3\,e$, $r_{int.} = 3\,r$

Emf of the battery (E) may be read from the graph (V_t –vs– I) corresponding to $I = 0$ A.

i.e. $E = 3\,e = 6$ V

So, emf of each cell may be given as:

$$e = \frac{6}{3} = 2 \text{ V}$$

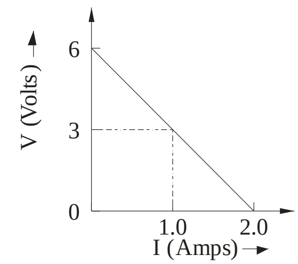

Fig. 3.145

The terminal voltage V_t is 3 V corresponding to the current $I = 1$ A.

Since, the terminal voltage at any current may be given as:

$$V_t = E - I \times r_{int}$$

or, $$r_{int} = \frac{E - V_t}{I} = \frac{6-3}{1} = 3\ \Omega$$

So, $$r = \frac{r_{int}}{3} = \frac{3}{3} = 1\ \Omega$$

The condition of maximum power dissipation in the external resistance may be given according to "*Maximum Power Transfer Theorem*", which is given below:

$$R_{ext} = r_{int} = 3\ \Omega$$

So, the current corresponding to the maximum power dissipation in the external resistance may be given as:

$$I_{max} = \frac{E}{R_{ext} + r_{int}} = \frac{E}{2\,r_{int}} = \frac{6}{2 \times 3} = 1 \text{ A}$$

3.30 Kirchhoff's Laws: The German physicist *Kirchhoff* extended the Ohm's law to complex circuits and gave two laws to solve the complex circuits / networks conveniently. Before stating the laws, let us define some technical and specific terms related to electrical networks.

Electric Network / Circuit: *A set of several electrical elements* (*active and passive elements*) *connected together to do some useful work is known as electrical network.*

Junction / Node: *Any point in a circuit / network where two or more than two branches are meeting is known as a node / junction.*

Branch: *Any element or set of elements connected between two junctions / nodes is known as a branch.*

Loop or Mesh: *A loop is a closed path of several electrical elements connected together. A loop may contain several other smaller loops or meshes inside it.*

Kirchhoff's Current Law / Junction Rule / First Law: "*Algebraic sum of all the currents meeting at a node* (*junction*) *is always zero*", or in other words "*the sum of all incoming currents is always equal to the sum of all outgoing currents at a node* (*junction*)". **(*Nodal analysis*)**

$$+ I_1 + I_2 - I_3 + I_4 - I_5 = 0 \qquad (3.171.1)$$

or, $$I_1 + I_2 + I_4 = I_3 + I_5 \qquad (3.171.2)$$

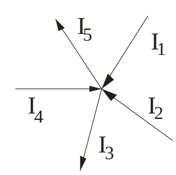

Fig. 3.146

Important: Note a thumb rule for writing down the *KCL* equation-

If incoming current is taken as (+ve), Outgoing current must be taken as (–ve).

Any arbitrary direction of current may be chosen for solving a problem, if the direction were assumed wrong initially, it automatically comes out to be negative on solving the problem.

Trick: If convenient, chose all the currents outgoing from a node / junction (assume that the node is at the highest potential in the network).

Kirchhoff's Voltage Law / Loop Rule / Second Law: "*Algebraic sum of all the voltages in a closed loop or mesh is always zero*". **(*Maxwell loop current method / Mesh Analysis*)**

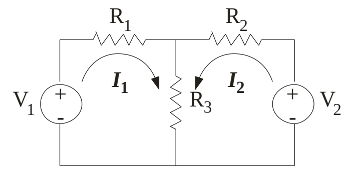

Fig. 3.147

$$+ V_1 - I_1 R_1 - (I_1 + I_2) R_3 = 0 \quad (3.172)$$

$$+ V_2 - I_2 R_2 - (I_1 + I_2) R_3 = 0 \quad (3.173)$$

Imp: Note few rules for writing down the *KVL* equation:

i) Voltage rise is taken as (+ve): - Moving from (–ve) plate of battery to (+ve) plate of battery.
- Moving in opposite direction of current in a resistance/passive element.

ii) Voltage drop is taken as (–ve): - Moving from (+ve) plate of battery to (–ve) plate of battery.
- Moving in the direction of current in a resistance/passive element.

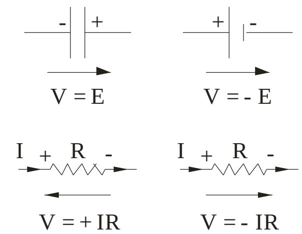

Fig. 3.148

Any arbitrary direction of current may be chosen for solving a problem, if the direction were assumed wrong initially, it automatically comes out to be negative on solving the problem.

Trick: If convenient, chose the directions of currents in such a way, that the current in the branch common between two loops / meshes is either downwards or upwards due to both the loops / meshes.

Problem 3.266: *A network PQRS is having following components in its branches: i) PQ has a battery of 4 V and negligible internal resistance, positive terminal connected to point P, ii) QR has a resistance of 60 Ω, iii) PS has a battery of 5 V and negligible internal resistance, positive terminal connected to point P, iv) RS has a resistance of 200 Ω. If a milli-ammeter, of 20 Ω resistance, is connected across the points P and R, determine the reading of the milli-ammeter.* [NCERT]

Solution: The required network is drawn in the Fig. 3.149. Assuming the directions of currents as shown in the figure (the central branch common to both meshes has same, towards right current due to both the meshes).

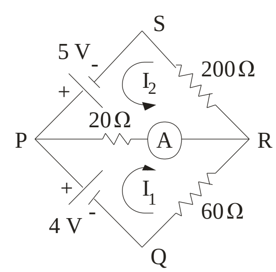

Fig. 3.149

Now applying *KVL* in mesh *PQRP*:

$$+ 4 - 20 \times (I_1 + I_2) - 60 I_1 = 0$$

or, $80 I_1 + 20 I_2 = 4$

or, $20 I_1 + 5 I_2 = 1 \quad (3.174)$

Now applying *KVL* in mesh *PRSP*:

$$+ 5 - 20 \times (I_1 + I_2) - 200 I_2 = 0$$

or, $20 I_1 + 220 I_2 = 5 \quad (3.175)$

Equation (3.175) – (3.174):

$215 I_2 = 4$ or, $I_2 = \dfrac{4}{215}$ A

Putting in equation (3.174):

$$I_1 = \frac{1-5I_2}{20} = \frac{1-5\times\frac{4}{215}}{20} = \frac{1-\frac{4}{43}}{20} = \frac{43-4}{43\times 20} = \frac{39}{860} \text{ A}$$

So reading of the ammeter may be given as:

$$I = I_1 + I_2 = \frac{4}{215} + \frac{39}{860} = \frac{16+39}{860} = \frac{55}{860} = \frac{11}{172} = 0.064 \text{ A}$$

Alternatively: Assume voltage at node *P* as zero volts, and at node *R* as *V* volts.

Now, the voltage at point *S*, $V_S = -5$ V

and, the voltage at point *Q*, $V_Q = -4$ V

Assuming all the branch currents going out from the node *R*, and applying *KCL* at node *R*:

$$\left[\frac{V-(-4)}{60} + \frac{V-(-5)}{200} + \frac{V-0}{20} = 0\right] \qquad \times 600$$

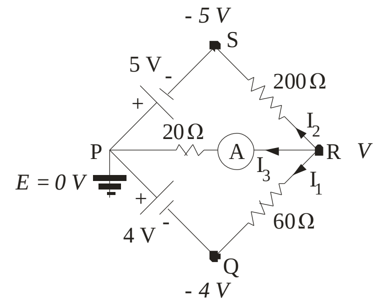

Fig. 3.150

or, $\quad 10\,V + 40 + 3\,V + 15 + 30\,V = 0$

or, $\quad V = -\dfrac{55}{43}$ V

Now, $I_3 = \dfrac{V-0}{20} = \dfrac{-(55/43)}{20} = -\dfrac{11}{172}$ A $= -0.064$ A

(–ve) sign is showing that the actual direction of flow of current is from point *P* to point *R*.

Problem 3.267: *Using Kirchhoff's laws in the electrical network shown in the Fig. 3.151, determine the currents in three resistors.* [CBSE 1993-94, 1999-2000]

Solution: Assuming the directions in two meshes, as shown in the figure, and applying *KVL* in two meshes:

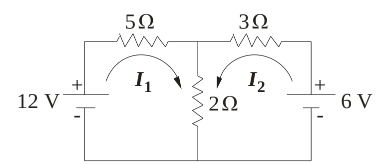

Fig. 3.151

Mesh 1:

$$+12 - 5\,I_1 - 2 \times (I_1 + I_2) = 0$$

$$7\,I_1 + 2\,I_2 = 12 \qquad (3.176)$$

Mesh 2:

$$+6 - 3\,I_2 - 2 \times (I_1 + I_2) = 0$$

$$2\,I_1 + 5\,I_2 = 6 \qquad (3.177)$$

Equation (3.177) × 7 – (3.176) × 2:

$$31\,I_2 = 18 \qquad \text{or,} \qquad I_2 = \frac{18}{31} \text{ A}$$

Putting in equation (3.176):

$$I_1 = \frac{12-2I_2}{7} = \frac{12-\frac{36}{31}}{7} = \frac{12\times 31-36}{7\times 31} = \frac{336}{7\times 31} = \frac{48}{31} \text{ A}$$

The current through 5 Ω resistor $= I_1 = \dfrac{48}{31}$ A

The current through 3 Ω resistor $= I_2 = \dfrac{18}{31}$ A

The current through 2 Ω resistor $= I_1 + I_2 = \dfrac{48}{31} + \dfrac{18}{31} = \dfrac{66}{31}$ A

Alternatively: There are only two nodes in the network as shown in the Fig. 3.152. Assume the node E at zero Volt and the voltage at another node as V. The assumed current directions are shown on the network. Applying KCL at the node V:

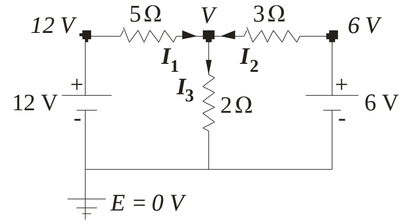

Fig. 3.152

$$\left[\frac{12-V}{5}+\frac{6-V}{3}=\frac{V-0}{2}\right] \qquad \times 30$$

or, $\quad 72 - 6\,V + 60 - 10\,V = 15\,V$

or, $\quad V = \dfrac{132}{31}$ V

The current through 5 Ω resistor $= I_1 = \dfrac{12-V}{5} = \dfrac{12-\left(\dfrac{132}{31}\right)}{5} = \dfrac{240}{5\times 31} = \dfrac{48}{31}$ A

The current through 3 Ω resistor $= I_2 = \dfrac{6-V}{3} = \dfrac{6-\left(\dfrac{132}{31}\right)}{3} = \dfrac{54}{3\times 31} = \dfrac{18}{31}$ A

The current through 2 Ω resistor $= I_3 == \dfrac{V-0}{2} = \dfrac{\left(\dfrac{132}{31}\right)-0}{2} = \dfrac{66}{31}$ A

Problem 3.268: *Using Kirchhoff's laws in the electrical network shown in the Fig. 3.153, determine the potential difference across 8 Ω resistor.* [CBSE 2005-06]

Solution: Assuming the directions in two meshes, as shown in the Fig. 3.153, and applying KVL in two meshes:

Mesh 1:

$$+\,4 - 8\,(I_1 + I_2) - 6\,I_1 - 2\,I_1 = 0$$

or, $\quad 16\,I_1 + 8\,I_2 = 4$

or, $\quad 4\,I_1 + 2\,I_2 = 1 \qquad (3.178)$

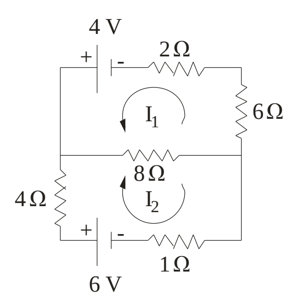

Fig. 3.153

Mesh 2:

$$+\,6 - 4\,I_2 - 8\,(I_1 + I_2) - 1\,I_2 = 0$$

or, $\quad 8\,I_1 + 13\,I_2 = 6 \qquad (3.179)$

Equation (3.179) – (3.178) × 2:

$9\,I_2 = 4 \qquad$ or, $\qquad I_2 = \dfrac{4}{9}$ A

Putting in equation (3.178):

$$I_1 = \frac{1-2I_2}{4} = \frac{1-\dfrac{8}{9}}{4} = \frac{9-8}{4\times 9} = \frac{1}{36}\ \text{A}$$

The current flowing through the 8 Ω resistor $= I_1 + I_2 = \dfrac{4}{9}+\dfrac{1}{36} = \dfrac{4\times 4+1}{36} = \dfrac{17}{36}$ A

So, the potential difference across the 8 Ω resistor may be given as:

$$V_{8\,\Omega} = \frac{17}{36} \times 8 = \frac{34}{9} \text{ V} = 3.778 \text{ V}$$

Alternatively: There are only two nodes in the network as shown in the Fig. 3.154. Assume the node *E* at zero Volt and the voltage at another node as *V*. The assumed current directions are shown on the network. Applying *KCL* at the node *V*:

$$\left[\frac{0-V+6}{1+4} + \frac{0-V+4}{6+2} = \frac{V-0}{8}\right] \times 40$$

or, $\quad -8\,V + 48 - 5\,V + 20 = 5\,V$

or, $\quad V = \frac{68}{18} = \frac{34}{9} \text{ V} = 3.778 \text{ V}$

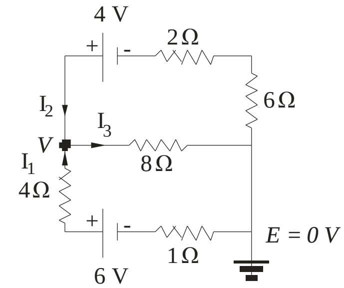

Fig. 3.154

So, the potential difference across the 8 Ω resistor may be given as:

$$V_{8\,\Omega} = V - 0 = \frac{34}{9} \text{ V} = 3.778 \text{ V}$$

Problem 3.269: *Two cells of emf 1.5 V and 2 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω. i) Draw the required circuit diagram. ii) Using Kirchhoff's laws determine:*

a) the current flowing through each branch of the circuit, b) the potential difference across the 5 Ω resistor. [CBSE 1994-95, 2004-05]

Solution: The required circuit diagram is drawn in the Fig. 3.155.

Assuming the directions in two meshes, as shown in the Fig. 3.155, and applying *KVL* in two meshes:

Mesh 1:

$$+1.5 - 1\,I_1 - 5\,(I_1 + I_2) = 0$$

or, $\quad 6\,I_1 + 5\,I_2 = 1.5 \qquad (3.180)$

Mesh 2:

$$+2 - 2\,I_2 - 5\,(I_1 + I_2) = 0$$

or, $\quad 5\,I_1 + 7\,I_2 = 2 \qquad (3.181)$

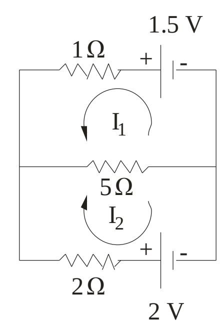

Fig. 3.155

Equation (3.181) × 6 – (3.180) × 5:

$$17\,I_2 = 4.5 \qquad \text{or,} \qquad I_2 = \frac{9}{34} \text{ A}$$

Putting in equation (3.181):

$$I_1 = \frac{2-7I_2}{5} = \frac{2-7\times\frac{9}{34}}{5} = \frac{68-63}{5\times 34} = \frac{1}{34} \text{ A}$$

The current through branch having 1.5 V battery = $I_1 = \frac{1}{34}$ A (right → left)

The current through branch having 2 V battery = $I_2 = \frac{9}{34}$ A (right → left)

The current through branch having 5 Ω resistor = $I_1 + I_2 = \frac{1+9}{34} = \frac{5}{17}$ A (left → right)

The potential difference across the 5 Ω resistor may be given as:

$$V_{5\,\Omega} = \frac{5}{17} \times 5 = \frac{25}{17} \text{ V} = 1.47 \text{ V}$$

Alternatively: There are only two nodes in the network as shown in the Fig. 3.156. Assume the node E at zero Volt and the voltage at another node as V. The assumed current directions are shown on the network. Applying KCL at the node V:

$$\left[\frac{1.5-V}{1} + \frac{2-V}{2} = \frac{V-0}{5}\right] \qquad \times 10$$

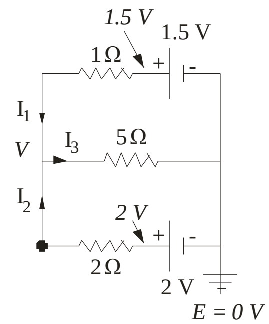

Fig. 3.156

or, $15 - 10\,V + 10 - 5\,V = 2\,V$

So, $V = \frac{25}{17}$ V = potential difference across the 5 Ω resistor

The current through branch having 1.5 V battery may be given as:

$$I_1 = \frac{1.5-V}{1} = \frac{\frac{3}{2} - \frac{25}{17}}{1} = \frac{1}{34} \text{ A (right → left)}$$

The current through branch having 2 V battery may be given as:

$$I_2 = \frac{2-V}{2} = \frac{2-\left(\frac{25}{17}\right)}{2} = \frac{9}{34} \text{ A (right → left)}$$

The current through branch having 5 Ω resistor may be given as:

$$I_3 = \frac{V-0}{5} = \frac{\left(\frac{25}{17}\right)-0}{5} = \frac{5}{17} \text{ A (left → right)}$$

Problem 3.270: *Determine the current flowing through each battery in the circuit shown in the Fig. 3.157. Also determine the potential difference across each battery.*

Solution: Assuming the directions in two meshes, as shown in the Fig. 3.157, and applying KVL in two meshes:

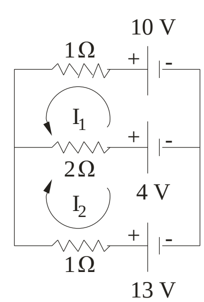

Fig. 3.157

Mesh 1:

$$+\,10 - 1\,I_1 - 2\,(I_1 + I_2) - 4 = 0$$

or, $3\,I_1 + 2\,I_2 = 6$ (3.182)

Mesh 2:

$$+\,13 - 1\,I_2 - 2\,(I_1 + I_2) - 4 = 0$$

or, $2\,I_1 + 3\,I_2 = 9$ (3.183)

Equation (3.183) × 3 – (3.182) × 2:

$5\,I_2 = 15$ or, $I_2 = \frac{15}{5} = 3$ A

Putting in equation (3.183):

$$I_1 = \frac{9-3I_2}{2} = \frac{9-3\times3}{2} = 0 \text{ A}$$

The current flowing through 10 V battery = I_1 = 0 A

The current Flowing through 4 V battery = $I_1 + I_2 = 3 + 0 = 3$ A

The current flowing through 13 V battery = I_2 = 3 A

The terminal voltage across three batteries will be same and may be given as:

$$V_t = 13 - 3\times1 = 10 \text{ V}$$
$$= 10 - 0\times1 = 10 \text{ V}$$
$$= 4 + 3\times2 = 10 \text{ V}$$

Alternatively: There are only two nodes in the network as shown in the Fig. 3.158. Assume the node *E* at zero Volt and the voltage at another node as *V*. The assumed current directions are shown on the network. Applying *KCL* at the node *V*:

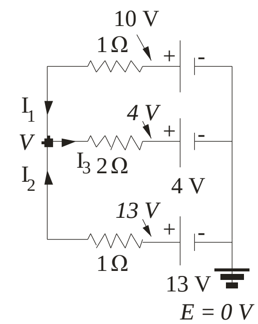

Fig. 3.158

$$\left[\frac{13-V}{1} + \frac{10-V}{1} = \frac{V-4}{2}\right] \times 2$$

or, $26 - 2V + 20 - 2V = V - 4$

or, $V = \frac{50}{5} = 10$ V

The current flowing through 10 V battery = $\frac{10-V}{1} = \frac{10-10}{1} = 0$ A

The current Flowing through 4 V battery = $\frac{V-4}{2} = \frac{10-4}{2} = 3$ A

The current flowing through 13 V battery = $\frac{13-V}{1} = \frac{13-10}{1} = 3$ A

The terminal voltage across three batteries = $V - 0 = V = 10$ V

Problem 3.271: *Three batteries of emf's 9 V, 10 V and 12 V with internal resistances 1 Ω, 2 Ω and 3 Ω respectively are connected in parallel with each other. The combination sends current through an external resistance of 6 Ω. Determine the current through the external resistance and each battery.*

Solution: There are only two nodes in the network as shown in the Fig. 3.159. Assume the node *E* at zero Volt and the voltage at another node as *V*. The assumed current directions are shown on the network. Applying *KCL* at the node *V*:

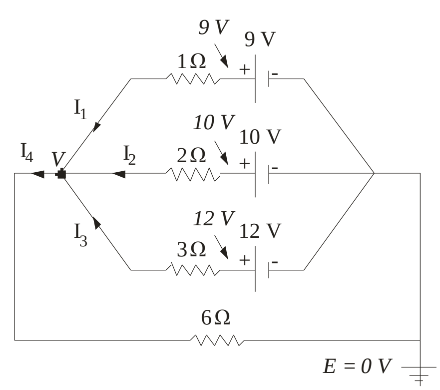

Fig. 3.159

$$\left[\frac{9-V}{1} + \frac{10-V}{2} + \frac{12-V}{3} = \frac{V-0}{6}\right] \times 18$$

or, $162 - 18V + 90 - 9V + 72 - 6V = 3V$

or, $V = \frac{324}{36} = 9$ V

The current through 6 Ω resistor may be given as:

$$I_4 = \frac{V-0}{6} = \frac{9-0}{6} = 1.5 \text{ A}$$

The current through the 9 V battery = $I_1 = \dfrac{9-V}{1} = \dfrac{9-9}{1} = 0$ A

The current through the 10 V battery = $I_2 = \dfrac{10-V}{2} = \dfrac{10-9}{2} = 0.5$ A

The current through the 12 V battery = $I_3 = \dfrac{12-V}{3} = \dfrac{12-9}{3} = 1$ A

Problem 3.272: *Determine the current in each branch of the network shown in the Fig. 3.160.*

Solution: There are four nodes in the network as shown in the Fig. 3.160. Assume the node E at zero Volt, the node at the top will be at 5 V due to the emf of battery connected there. The other two nodes are assumed at voltages V_1 and V_2, as shown in the figure. Now assuming the directions of current as shown in the figure and applying the KCL at two nodes with unknown voltages:

Node 1:

$$\left[\frac{V_2 - V_1 + 10}{1} = \frac{V_1 - 5}{4} + \frac{V_1 - 0}{4}\right] \qquad \times 4`$$

or, $4\,V_2 - 4\,V_1 + 40 = V_1 - 5 + V_1$

or, $6\,V_1 - 4\,V_2 = 45$ (3.184)

Node 2:

$$\left[\frac{5 - V_2}{2} = \frac{V_2 - V_1 + 10}{1} + \frac{V_2 - 0}{2}\right] \qquad \times 2$$

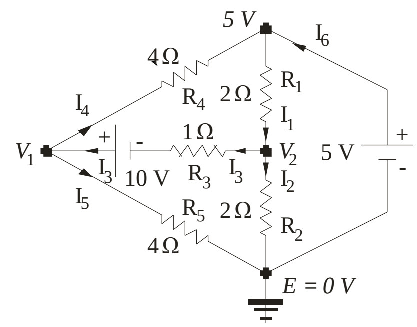

Fig. 3.160

or, $5 - V_2 = 2\,V_2 - 2\,V_1 + 20 + V_2$

or, $2\,V_1 - 4\,V_2 = 15$ (3.185)

Equation (3.184) – (3.185):

$4\,V_1 = 30$ or, $V_1 = \dfrac{30}{4} = 7.5$ V

Putting in equation (3.185):

$$V_2 = \frac{2V_1 - 15}{4} = \frac{2 \times 7.5 - 15}{4} = 0 \text{ V}$$

The current through resistor R_1, $I_1 = \dfrac{5-V_2}{2} = \dfrac{5-0}{2} = 2.5$ A

The current through resistor R_2, $I_2 = \dfrac{V_2-0}{2} = \dfrac{0-0}{2} = 0$ A

The current through resistor R_3, $I_3 = \dfrac{V_2-V_1+10}{1} = \dfrac{0-7.5+10}{1} = 2.5$ A

The current through resistor R_4, $I_4 = \dfrac{V_1-5}{4} = \dfrac{7.5-5}{4} = 0.625$ A

The current through resistor R_5, $I_5 = \dfrac{V_1 - 0}{4} = \dfrac{7.5 - 0}{4} = 1.875$ A

The current through the 5 V battery, $I_6 = I_1 - I_4 = 2.5 - 0.625 = 1.875$ A (*KCL*)

Problem 3.273: *The batteries E, F, G and H are having emf's of 2 V, 1 V, 3 V and 1 V respectively. Their internal resistances are 2 Ω, 1 Ω, 3 Ω and 1 Ω respectively. The circuit diagram having these batteries is shown in the Fig. 3.161. Determine: i) the potential difference across the 2 Ω resistor along the diagonal of the rectangle, ii) the potential difference across the terminals of the batteries G and H.* [CBSE 2003-04]

Solution: There are only two nodes in the network as shown in the Fig. 3.161. Assume the node *E* at zero Volt and the voltage at another node as *V*. The assumed current directions are shown on the network. Applying *KCL* at the node *V*:

$$\left[\frac{0 - V + 3 - 1}{3+1} = \frac{V - 0}{2} + \frac{V - 0 + 2 - 1}{2+1}\right] \quad \times 12$$

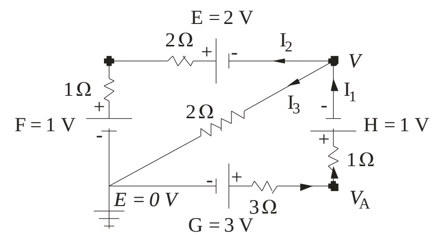

Fig. 3.161

or, $\quad -3V + 6 = 6V + 4V + 4$

or, $\quad V = \dfrac{2}{13}$ V

The potential difference across diagonal branch $= V - 0 = \dfrac{2}{13}$ V

The potential difference across the terminals of the battery *G* and *H* may be determined by applying *KCL* at point *A*:

$$\frac{0 - V_A + 3}{3} = \frac{V_A - \frac{2}{13} - 1}{1}$$

or, $$\left[\frac{3 - V_A}{3} = \frac{13V_A - 2 - 13}{13}\right] \quad \times 39$$

or, $\quad 39 - 13\,V_A = 39\,V_A - 45$

or, $\quad V_A = \dfrac{84}{52} = \dfrac{21}{13}$ V

The potential difference across the terminals of battery $G = V_A - 0 = \dfrac{21}{13}$ V = 1.615 V

The potential difference across the terminals of battery $H = V_A - V = \dfrac{21}{13} - \dfrac{2}{13}$ V $= \dfrac{19}{13}$ V

$= 1.462$ V

Problem 3.274: *In a Wheatstone bridge, P = 1 Ω, Q = 2 Ω, R = 2 Ω, S = 3 Ω and R_g = 4 Ω. Determine the current through the galvanometer in the unbalanced position of the bridge, when a battery of 2 V and internal resistance of 2 Ω is used across it as shown in the Fig. 3.162.*

Solution: There are four nodes in the network as shown in the Fig. 3.162. Assume the node *E* at zero Volt, the node at the right side will be at 0 V. The other three nodes are assumed at voltages V_1, V_2 and V_3, as shown in the figure. Now assuming the directions of current as shown in the figure and applying the *KCL* at three nodes with unknown voltages:

Node 1:

$$\left[\frac{2-V_1}{2} = \frac{V_1-V_2}{1} + \frac{V_1-V_3}{2}\right] \qquad \times 2$$

or, $2 - V_1 = 2\,V_1 - 2\,V_2 + V_1 - V_3$

or, $4\,V_1 - 2\,V_2 - V_3 = 2$ (3.186)

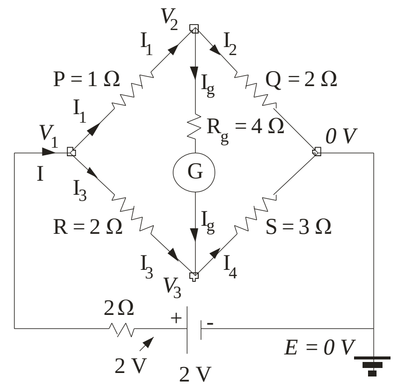

Fig. 3.162

Node 2:

$$\left[\frac{V_1-V_2}{1} = \frac{V_2-V_3}{4} + \frac{V_2-0}{2}\right] \qquad \times 4$$

or, $4\,V_1 - 4\,V_2 = V_2 - V_3 + 2\,V_2$

or, $4\,V_1 - 7\,V_2 + V_3 = 0$ (3.187)

Node 3:

$$\left[\frac{V_1-V_3}{2} + \frac{V_2-V_3}{4} = \frac{V_3-0}{3}\right] \qquad \times 12$$

or, $6\,V_1 - 6\,V_3 + 3\,V_2 - 3\,V_3 = 4\,V_3$

or, $6\,V_1 + 3\,V_2 - 13\,V_3 = 0$ (3.188)

Equation (3.186) – (3.187):

$5\,V_2 - 2\,V_3 = 2$ (3.189)

Equation (3.188) × 4 – (3.187) × 6:

$54\,V_2 - 58\,V_3 = 0$

or, $27\,V_2 - 29\,V_3 = 0$ (3.190)

Equation (3.189) × 29 – (3.190) × 2:

$91\,V_2 = 58$ or, $V_2 = \dfrac{58}{91}$ V

Putting in equation (3.189):

$$V_3 = \frac{5V_2-2}{2} = \frac{5\times\frac{58}{91}-2}{2} = \frac{290-182}{2\times 91} = \frac{54}{91}\ \text{V}$$

The current through the galvanometer $= I_g = \dfrac{V_2-V_3}{R_g} = \dfrac{\left(\frac{58}{91}-\frac{54}{91}\right)}{4} = \dfrac{1}{91}$ A

Problem 3.275: *In a Wheatstone bridge, $P = 100\ \Omega$, $Q = 10\ \Omega$, $R = 60\ \Omega$, $S = 5\ \Omega$ and $R_g = 15\ \Omega$. Determine the current through the galvanometer in the unbalanced position of the bridge, when a potential difference of 10 V is maintained across it, as shown in the Fig. 3.163.*

Solution: There are four nodes in the network as shown in the Fig. 3.163. Assume the node *E* at zero Volt, the node at the right side will be at 0 V and that on the left side is at 10 V due to the emf of battery connected there. The other two nodes are assumed at voltages V_1 and V_2, as shown in the figure. Now assuming the directions of current as shown in the figure and applying the *KCL* at two nodes with unknown voltages:

Node 1:

$$\left[\frac{10-V_1}{100}=\frac{V_1-V_2}{15}+\frac{V_1-0}{10}\right] \qquad \times 600$$

or, $60 - 6V_1 = 40V_1 - 40V_2 + 60V_1$

or, $106V_1 - 40V_2 = 60$

or, $53V_1 - 20V_2 = 30$ (3.191)

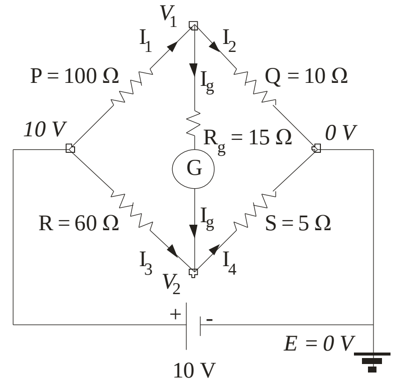

Fig. 3.163

Node 2:

$$\left[\frac{10-V_2}{60}+\frac{V_1-V_2}{15}=\frac{V_2-0}{5}\right] \qquad \times 60$$

or, $10 - V_2 + 4V_1 - 4V_2 = 12V_2$

or, $4V_1 - 17V_2 = -10$ (3.192)

Equation (3.191) × 4 – (3.192) × 53:

$$821\,V_2 = 650 \qquad \text{or,} \qquad V_1 = \frac{650}{821}\text{ V}$$

Putting in equation (3.192):

$$V_1 = \frac{17V_2-10}{4} = \frac{17\times\left(\frac{650}{821}\right)-10}{4} = \frac{11050-8210}{4\times 821} = \frac{710}{821}\text{ V}$$

The current through the galvanometer May be given as:

$$I_g = \frac{V_1-V_2}{R_g} = \frac{\left(\frac{710}{821}-\frac{650}{821}\right)}{15} = \frac{4}{821} = 4.872\text{ mA}$$

Problem 3.276: *Two cells of emf's 1.5 V and 2 V and internal resistances 2 Ω and 1 Ω respectively have their negative terminals connected with a wire of 6 Ω and positive terminals by a wire of 4 Ω resistance. A third wire of resistance 8 Ω is connected at the middle points of above two wires. Draw the circuit diagram and determine the potential difference across the 8 Ω resistor using Kirchhoff's laws.* [CBSE 1999-2000]

Solution: The required circuit is drawn in the Fig. 3.164.

There are only two nodes in the network as shown in the Fig. 3.164. Assume the node *E* at zero Volt and the voltage at another node as *V*. The assumed current directions are shown on the network. Applying *KCL* at the node *V*:

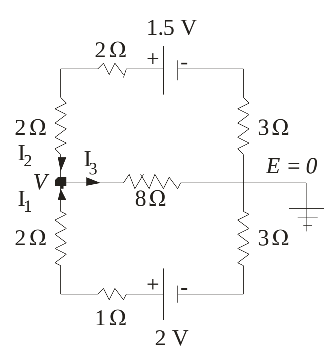

Fig. 3.164

$$\left[\frac{0-V+2}{3+1+2}+\frac{0-V+1.5}{3+2+2}=\frac{V-0}{8}\right] \qquad \times 168$$

or, $-28V + 56 - 24V + 36 = 21V$

or, $V = \dfrac{92}{73}\text{ V} = 1.26\text{ V}$

Problem 3.277: *Determine the current in each branch of the circuit shown in the Fig. 3.165.*

Solution: Assuming the directions of the current as shown in the figure and applying the *KVL* in three meshes of the circuit.

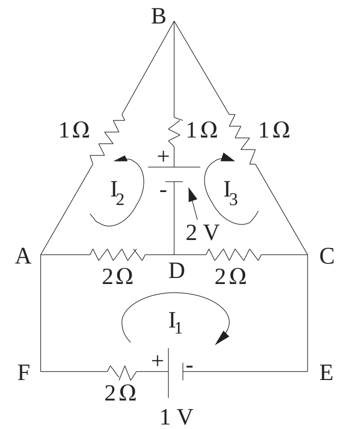

Fig. 3.165

Mesh 1:

$$+1 - 2I_1 - 2(I_1 + I_2) - 2(I_1 - I_3) = 0$$

$$6I_1 + 2I_2 - 2I_3 = 1 \quad (3.193)$$

Mesh 2:

$$+2 - 1(I_2 + I_3) - 1I_2 - 2(I_1 + I_2) = 0$$

$$2I_1 + 4I_2 + I_3 = 2 \quad (3.194)$$

Mesh 3:

$$+2 - 1(I_2 + I_3) - 1I_3 - 2(I_3 - I_1) = 0$$

$$2I_1 - I_2 - 4I_3 = -2 \quad (3.195)$$

Equation (3.194) × 3 – (3.193):

$$10I_2 + 5I_3 = 5 \quad (3.196)$$

Equation (3.194) – (3.195):

$$5I_2 + 5I_3 = 4 \quad (3.197)$$

Equation (3.196) – (3.197):

$$5I_2 = 1 \qquad \text{or,} \qquad I_2 = \frac{1}{5}\ \text{A}$$

Putting this value in equation (3.197):

$$I_3 = \frac{4 - 5I_2}{5} = \frac{4 - 5\left(\frac{1}{5}\right)}{5} = \frac{3}{5}\ \text{A}$$

Putting values of I_2 and I_3 in equation (3.193):

$$I_1 = \frac{1 - 2I_2 + 2I_3}{6} = \frac{1 - 2\left(\frac{1}{5}\right) + 2\left(\frac{3}{5}\right)}{6} = \frac{5 - 2 + 6}{30} = \frac{9}{30} = \frac{3}{10}\ \text{A}$$

So, $I_{AB} = I_2 = \frac{1}{5} = 0.2$ A

$$I_{BC} = I_3 = \frac{3}{5} = 0.6\ \text{A}$$

$$I_{CD} = I_3 - I_1 = \frac{3}{5} - \frac{3}{10} = \frac{3}{10} = 0.3\ \text{A}$$

$$I_{DA} = I_1 + I_2 = \frac{3}{10} + \frac{1}{5} = \frac{5}{10} = 0.5\ \text{A}$$

$$I_{FE} = I_1 = \frac{3}{10} = 0.3\ \text{A}$$

Problem 3.278: *Determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.166. Each resistance R = 10 Ω.*

Solution: Applying an imaginary d.c. source of constant terminal voltage V across the points A and B.

The point B is at zero Volts and point A is at V Volts due to the d.c. source applied there. Assuming the voltage of two other nodes as V_1 and V_2 and applying KCL at both the nodes:

Node 1:

$$\frac{V - V_1}{R} = \frac{V_1 - V_2}{R} + \frac{V_1 - 0}{2R}$$

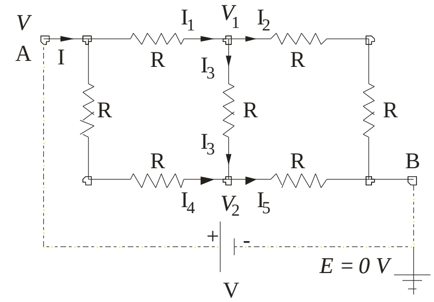

Fig. 3.166

or, $2\,V - 2\,V_1 = 2\,V_1 - 2\,V_2 + V_1$

or, $5\,V_1 - 2\,V_2 = 2\,V$ (3.198)

Node 2:

$$\frac{V - V_2}{2R} + \frac{V_1 - V_2}{R} = \frac{V_2 - 0}{R}$$

or, $V - V_2 + 2\,V_1 - 2\,V_2 = 2\,V_2$

or, $2\,V_1 - 5\,V_2 = -\,V$ (3.199)

Equation (3.198) × 2 – (3.199) × 5:

$$21\,V_2 = 9\,V \qquad \text{or,} \qquad V_2 = \frac{9}{21}\,V = \frac{3}{7}\,V$$

Putting this value in equation (3.198):

$$V_1 = \frac{2V + 2V_2}{5} = \frac{2V + 2\times\left(\frac{3}{7}V\right)}{5} = \frac{(14+6)V}{5\times 7} = \frac{4}{7}\,V$$

So, the current from the d.c. source may be given as:

$$I = \frac{V - V_1}{R} + \frac{V - V_2}{2R} = \frac{V - \left(\frac{4}{7}V\right)}{R} + \frac{V - \left(\frac{3}{7}V\right)}{2R} = \frac{(14 - 8 + 7 - 3)V}{14R}$$

$$= \frac{10V}{14R} = \frac{V}{(14R/10)} = \frac{V}{(7R/5)} = \frac{V}{R_{eq}}$$

So, $R_{eq} = \frac{7}{5}\,R = \frac{7}{5} \times 10 = 14\ \Omega$

Problem 3.279: *Determine the current in each branch of the network shown in the Fig. 3.167 using Kirchhoff's voltage law.*

Solution: Assuming the directions of the currents as shown in the figure, and applying the KVL in three meshes:

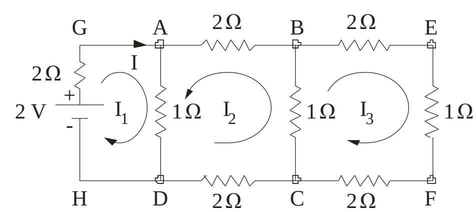

Fig. 3.167

Mesh 1:

$$+\,2 - 2\,I_1 - 1\,(I_1 + I_2) = 0$$

or, $3\,I_1 + I_2 = 2$ (3.200)

Mesh 2:

$$-1\,(I_1 + I_2) - 2\,I_2 - 1\,(I_2 + I_3) - 2\,I_2 = 0$$

or, $$I_1 + 6\,I_2 + I_3 = 0 \qquad (3.201)$$

Mesh 3:

$$(-2 - 1 - 2)\,I_3 - 1\,(I_2 + I_3) = 0$$

or, $$I_2 + 6\,I_3 = 0 \qquad (3.202)$$

Equation (3.201) × 3 – (3.200):

$$17\,I_2 + 3\,I_3 = -2 \qquad (3.203)$$

Equation (3.202) × 17 – (3.203):

$$99\,I_3 = 2 \qquad \text{or,} \qquad I_3 = \frac{2}{99}\text{ A}$$

Putting this value in equation (3.202):

$$I_2 = -6\,I_3 = -\frac{12}{99}\text{ A}$$

[(– ve) sign shows clockwise direction.]

Putting this value in equation (3.200):

$$I_1 = \frac{2 - I_2}{3} = \frac{2 - \left(-\frac{12}{99}\right)}{3} = \frac{198 + 12}{3 \times 99} = \frac{70}{99}\text{ A}$$

Now, $I_{AB} = I_2 = \frac{12}{99}$ A, $\qquad I_{BC} = I_2 - I_3 = \frac{12}{99} - \frac{2}{99} = \frac{10}{99}$ A

$I_{CD} = I_2 = \frac{12}{99}$ A, $\qquad I_{DA} = I_1 - I_2 = \frac{70}{99} - \frac{12}{99} = \frac{58}{99}$ A

$I_{BEFC} = I_3 = \frac{2}{99}$ A, $\qquad I_{GH} = I_1 = \frac{70}{99}$ A

Problem 3.280: *Determine the current in each branch of the network shown in the Fig. 3.168 using Kirchhoff's laws. Also determine potential difference across the points AC and BC.*

Solution: There are only three nodes in the network, assuming node *B* to be at zero Volts, the node *A* will be at 110 V due to the d.c. source connected there. Assuming the voltage at node *C* as *V* Volts and applying *KCL* at node *C*:

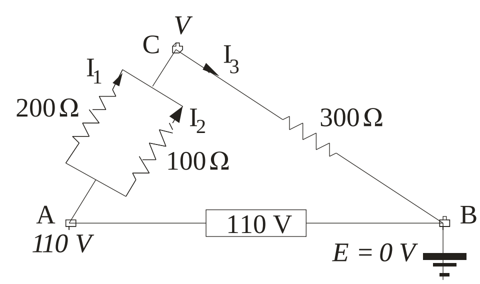

Fig. 3.168

$$\left[\frac{110 - V}{200} + \frac{110 - V}{100} = \frac{V - 0}{300}\right] \times 600$$

or, $$330 - 3\,V + 660 - 6\,V = 2\,V$$

or, $$V = \frac{990}{11} = 90\text{ V}$$

So, $I_1 = \frac{110 - V}{200} = \frac{110 - 90}{200} = 0.1$ A, $\qquad I_2 = \frac{110 - V}{100} = \frac{110 - 90}{100} = 0.2$ A

$$I_3 = \frac{V-0}{300} = \frac{90-0}{300} = 0.3 \text{ A}$$

and, $V_{AC} = V_A - V_C = 110 - 90 = 20$ V, $\quad V_{BC} = V_B - V_C = 90 - 0 = 90$ V

Problem 3.281: *A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω. Determine the equivalent resistance of the network and the current along each edge of the cube.* [NCERT]

Solution: The required network and the current distribution is shown in the Fig. 3.169. The reader may observe the pattern of current division between various branches due to the symmetry of the cube. The current entering at node *A* is 6*I* Amps, dividing into three equal parts of 2 *I* Amps for three edges. Then at each of the nodes *B*, *E* and *D*; this 2 *I* Amps current is being divided in two equal parts of *I* Amps each. Then at the nodes *C*, *F* and *H* two incoming currents of *I* Amps are meeting to make 2 *I* Amps of current in the next branch. Then at the node *G*, three 2 *I* Amps currents are meeting to make it 6 *I* Amps and to leave the cube towards the d.c. source applied.

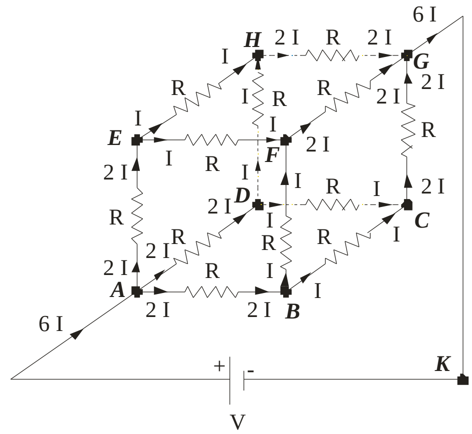

Fig. 3.169

Applying *KVL* in the mesh *ABCGKA*:

$$+ V - 2\,I \times R - I \times R - 2\,I \times R = 0$$

or, $\quad V = 5\,I\,R$

So, $\quad R_{eq} = \dfrac{V}{6I} = \dfrac{5IR}{6I} = \dfrac{5}{6}R = \dfrac{5}{6} \times 1 = \dfrac{5}{6}\ \Omega$

The current supplied by the source may be given by the relationship:

$$6\,I = \frac{V}{R_{eq}} = \frac{10}{(5/6)} = 12 \text{ A} \qquad \text{or,} \qquad I = \frac{12}{6} = 2 \text{ A}$$

(The current through the each branch may now easily be read from the network)

Problem 3.282: *Twelve wires of resistance r Ω are connected to form the structure of a cube. Determine the equivalent resistance of the cube across the two vertices of same edge.*

Solution: The required network and the current distribution is shown in the Fig. 3.170. The reader may easily recognize the current distribution due to the symmetry of the cube.

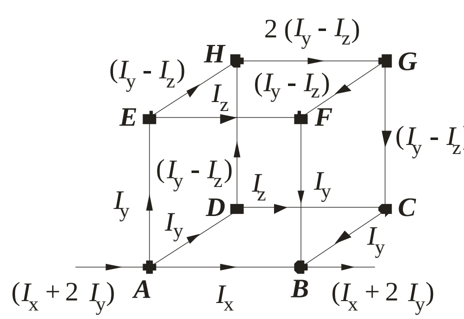

Fig. 3.170

Applying *KVL* in the mesh *CDHGC*:

$$+ I_z\,r - (I_y - I_z)\ r - 2\,(I_y - I_z)\,r - (I_y - I_z)\,r = 0$$

or, $\quad 5\,I_z\,r = 4\,I_y\,r$

or, $\quad I_z = \dfrac{4}{5}\,I_y$

Applying *KVL* in the mesh *ABCDA*:

$$+ I_x\,r - I_y\,r - I_z\,r - I_y\,r = 0$$

or, $\quad I_x = 2\,I_y + I_z = 2\,I_y + \dfrac{4}{5}\,I_y = \dfrac{14}{5}\,I_y \qquad$ or, $\quad I_y = \dfrac{5}{14}\,I_x$

The potential difference across points *AB* may be given as:

$$V_{AB} = (I_x + 2\ I_y) \times R_{eq} = I_x \times r$$

or, $$\left(I_x + 2 \times \frac{5}{14} I_x \right) \times R_{eq} = I_x \times r$$

or, $$\frac{12}{7} \times R_{eq} = r, \qquad \text{or,} \qquad R_{eq} = \frac{7}{12}\ r\ \Omega$$

Problem 3.283: *Eleven wires of resistance r Ω are connected to form the structure of an incomplete cube as shown in the Fig. 3.171. Determine the equivalent resistance of the cube across the two vertices of missing edge (points AB).*

Solution: The required network and the current distribution is shown in the Fig. 3.171. The reader may easily recognize the current distribution due to the symmetry of the cube.

Applying *KVL* in mesh *ABCDA*:

$$+ V - I_x\, r - I_y\, r - I_x\, r = 0$$

or, $$2\ I_x\, r + I_y\, r = V$$

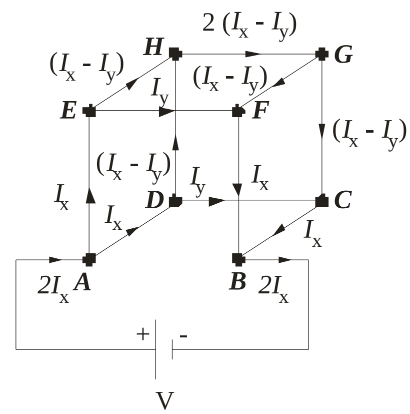

Fig. 3.171

Applying *KVL* to mesh *EFGHE*:

$$I_y\, r - (I_x - I_y)\, r - 2\ (I_x - I_y)\, r - (I_x - I_y)\, r = 0$$

or, $$5\ I_y\, r = 4\ I_x\, r \qquad \text{or,} \qquad I_y = \frac{4}{5}\ I_x$$

According to Ohm's Law:

$$V = 2\ I_x \times R_{eq}$$

or, $$2\ I_x\, r + I_y\, r = 2\ I_x \times R_{eq}$$

or, $$R_{eq} = \left(\frac{2 I_x + I_y}{2 I_x} \right) \times r = \left(\frac{2 I_x + \frac{4}{5} I_x}{2 I_x} \right) \times r = \frac{7}{5}\ r = 1.4\ r\ \Omega$$

Problem 3.284: *Twelve wires of resistance 1 Ω each are connected to form the structure of a cube. Determine the equivalent resistance of the cube across the two corners of a diagonal of one face of the cube.*

Solution: The required network and the current distribution is shown in the Fig. 3.172, for an imaginary battery across the points *E* and *B*, which sends a current of 1 A into the network. The reader may easily recognize the current distribution due to the symmetry of the cube.

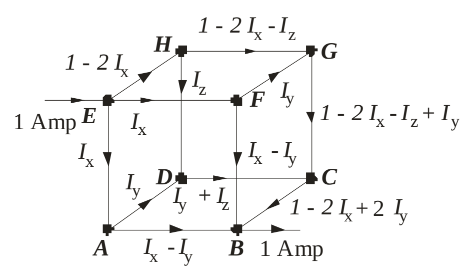

Fig. 3.172

Applying *KVL* in the mesh *ADHEA*:

$$I_y\, r + I_x\, r - (1 - 2\ I_x)\, r - I_z\, r = 0$$

or, $$3\ I_x + I_y - I_z = 1 \qquad (3.204)$$

Applying *KVL* in mesh *BCGFB*:

$$(I_x - I_y)\, r - I_y\, r - (1 - 2\ I_x - I_z + I_y)\, r - (1 - 2\ I_x + 2\ I_y)\, r = 0$$

or, $$5\ I_x - 5\ I_y + I_z = 2 \qquad (3.205)$$

Applying *KVL* in mesh *ABCDA*:

$$(I_x - I_y)\, r - I_y\, r - (I_y + I_z)\, r - (1 - 2\, I_x + 2\, I_y)\, r = 0$$

or, $$3\, I_x - 5\, I_y - I_z = 1 \qquad (3.206)$$

Equation (3.204) – (3.206):

$$6\, I_y = 0 \qquad \text{or,} \qquad I_y = 0 \text{ A}$$

Putting this value in equation (3.204) and (3.205):

$$3\, I_x - I_z = 1 \qquad (3.207)$$

$$5\, I_x + I_z = 2 \qquad (3.208)$$

Equation (3.207) + (3.208):

$$8\, I_x = 3 \qquad \text{or,} \qquad I_x = \frac{3}{8} \text{ A}$$

Putting this value in equation (3.207):

$$I_z = 3\, I_x - 1 = \frac{9}{8} - 1 = \frac{1}{8} \text{ A}$$

Now, the potential difference across the points E and B may be given as:

$$V_{\mathrm{EB}} = V_{\mathrm{EA}} + V_{\mathrm{AB}} = I_x \times r + (I_x - I_y) \times r = (2\, I_x - I_y) \times r$$

$$= \left(2 \times \frac{3}{8} - 0\right) \times 1 = \frac{3}{4} \text{ V}$$

The equivalent resistance across the points E and B may be given as:

$$R_{\mathrm{eq}} = \frac{V_{EB}}{1} = \frac{3}{4}\ \Omega$$

Problem 3.285: *Determine the equivalent resistance of the network, shown in the Fig. 3.173, across its terminals A and B.*

Solution: Let us connect an imaginary battery of constant terminal voltage V across the points A and B. The voltage at point B is zero Volts and at point A is V Volts. Assume the potentials at points F, G and C as V_1, V_2 and V_3 respectively. Now, apply the KCL at three nodes with unknown voltages.

Node 1:

$$\frac{V - V_1}{r} = \frac{V_1 - V_2}{r} + \frac{V_1 - V_3}{r}$$

or, $$V - V_1 = V_1 - V_2 + V_1 - V_3$$

or, $$3\, V_1 - V_2 - V_3 = V \qquad (3.209)$$

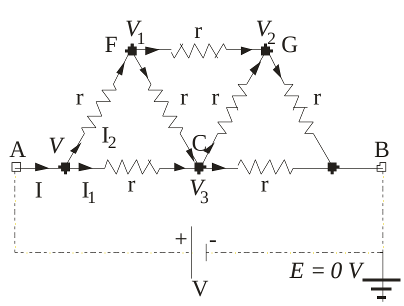

Fig. 3.173

Node 2:

$$\frac{V_1 - V_2}{r} + \frac{V_3 - V_2}{r} = \frac{V_2 - 0}{r}$$

or, $$V_1 - V_2 + V_3 - V_2 = V_2$$

or, $$V_1 - 3\, V_2 + V_3 = 0 \qquad (3.210)$$

Node 3:

$$\frac{V-V_3}{r}+\frac{V_1-V_3}{r}=\frac{V_3-V_2}{r}+\frac{V_3-0}{r}$$

or, $V - V_3 + V_1 - V_3 = V_3 - V_2 + V_3$

or, $V_1 + V_2 - 4\,V_3 = -\,V$ (3.211)

Equation (3.209) + (3.211):

$4\,V_1 - 5\,V_3 = 0$ (3.212)

Equation (3.209) × 3 – (3.210):

$8\,V_1 - 4\,V_3 = 3\,V$

or, $4\,V_1 - 2\,V_3 = 1.5\,V$ (3.213)

Equation (3.213) – (3.212):

$3\,V_3 = 1.5\,V$ or, $V_3 = \frac{1.5}{3}\,V = \frac{1}{2}\,V$

Putting this value in equation (3.212):

$$V_1 = \frac{5}{4}\,V_3 = \frac{5}{4}\times\frac{1}{2}\,V = \frac{5}{8}\,V$$

Now the current I may be given as:

$$I = \frac{V-V_1}{r}+\frac{V-V_3}{r} = \frac{V-\left(\frac{5}{8}\right)V}{2}+\frac{V-\left(\frac{1}{2}\right)V}{2} = \frac{8V-5V}{16}+\frac{2V-V}{4}$$

$$= \left(\frac{3}{16}+\frac{1}{4}\right)V = \frac{7}{16}\times V = \frac{V}{(16/7)} = \frac{V}{R_{eq}}$$

So, $R_{eq} = \frac{16}{7}\ \Omega$

Problem 3.286: *Determine the equivalent resistance across the points A and B in the network shown in the Fig. 3.174.*

Solution: Let us connect an imaginary battery of constant terminal voltage V across the points A and B. The voltage at point B is zero Volts and at point A is V Volts. Assume the potentials at two other nodes as V_1 and V_2 respectively. Now, apply the KCL at two nodes with unknown voltages.

Node 1:

$$\left[\frac{V-V_1}{r}=\frac{V_1-V_2}{r}+\frac{V_1-0}{R}\right] \times r\,R$$

or, $R\,V - R\,V_1 = R\,V_1 - R\,V_2 + r\,V_1$

or, $(2\,R + r)\,V_1 - R\,V_2 = R\,V$ (3.214)

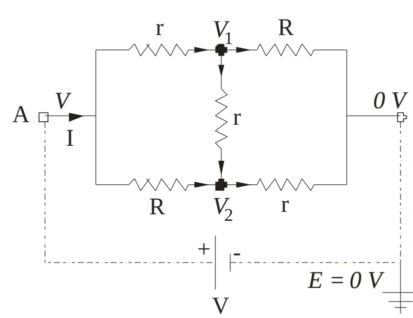

Fig. 3.174

Node 2:

$$\left[\frac{V-V_2}{R}+\frac{V_1-V_2}{r}=\frac{V_2-0}{r}\right] \times r\,R$$

or, $r\,V - r\,V_2 + R\,V_1 - R\,V_2 = R\,V_2$

or, $R\,V_1 - (2\,R + r)\,V_2 = -\,r\,V$ (3.215)

Equation (3.214) × R – (3.215) × $(2\,R + r)$:

$$[(2\,R + r)^2 - R^2]\,V_2 = [R^2 + (2\,R + r)\,r]\,V$$

or, $$V_2 = \frac{R^2 + 2Rr + r^2}{4R^2 + r^2 + 4Rr - R^2}\,V = \frac{(R+r)^2}{3R^2 + r^2 + 4Rr}\,V$$

Equation (3.214) × $(2\,R + r)$ – (3.215) × R:

$$[(2\,R + r)^2 - R^2]\,V_1 = [(2\,R + r)\,R + r\,R]\,V$$

or, $$V_1 = \frac{2R^2 + rR + rR}{4R^2 + r^2 + 4Rr - R^2}\,V = \frac{2R(R+r)}{3R^2 + r^2 + 4Rr}\,V$$

Now the current I drawn from the voltage source may be given as:

$$I = \frac{V - V_1}{r} + \frac{V - V_2}{R} = \frac{V - \left(\dfrac{2R(R+r)}{3R^2 + r^2 + 4Rr}\right)V}{r} + \frac{V - \left(\dfrac{(R+r)^2}{3R^2 + r^2 + 4Rr}\right)V}{R}$$

$$= \frac{(3R^2 + r^2 + 4Rr - 2R^2 - 2Rr)V}{r(3R^2 + r^2 + 4Rr)} + \frac{(3R^2 + r^2 + 4Rr - R^2 - r^2 - 2Rr)V}{R(3R^2 + r^2 + 4Rr)}$$

$$= \frac{(R^2 + r^2 + 2Rr)V}{r(3R^2 + r^2 + 4Rr)} + \frac{(2R^2 + 2Rr)V}{R(3R^2 + r^2 + 4Rr)}$$

$$= \frac{[R(R+r)^2 + 2r\,R(R+r)]V}{r\,R(3R^2 + r^2 + 4Rr)} = \frac{(R+r)[R(R+r) + 2r\,R]V}{r\,R(3R^2 + r^2 + 3Rr + Rr)}$$

$$= \frac{(R+r)(R^2 + 3r\,R)V}{r\,R[3R(R+r) + r(R+r))} = \frac{R(R+r)(R+3r)V}{r\,R(R+r)(3R+r)} = \frac{(R+3r)V}{r\,(3R+r)}$$

$$= \frac{V}{\left[\dfrac{r\,(3R+r)}{(R+3r)}\right]} = \frac{V}{R_{eq}}$$

So, $$R_{eq} = \frac{r\,(3R+r)}{(R+3r)}$$

Problem 3.287: *Determine the current in each resistor in the circuit shown in the Fig. 3.175. The internal resistances of the batteries are negligible.*

Solution: Assume voltage at node P as zero Volts, and at node R as V volts.

Now, the voltage at point S, $V_S = 2$ V

and, the voltage at point Q, $V_Q = 1$ V

Assuming all the branch currents at the node R, and applying KCL at node R:

$$\left[\frac{2 - V}{1} + \frac{1 - V}{1} = \frac{V - 0}{2}\right] \qquad \times 2$$

or, $4 - 2V + 2 - 2V = V$

or, $V = \frac{6}{5}$ V

Now, $I_1 = \frac{1-V}{1} = \frac{1-(6/5)}{1} = -\frac{1}{5}$ A = – 0.2 A

(–ve) sign is showing that the actual direction of flow of current is from point R to point Q.

and, $I_2 = \frac{2-V}{1} = \frac{2-(6/5)}{1} = \frac{4}{5}$ A = 0.8 A

and, $I_3 = \frac{V-0}{2} = \frac{(6/5)-0}{2} = \frac{3}{5}$ A = 0.6 A

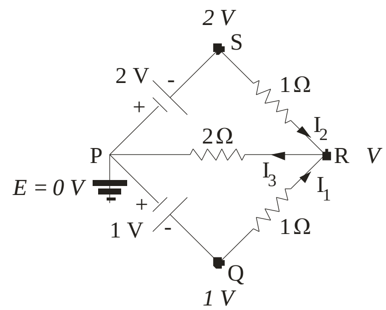

Fig. 3.175

Problem 3.288: *Determine the current I_1, I_2 and I_3 in the network shown in the Fig. 3.176.*

Solution: Assume voltage at node E as zero Volts, and applying KCL at node V:

$$\left[\frac{0-V+80}{1+20} + \frac{0-V}{30} = \frac{V-0+45}{1+40}\right] \quad \times 8610$$

or, $-410V + 32800 - 287V = 210V + 9450$

or, $V = \frac{23350}{907} = 25.744$ V

So, $I_1 = \frac{0-V}{30} = \frac{0-25.744}{30} = 0.858$ A

and, $I_2 = \frac{0-V+80}{1+20} = \frac{0-25.744+80}{1+20} = 2.584$ A

and, $I_3 = \frac{V-0+45}{1+40} = \frac{25.744-0+45}{1+40} = 1.725$ A

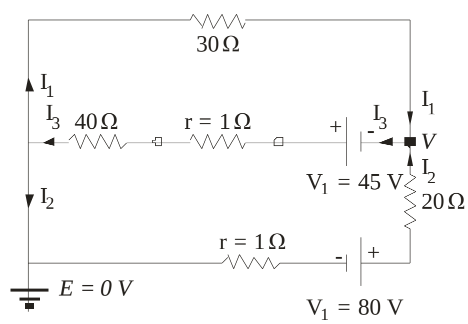

Fig. 3.176

Problem 3.289: *Determine the current I_1, I_2 and I_3 in the network shown in the Fig. 3.177, using Kirchhoff's Laws.* [CBSE 1998-99]

Solution: Assume voltage at node E as zero Volts, and applying KCL at node V:

$$\left[\frac{0-V+24}{2} = \frac{V-0-27}{6} + \frac{V-0}{4}\right] \quad \times 12$$

or, $-6V + 144 = 2V - 54 + 3V$

or, $V = \frac{198}{11} = 18$ V

So, $I_1 = \frac{0-V+24}{2} = \frac{0-18+24}{2} = 3$ A

and, $I_2 = \frac{V-0-27}{6} = \frac{18-0-27}{6} = -1.5$ A

and, $I_3 = \frac{V-0}{4} = \frac{18-0}{4} = 4.5$ A

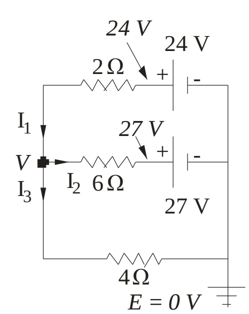

Fig. 3.177

Problem 3.290: *The circuit diagram shown in the Fig. 3.178 has two cells E_1 and E_2 with emf's 4 V and 2 V respectively, each one having an internal resistance of 2 Ω. The external resistance is of 8 Ω. Determine the magnitude and direction of current flowing through each cells.* [ISCE 1997-98]

Solution: Assume voltage at node E as zero Volts, and applying KCL at node V:

$$\left[\frac{4-V}{2}+\frac{2-V}{2}=\frac{V-0}{8}\right] \qquad \times 8$$

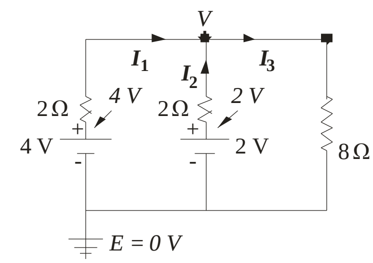

Fig. 3.178

or, $16 - 4V + 8 - 4V = V$

or, $V = \dfrac{24}{9}$ V

So, $I_1 = \dfrac{4-V}{2} = \dfrac{4-\left(\dfrac{24}{9}\right)}{2} = \dfrac{36-24}{2\times 9} = \dfrac{2}{3}$ A

and, $I_2 = \dfrac{2-V}{2} = \dfrac{2-\left(\dfrac{24}{9}\right)}{2} = \dfrac{18-24}{2\times 9} = -\dfrac{1}{3}$ A

Problem 3.291: *A series combination of n cells is shown in the Fig. 3.179. The internal resistances of the cells are related to their emf's as $r_i = \alpha E_i$, where α is a constant. Determine: i) the current through the circuit, ii) the potential difference across the terminals of the i^{th} cell.*

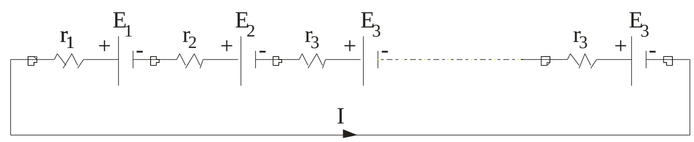

Fig. 3.179

Solution: The current flowing through the n series connected batteries without any external resistance may be given as:

$$I = \frac{E_1+E_2+E_3+\ldots\ldots\ldots\ldots+E_n}{r_1+r_2+r_3+\ldots\ldots\ldots\ldots+r_n} = \frac{E_1+E_2+E_3+\ldots\ldots\ldots+E_n}{\alpha E_1+\alpha E_2+\alpha E_3+\ldots\ldots\ldots+\alpha E_n}$$

$$= \frac{E_1+E_2+E_3+\ldots\ldots\ldots+E_n}{\alpha(E_1+E_2+E_3+\ldots\ldots\ldots+E_n)} = \frac{1}{\alpha}$$

The terminal voltage across i^{th} cell may be given as:

$$V_t = E_i - I \times r_i = E_i - \frac{1}{\alpha} \times \alpha E_i = E_i - E_i = 0 \text{ Volts}$$

Problem 3.292: *Two cells of emf's 3 V and 4 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel, so as to send the current in same direction through an external resistance of 5 Ω. i) Draw the circuit diagram, ii) Determine the current through each branch of the circuit and the potential difference across the 5 Ω resistance.* [CBSE 1994-95, 1995-96]

Solution: Assuming voltage at node E as zero Volts, and applying KCL at node V:

$$\left[\frac{3-V}{1}+\frac{4-V}{2}=\frac{V-0}{5}\right] \qquad \times 10$$

or, $30 - 10V + 20 - 5V = 2V$

or, $V = \dfrac{50}{17}$ V

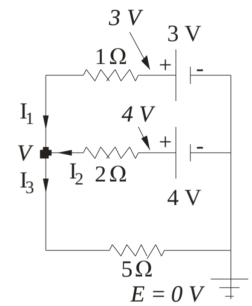

Fig. 3.180

So, $I_1 = \dfrac{3-V}{1} = \dfrac{3-\left(\dfrac{50}{17}\right)}{1} = \dfrac{1}{17}$ A

and, $I_2 = \dfrac{4-V}{2} = \dfrac{4-\left(\dfrac{50}{17}\right)}{2} = \dfrac{9}{17}$ A

and, $I_3 = \dfrac{V-0}{5} = \dfrac{\left(\dfrac{50}{17}\right)-0}{5} = \dfrac{10}{17}$ A

Problem 3.293: *Determine the potential difference across 2 kΩ resistor in the circuit shown in the Fig. 3.181. The internal resistances of the cells are negligible.*

Solution: Assuming voltage at node *E* as zero Volts, and applying *KCL* at node *V*:

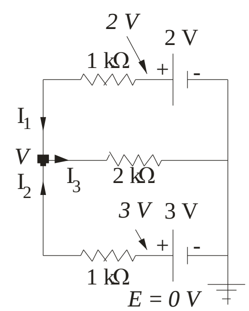

Fig. 3.181

$$\left[\frac{3-V}{1} + \frac{2-V}{1} = \frac{V-0}{2}\right] \qquad \times 2$$

or, $6 - 2V + 4 - 2V = V$

or, $V = \dfrac{10}{5} = 2$ V

Problem 3.294: *A network of resistors is connected across a battery of negligible internal resistance, as shown in the Fig. 3.182. Determine the equivalent resistance across the points A and D, and the value of current I_3.*

Solution: Assuming the voltage of the source as *V* Volts, voltage at node *E* as zero Volts, and applying *KCL* at node V_1:

$$\left[\frac{V-V_1}{2+2} + \frac{V-V_1}{2} = \frac{V_1-0}{2}\right] \qquad \times 4$$

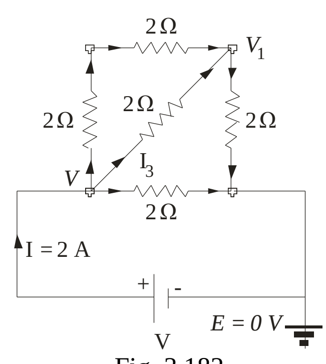

Fig. 3.182

or, $V - V_1 + 2V - 2V_1 = 2V_1$

or, $V_1 = \dfrac{3}{5}\ V$

Now, applying *KCL* at node having voltage *V*:

$$\left[\frac{V-V_1}{2+2} + \frac{V-V_1}{2} + \frac{V-0}{2} = 2\right] \qquad \times 4$$

or, $V - V_1 + 2V - 2V_1 + 2V = 8$

or, $5V - 3V_1 = 8$

or, $5V - 3 \times \dfrac{3}{5}\ V = 8$

or, $\dfrac{16}{5}\ V = 8$ or, $V = 8 \times \dfrac{5}{16} = \dfrac{5}{2}$ V

So, $R_{eq} = \dfrac{V}{I} = \dfrac{(5\backslash 2)}{2} = \dfrac{5}{4}\ \Omega = 1.25\ \Omega$

and, $I_3 = \dfrac{V - V_1}{2} = \dfrac{V-(3/5)V}{2} = \dfrac{2V}{2\times 5} = \dfrac{2\times(5/2)}{2\times 5} = \dfrac{1}{2}$ A = 0.5 A

Problem 3.295: *Three cells are connected in parallel with their like terminals connected together with wires of negligible resistances. If the emf of the cells are 2 V, 1 V and 4 V; and their internal resistances are 4 Ω, 3 Ω and 2 Ω respectively, determine the current flowing through each cell.*

Solution: The required circuit diagram is drawn in the Fig. 3.183. Assuming the voltage at node E as zero Volts and at the other node as V Volts, apply the KCL at node V:

$$\left[\frac{4-V}{2} = \frac{V-2}{4} + \frac{V-1}{3}\right] \qquad \times 12$$

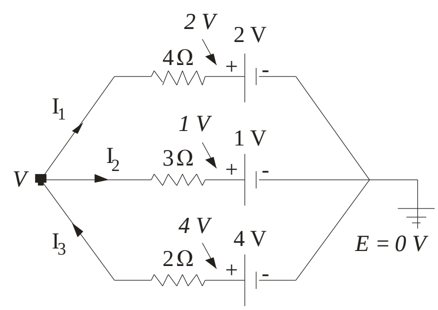

Fig. 3.183

or, $24 - 6\,V = 3\,V - 6 + 4\,V - 4$

or, $V = \dfrac{34}{13}$ V

$$I_1 = \frac{V-2}{4} = \frac{\left(\frac{34}{13}\right)-2}{4} = \frac{8}{4\times 13} = \frac{2}{13}\ \text{A}$$

$$I_2 = \frac{V-1}{3} = \frac{\left(\frac{34}{13}\right)-1}{3} = \frac{21}{3\times 13} = \frac{7}{13}\ \text{A}$$

$$I_3 = \frac{4-V}{2} = \frac{4-\left(\frac{34}{13}\right)}{2} = \frac{18}{2\times 13} = \frac{9}{13}\ \text{A}$$

Problem 3.296: *Determine i) the current in 6 Ω resistor, ii) terminal voltage across the 4 V battery in the circuit shown in the Fig. 3.184.*

Solution: Assuming the voltage at node E as zero Volts and at other node as V, and applying KCL at node V:

$$\left[\frac{0-V+4}{4+3+5} + \frac{0-V+2}{4+1+5} = \frac{V-0}{6}\right] \qquad \times 60$$

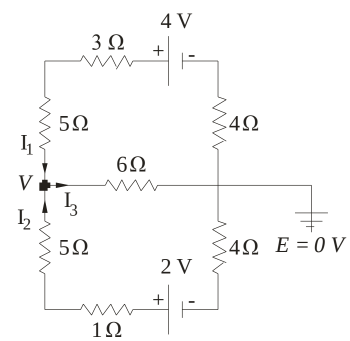

Fig. 3.184

or, $-\,5\,V + 20 - 6\,V + 12 = 10\,V$

or, $V = \dfrac{32}{21}$ V

The current in 6 Ω resistor may be given as:

$$I_3 = \frac{V-0}{6} = \frac{(32/21)-0}{6} = \frac{16}{63}\ \text{A}$$

The current flowing through the 4 V battery may be given as:

$$I_1 = \frac{0-V+4}{4+3+5} = \frac{0-\left(\frac{32}{21}\right)+4}{4+3+5} = \frac{52}{12\times 21} = \frac{13}{63}\ \text{A}$$

So, the terminal voltage across the 4 V battery may be given as:

$$V_t = E - I_1 \times r = 4 - \frac{13}{63} \times 3 = \frac{252-39}{63} = \frac{213}{63} = 3.381 \text{ V}$$

Problem 3.297: *Determine i) the current flowing through the 6 V battery, ii) potential difference across the points A and B, in the circuit shown in the Fig. 3.185.*

Solution: Assuming the voltage at node *E* as zero Volts. The reader may easily observe that the potential at the point *C* is 6 V, at the point *A* is (6 + 2 =) 8 V and at the point *B* is 4 V.

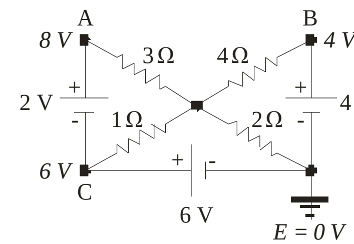

Fig. 3.185

So, the potential difference across the points A and B may be given as:

$$V_{AB} = 8 - 4 = 4 \text{ V}$$

The current through the 6 V battery may be given as:

$$I_{6\text{ V}} = \frac{6-0}{1+2} = 2 \text{ A (Since the potential difference across the battery is constant)}$$

Problem 3.298: *Determine i) the currents I_1, I_2 and I_3, ii) potential difference across the points B and E, in the circuit shown in the Fig. 3.185.*

Solution: Assuming the voltage at node *E* as zero Volts and at the node *B* as *V* Volts. Apply the *KCL* at node *B*:

$$\left[\frac{14-V}{4} = \frac{V-(-10)}{6} + \frac{V-0}{2}\right] \times 12$$

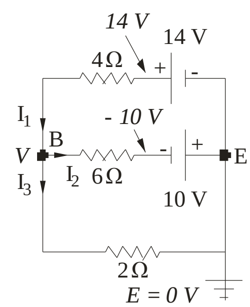

Fig. 3.186

or, $42 - 3V = 2V + 20 + 6V$

or, $V = \dfrac{22}{11} = 2 \text{ V}$

Now, $I_1 = \dfrac{14-V}{4} = \dfrac{14-2}{4} = 3 \text{ A}$

and, $I_2 = \dfrac{V-(-10)}{6} = \dfrac{2+10}{6} = 2 \text{ A}$

and, $I_3 = \dfrac{V-0}{2} = \dfrac{2-0}{2} = 1 \text{ A}$

The potential difference across the points *B* and *E*, $V_{BE} = V - 0 = V = 2$ V

Problem 3.299: *In the networks given in the Fig. 3.187 (a) and (b), identical cells each of emf E, are giving same current I. Determine the value of the resistors R_1 and R_2.*

Solution: The current through the 11 Ω resistor may be given as (current division in parallel branches):

$$I_{11\,\Omega} = \frac{R_2}{11 + R_2} \times I = \frac{I}{10}$$

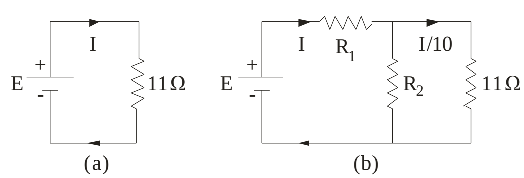

Fig. 3.187

or, $10 R_2 = 11 + R_2$

or, $R_2 = \dfrac{11}{9}\ \Omega$

The two combinations are drawing sane current from the battery, So, $R_1 + (R_2 \parallel 11) = 11$

or, $R_1 + \dfrac{\frac{11}{9}\times 11}{\frac{11}{9}+11} = 11$ or, $R_1 = 11 - \dfrac{121}{110} = \dfrac{1210-121}{110} = \dfrac{1089}{110} = 9.9\ \Omega$

Problem 3.300: *Calculate the potential difference between the points B and D in the Wheatstone's bridge shown in the Fig. 3.188.* [Roorkee 1989]

Solution: Assuming the voltage at node *C* as zero Volts, the node *A* is at 2 V due to the battery connected there. Now assume the voltage at point *B* as V_B and at the point *D* as V_D.

The voltage at the point *B* is equal to the potential difference across the 1 Ω resistor connected across the points *B* and *C*, and may be given as:

$$V_B = \frac{1}{1+1} \times 2 = 1\text{ V}$$

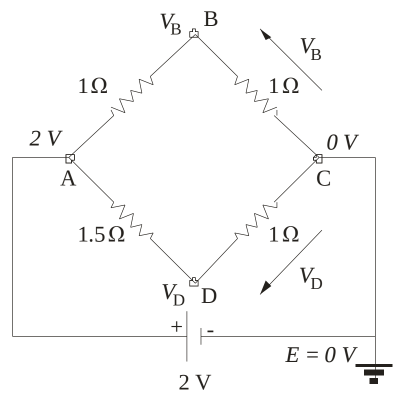

Fig. 3.188

The voltage at the point *D* is equal to the potential difference across the 1 Ω resistor connected across the points *D* and *C*, and may be given as:

$$V_D = \frac{1}{1+1.5} \times 2 = 0.8\text{ V}$$

So, the potential difference across the points *B* and *D* may be given as:

$$V_{BD} = 1 - 0.8 = 0.2\text{ V}$$

Problem 3.301: *Determine the reading of the ammeter in the circuit shown in the Fig. 3.189* (*a*). *What will be the reading, if the positions of cell and the ammeter are changed?*

Solution: Assume the node *E* at zero Volts and the directions of currents as shown in the Fig. 3.189 (a), now apply *KCL* at node *V*:

$$\left[\frac{5-V}{2} = \frac{V-0}{4} + \frac{V-0}{6}\right] \times 12$$

or, $30 - 6\,V = 3\,V + 2\,V$

or, $V = \dfrac{30}{11}$ V

So, $I_2 = \dfrac{V-0}{6} = \dfrac{\left(\frac{30}{11}\right)-0}{6} = \dfrac{5}{11}$ A

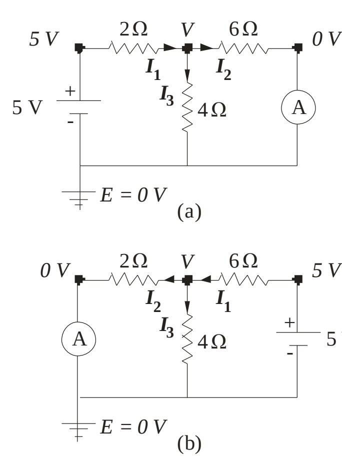

Fig. 3.189

Redraw the circuit by replacing ammeter and the cell as shown in the Fig. 3.189 (b). Assume the node *E* at zero Volts and the directions of currents as shown in the Fig. 3.189 (b), now apply *KCL* at node *V*:

$$\left[\frac{5-V}{6} = \frac{V-0}{4} + \frac{V-0}{2}\right] \times 12$$

or, $10 - 2\,V = 3\,V + 6\,V$

or, $V = \frac{10}{11}$ V So, $I_2 = \frac{V - 0}{2} = \frac{\left(\frac{10}{11}\right) - 0}{2} = \frac{5}{11}$ A

*This is known as "**Reciprocity Theorem**".*

Problem 3.302: *Determine the current flowing through the resistance connected across the points C and D. Also determine the equivalent resistance across the points A and B in the circuit shown in the Fig. 3.190.*

Solution: Assume the node *E* at zero Volts and the directions of currents as shown in the Fig. 3.190 (a). The voltages at points *C* and *D* are assumed as V_C and V_D respectively. Now apply the KCL at nodes *C* and *D* respectively.

Node C:

$$\left[\frac{14 - V_C}{5} = \frac{V_C - V_D}{5} + \frac{V_C - 0}{5+5}\right] \times 10$$

or, $28 - 2\,V_C = 2\,V_C - 2\,V_D + V_C$

or, $5\,V_C - 2\,V_D = 28$ (3.216)

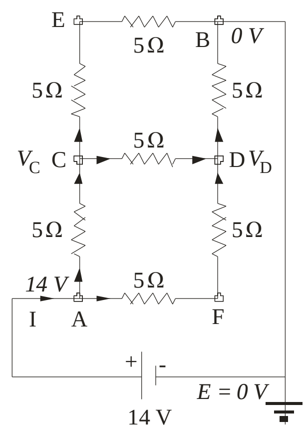

Fig. 3.190

Node D:

$$\left[\frac{14 - V_D}{5+5} + \frac{V_C - V_D}{5} = \frac{V_D - 0}{5}\right] \times 10$$

or, $14 - V_D + 2\,V_C - 2\,V_D = 2\,V_D$

or, $2\,V_C - 5\,V_D = -14$ (3.217)

Equation (3.216) × 2 – (3.217) × 5:

$21\,V_D = 126$

or, $V_D = \frac{126}{21} = 6$ V

Putting this value in equation (3.217):

$$V_C = \frac{5V_D - 14}{2} = \frac{5 \times 6 - 14}{2} = 8 \text{ V}$$

So, the current flowing through the 5 Ω resistance connected across the points *C* and *D* may be given as:

$$I_{CD} = \frac{V_C - V_D}{5} = \frac{8 - 6}{5} = 0.4 \text{ A}$$

Now, applying *KCL* at point *A*:

$$I = \frac{14 - V_C}{5} + \frac{14 - V_D}{5+5} = \frac{14 - 8}{5} + \frac{14 - 6}{5+5} = 2 \text{ A}$$

So, the equivalent resistance across the points *A* and *B* may be given as:

$$R_{eq} = \frac{E}{I} = \frac{14}{2} = 7\ \Omega$$

Problem 3.303: *A certain length of a uniform wire of resistance 2 Ω is bent into a circle and two points, a quarter of circumference apart, are connected to a battery of emf 4 V and internal resistance 1 Ω. Determine the current in various parts of the circuit.*

Solution: Assume the node E at zero Volts and the directions of currents as shown in the figure. Now apply KCL at the node having voltage V:

$$\left[\frac{4-V}{1}=\frac{V-0}{3}+\frac{V-0}{3+3+3}\right] \qquad \times 9$$

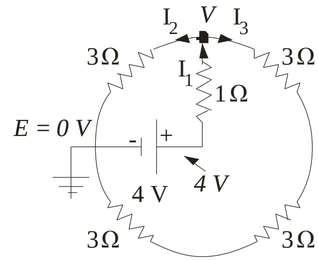

Fig. 3.191

or, $36 - 9\,V = 3\,V + V$

or, $V = \dfrac{36}{13}$ V

So, $I_1 = \dfrac{4-V}{1} = \dfrac{4-\left(\dfrac{36}{13}\right)}{1} = \dfrac{52-36}{1\times 13} = \dfrac{16}{13}$ A

and, $I_2 = \dfrac{V-0}{3} = \dfrac{\left(\dfrac{36}{13}\right)-0}{3} = \dfrac{12}{13}$ A

and, $I_3 = \dfrac{V-0}{3+3+3} = \dfrac{\left(\dfrac{36}{13}\right)-0}{9} = \dfrac{4}{13}$ A

Problem 3.304: *A uniform wire ABCDA of resistance 2 Ω is shown in the Fig. 3.192. Two wires AOC and BOD are along two perpendicular diameters of the circle, each having same resistance of 1 Ω. A battery of emf E and internal resistance r is connected across the points A and D. Determine the equivalent resistance of the network.*

Solution: Assume the directions of currents as shown in the figure. Since, we are interested to know the equivalent resistance of the network, so we are interested to evaluate the current I_1 only in the network.

Applying KVL in all the four meshes:

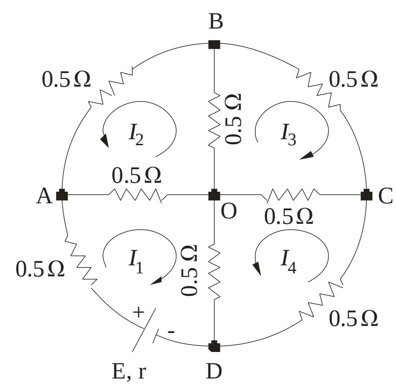

Fig. 3.192

Mesh 1:

$$E - I_1\,r - I_1 \times 0.5 - (I_1 + I_2) \times 0.5 - (I_1 + I_4) \times 0.5 = 0$$

or, $(1.5 + r)\,I_1 + 0.5\,I_2 + 0.5\,I_4 = E$

or, $(3 + 2\,r)\,I_1 + I_2 + I_4 = 2\,E$ (3.218)

Mesh 2:

$$-I_2 \times 0.5 - (I_1 + I_2) \times 0.5 - (I_2 + I_3) \times 0.5 = 0$$

or, $0.5\,I_1 + 1.5\,I_2 + 0.5\,I_3 = 0$

or, $I_1 + 3\,I_2 + I_3 = 0$ (3.219)

Mesh 3:

$$-I_3 \times 0.5 - (I_3 + I_4) \times 0.5 - (I_2 + I_3) \times 0.5 = 0$$

or, $0.5\,I_2 + 1.5\,I_3 + 0.5\,I_4 = 0$

or, $I_2 + 3\,I_3 + I_4 = 0$ (3.220)

Mesh 4:

$$-I_4 \times 0.5 - (I_3 + I_4) \times 0.5 - (I_1 + I_4) \times 0.5 = 0$$

or, $0.5\, I_1 + 0.5\, I_3 + 1.5\, I_4 = 0$

or, $I_1 + I_3 + 3\, I_4 = 0$ (3.221)

Equation (3.218) – (3.220):

$$(3 + 2\, r)\, I_1 - 3\, I_3 = 2\, E \quad (3.222)$$

Equation (3.221) – (3.220) × 3:

$$I_1 - 3\, I_2 - 8\, I_3 = 0 \quad (3.223)$$

Equation (3.219) + (3.223):

$$2\, I_1 - 7\, I_3 = 0 \quad (3.224)$$

Equation (3.222) × 7 – (3.224) × 3:

$$[7\,(3 + 2\, r) - 6]\, I_1 = 14\, E$$

or, $I_1 = \dfrac{14E}{15+14r} = \dfrac{E}{\left(\dfrac{15}{14}\right)+r}$

So, $R_{eq} + r = \left(\dfrac{15}{14}\right)+r$ or, $R_{eq} = \dfrac{15}{14}\ \Omega$

Problem 3.305: *One ampere of current enters the junction A of a Wheatstone Bridge ABCD with AB = 2 Ω, BC = 2 Ω, AC = 4 Ω, CD = 2 Ω and R_g = 4 Ω (galvanometer across B and C). Determine the reading of the galvanometer.*

Solution: Assume the directions of currents as shown in the figure. Now, apply *KVL* to two meshes:

Mesh 1:

$$-I_1 \times 2 - I_g \times 4 + (1 - I_1) \times 4 = 0$$

or, $6\, I_1 + 4\, I_g = 4$

or, $3\, I_1 + 2\, I_g = 2$ (3.225)

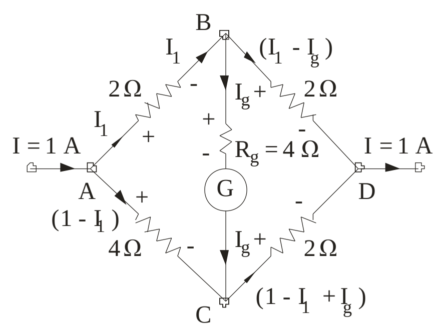

Fig. 3.193

Mesh 2:

$$-I_g \times 4 - (1 - I_1 + I_g) \times 2 + (I_1 - I_g) \times 2 = 0$$

or, $4\, I_1 - 8\, I_g = 2$

or, $2\, I_1 - 4\, I_g = 1$ (3.226)

Equation (3.225) × 2 – (3.226) × 3:

$(4 + 12)\, I_g = 4 - 3$ or, $I_g = \dfrac{1}{16} = 0.0625\text{ A} = 62.5\text{ mA}$

Problem 3.306: *Determine the current I_1, I_2 and I_3 through the three resistors in the circuit shown in the Fig. 3.194.*

Solution: The node *E* is assumed as zero Volts, and the voltages at various nodes are shown in the figure w.r.t. the zero volts at the node *E*.

So, the current through various resistors may be given as:

$$I_1 = \frac{6-6}{10} = 0 \text{ A}$$

and, $$I_2 = \frac{3-3}{10} = 0 \text{ A}$$

and, $$I_3 = \frac{0-0}{10} = 0 \text{ A}$$

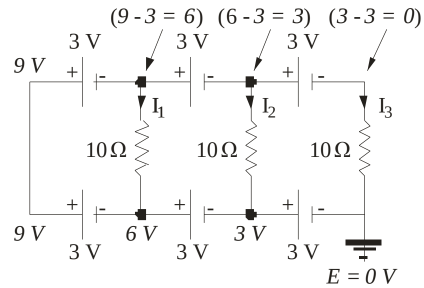

Fig. 3.195

3.31 Potentiometer: An *ideal voltmeter* must have an *infinite resistance* in order to read the actual reading of the *potential difference* across two points, if resistance of voltmeter is finite (but very large off course), *it will affect the actual reading of potential difference to be measured*.

Let Us Examine: If a current I is flowing through a resistor R, the actual *potential difference* across the resistance may be given as, $\boldsymbol{V = I \times R}$

If a voltmeter of infinite resistance is measuring the potential difference across the resistance, the current through the voltmeter may be given as:

$$I_m = \frac{V}{R_m} = \frac{V}{\infty} = 0$$

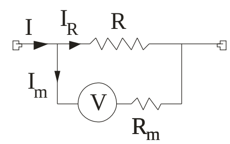

Fig. 3.196

So, $I_R = I$

Thus, potential difference measured by the voltmeter is,

$$V = I_R \times R = I \times R$$

On the other hand, if a voltmeter of finite resistance is measuring the potential difference across the resistance, the current through the voltmeter may be given as:

$$I_m = \frac{V}{R_m}$$

So, $$I_R = I - \frac{V}{R_m}$$

Thus, potential difference measured by the voltmeter is,

$$V = I_R \times R = \left(I - \frac{V}{R_m}\right) \times R$$

So, the measured value is slightly less than the actual value, if $R_m \gg R$.

An actual voltmeter can never be designed with an infinite resistance, if it is so than it will not be able give any reading on its display. This is due to the reason that a voltmeter reads potential difference proportional to the current drawn from the circuit.

A potentiometer is a device which does not draw any current from the circuit, but still measures the potential difference. So, *it acts as an ideal voltmeter*.

"***A potentiometer is a device, which can measure an unknown emf or potential difference accurately, i.e without introducing any error***".

Construction: A potentiometer consists of a long wire *AB* (usually 4 m to 10 m long) having uniform cross sectional area, high resistivity and a low temperature coefficient such as *constantan* or *manganin*. Since the wire is too long to accommodate on a table (or fixture) for measurement, so pieces of 1 m long wire are fixed side by side on a wooden board but connected in series with the help of thick copper strips, as shown in the Fig. 3.197. A meter scale is fixed parallel to the wires to read the length of corresponding wire to calculate the potential difference to be measured. The long wire of the potentiometer is connected to a very strong battery through a switch (S) and a rheostat (R_h) for flow of a constant current (I) through the potentiometer wire AB. So, the potential decreases gradually from point A to point B along the wire. A *jockey* (J) can slide along the wire to detect the null point or to read accurate reading of voltage.

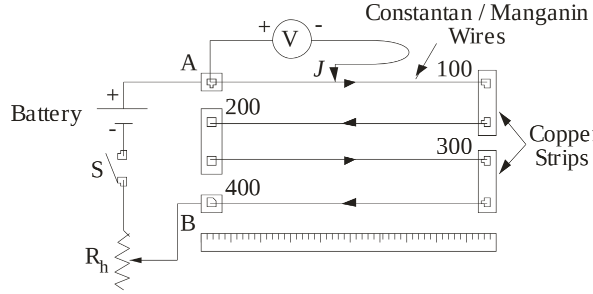

Fig. 3.197

Principle: "*The basic principle of working of the potentiometer is based upon the Ohm's law. When a constant current flows through a wire of uniform cross sectional area and composition, the potential drop across any length of the wire is directly proportional to the corresponding length of the wire*".

Consider the Fig. 3.197, if a voltmeter is connected across the end A and the jockey (J), it will display the potential difference (V) across the length (l) of the wire AJ. According to Ohm's law:

$$V = I\,R = \mathrm{I} \times \frac{\rho l}{A} \tag{3.227}$$

Since, the resistivity (ρ), area of cross section (A) and the current (I) flowing through the wire is constant for a *potentiometer*,

So, $\quad V = k\,l \quad$ or, $\quad V \propto l \qquad$ (3.228)

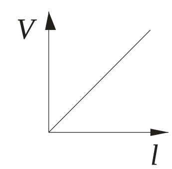

Fig. 3.198

The graph of V –vs– l is drawn in the Fig. 3.198, which is similar to the graph for *Ohm's law*.

Potential Gradient along the Wire of Potentiometer: "*The potential drop per unit length of the potentiometer wire is known as potential gradient*". The smaller is the potential gradient the more is the resolution of the potentiometer. The potential gradient may be given as:

$$k = \frac{V}{l}\ \mathrm{Vm^{-1}} \qquad \text{(or Vcm}^{-1}\text{, this is more practical unit, although SI unit is Vm}^{-1}\text{)}$$

3.32 Applications of Potentiometer: The main applications of the *potentiometer* are to measure the potential difference accurately and to determine the internal resistance of a cell (battery). The working of a potentiometer in both the cases is given below.

Comparison of EMF's of Two Primary Cells: A suitable circuit diagram is shown in the Fig. 3.199 for comparison of emf's of two primary cells. A constant current (I) is maintained in the potentiometer wire (AB) with the help of a battery of emf (E_S) through the switch (S) and a rheostat (R_h). Let emf of two primary cells to be compared are E_1 and E_2. The positive terminals of the cells, to be compared, are connected to the end A of potentiometer wire and their negative terminals are connected to a high resistance box ($R.B.$), a galvanometer (G) and the jockey (J) through a two way switch S_1S_2. A high resistance (R) must be inserted with the help of $R.B.$ in the circuit, to prevent high currents through the galvanometer for its safety.

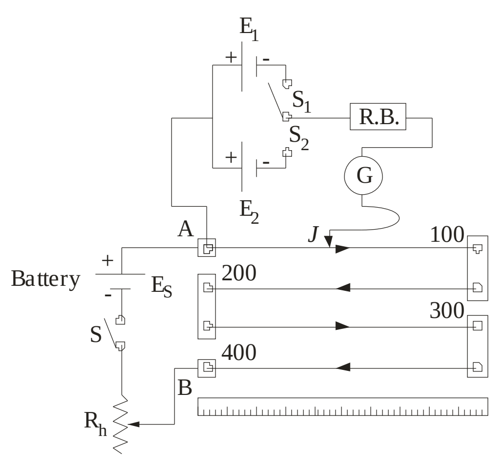

Fig. 3.199

As soon as the switch is closed in S_1 position, the cell E_1

gets introduced in the circuit. The jockey (J) is moved along the wire AB till the galvanometer shows null deflection. Let the position of jockey be J_1 and the corresponding length of the wire is $AJ_1 = l_1$. If k is the potential gradient along the wire AB, then the emf E_1 corresponding to null point may be given as:

$$E_1 = k\, l_1 \tag{3.229}$$

Now, when the switch is closed in S_2 position, the cell E_2 gets introduced in the circuit. The null point is again obtained for cell E_2. Let the position of jockey be J_2 and the corresponding length of the wire is $AJ_2 = l_2$, then the emf E_2 corresponding to null point may be given as:

$$E_2 = k\, l_2 \tag{3.230}$$

So, the ratio of emf of two cells may be given as:

$$\frac{E_2}{E_1} = \frac{l_2}{l_1} \tag{3.231}$$

If one of the two cells is a standard cell of known emf E_1, then emf of another cell may be determined as:

$$E_2 = \frac{l_2}{l_1} \times E_1 \tag{3.232}$$

It is mandatory to obtain a null point corresponding to each cell, so E_S must be greater than both the emf's E_1 and E_2.

To determine the Internal Resistance of a Primary Cell: A suitable circuit diagram is shown in the Fig. 3.200 for determination of internal resistance of a primary cell. A constant current (I) is maintained in the potentiometer wire (AB) with the help of a battery of emf (E_S) through the switch (S) and a rheostat (R_h). The positive terminal of the cell, whose internal resistance is to be measured, is connected to the end A of the potentiometer wire and the negative terminal to a galvanometer (G) and the jockey (J). A resistance box ($R.B.$) is connected across the cell through the switch S_1. First we have to determine the open circuit voltage (emf), while switch S is open, by moving the jockey (J) along the wire AB till the galvanometer shows null deflection. Let the position of jockey be J_1 and the corresponding length of the wire is $AJ_1 = l_1$. If k is the potential gradient along the wire AB, then the emf of the cell E corresponding to null point may be given as:

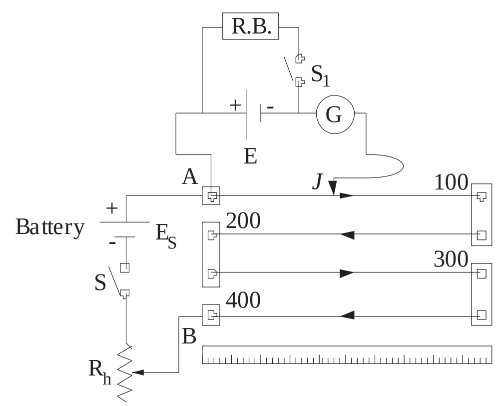

Fig. 3.200

$$E = k\, l_1 \tag{3.233}$$

Now, introduce a high resistance (R) with the help of $R.B.$ to flow a small current in the circuit of the cell, whose internal resistance is to be measured. The null point is again obtained for the terminal voltage of the shunted cell (V_t). Let the position of jockey be J_2 and the corresponding length of the wire is $AJ_2 = l_2$, then the terminal voltage (V_t) corresponding to null point may be given as:

$$V_t = k\, l_2 \tag{3.234}$$

Let r be the internal resistance of the cell. If a current I is flowing through the cell E while shunted with the resistance R, the emf of the cell and the terminal voltage of the cell may respectively be given according to *Ohm's law* as:

$$E = I\,(R + r) \tag{3.235}$$

and, $$V_t = I\, R \tag{3.236}$$

Now, the ratio of the emf of the cell to its terminal voltage may be given as:

$$\frac{E}{V_t} = \frac{I\,(R+r)}{I\,R} = \frac{(R+r)}{R} = \frac{l_1}{l_2}$$

or, $$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

or, $$r = \left(\frac{l_1}{l_2} - 1\right) \times R = \left(\frac{l_1 - l_2}{l_2}\right) \times R \qquad (3.237)$$

3.33 Sensitivity of a Potentiometer: *A potentiometer is said to be more sensitive if*:

i) *It is capable to measure the potential difference how much smaller it may be*.

ii) *It shows a significant change in the balancing length for a small change in the potential difference being measured*.

So, the sensitivity of a potentiometer depends on the potential gradient (k) along the wire *AB*. *Smaller the potential gradient, greater will be the sensitivity* (*resolution*) *of the potentiometer*.

The sensitivity of a potentiometer may be increased by reducing the potential gradient along the length of wire *AB*. This may be achieved in following two ways:

i) *For a given potential difference, the potential gradient can be reduced by increasing the length of the potentiometer wire AB.*

ii) *For a given length of the wire, the potential gradient can be reduced by reducing the current I flowing through the potentiometer wire with the help of a rheostat* (R_h).

Problem 3.307: *A Potentiometer wire is 10 m long and has a resistance of 18 Ω. It is connected to a battery of emf 5 V and internal resistance 2 Ω. Determine the potential gradient along the wire.*

Solution: $l = 10$ m, $R_{AB} = 18\ \Omega$, $E = 5$ V, $r = 2\ \Omega$

The current flowing through the potentiometer wire *AB* may be given as:

$$I = \frac{E}{r+R} = \frac{5}{2+18} = 0.25 \text{ A}$$

The potential gradient along the wire may be given as:

$$k = \frac{V_t}{l} = \frac{I\,R}{l} = \frac{0.25 \times 18}{10} = 0.45 \text{ V/m}$$

$$= \frac{E - I\,r}{l} = \frac{5 - 0.25 \times 2}{10} = 0.45 \text{ V/m}$$

Problem 3.308: *A potentiometer wire is supplied by a constant voltage of 3 V. A cell of emf 1.08 V is balanced by the voltage drop across 216 cm of the wire. Determine the total length of the wire.*

Solution: $E = 3$ V, $E_1 = 1.08$ V, $l_1 = 216$ cm

The ratio of two emf's may be given as:

$$\frac{E}{E_1} = \frac{l}{l_1}$$

or, $l = \frac{E}{E_1} \times l_1 = \frac{3}{1.08} \times 2.16 = 6 \text{ m}$

Problem 3.309: *Two cells of emf E_1 and E_2 ($E_1 > E_2$) are connected as shown in the Fig. 3.201. When a potentiometer is connected across the points A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer across the points A and C, the balancing length of the potentiometer wire is 100 cm. Determine the ratio of emf's E_1 and E_2.* [CBSE 1993-94]

Solution: $l_{1(AB)} = 300$ cm, $l_{2(AC)} = 100$ cm

The ratio of the emf's in two cases may be given as:

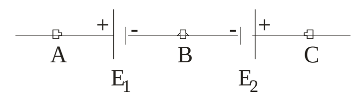

Fig. 3.201

$$\frac{E_1 - E_2}{E_1} = \frac{l_{2(AC)}}{l_{1(AB)}} = \frac{100}{300} = \frac{1}{3}$$

or, $\frac{E_2}{E_1} = 1 - \frac{1}{3} = \frac{2}{3}$

So, $E_1 : E_2 = 3 : 2$

Problem 3.310: *A circuit using a potentiometer and battery of negligible internal resistance is set up as shown in the Fig. 3.202. Two cells of emf E_1 and E_2 are connected in series as shown in combination (1) and (2). The balance points are obtained respectively at 400 cm and 240 cm from the point A. Determine: i) E_1 / E_2, ii) the balancing length for E_1 alone.* [CBSE 2008-09]

Solution: $l_{1(A+B)} = 400$ cm, $l_{2(A-B)} = 240$ cm

The ratio of the emf's in two cases may be given as:

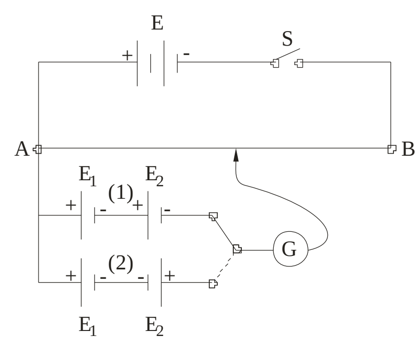

Fig. 3.202

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{l_{1(A+B)}}{l_{1(A-B)}} = \frac{400}{240} = \frac{5}{3}$$

or, $3E_1 + 3E_2 = 5E_1 - 5E_2$

or, $\frac{E_1}{E_2} = \frac{8}{2} = 4$

So, $E_1 : E_2 = 4 : 1$

The ratio of emf E_1 and ($E_1 + E_2$) may be given as:

$$\frac{E_1}{E_1 + E_2} = \frac{l_{E1}}{l_{1(A+B)}}$$

or, $l_{E1} = \frac{E_1}{E_1 + E_2} \times l_{1(A+B)} = \frac{1}{\left(1 + \frac{E_2}{E_1}\right)} \times l_{1(A+B)} = \frac{1}{\left(1 + \frac{1}{4}\right)} \times 400 = 320 \text{ cm}$

Problem 3.311: *A standard cell of emf 5 V and negligible internal resistance maintains a steady current through the potentiometer wire of length 5 m. Two primary cells of emf's E_1 and E_2 are joined in series with: i) same polarity, ii) opposite polarity. The combinations are connected through a galvanometer and jockey to the potentiometer. The balancing lengths in the two cases are found to be 350 cm and 50 cm respectively. i) Draw the necessary circuit diagram. ii) determine the value of emf's of individual cells.*

Solution: $E = 5$ V, $l = 5$ m, $l_{1(A+B)} = 350$ cm, $l_{2(A-B)} = 50$ cm

The required circuit diagram is drawn in the Fig. 3.203.

The potential gradient along the potentiometer wire *AB* may be given as:

$$k = \frac{E}{l} = \frac{5}{500} = 0.01 \text{ V/cm}$$

The emf's of two cells connected in same polarity may be given as:

$$E_1 + E_2 = k\, l_{1(A+B)} = 0.01 \times 350$$

or, $$E_1 + E_2 = 3.5 \qquad (3.238)$$

The emf's of two cells connected in opposite polarity may be given as:

$$E_1 - E_2 = k\, l_{1(A-B)} = 0.01 \times 50$$

or, $$E_1 - E_2 = 0.5 \qquad (3.239)$$

Equation (3.238) + (3.239):

$$2\,E_1 = 4 \qquad \text{or,} \qquad E_1 = \frac{4}{2} = 2 \text{ V}$$

Equation (3.238) – (3.239):

$$2\,E_2 = 3 \qquad \text{or,} \qquad E_2 = \frac{3}{2} = 1.5 \text{ V}$$

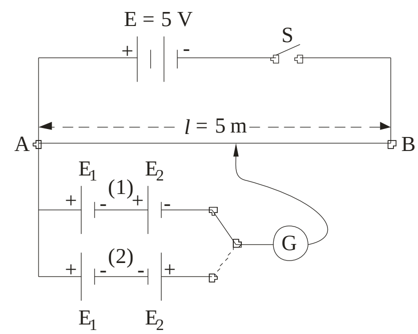

Fig. 3.203

Problem 3.312: *A 10 m long wire of uniform cross sectional area and of resistance 20 Ω is used as a potentiometer wire. This wire is connected in series with a battery of 5 V, along with an external resistance of 480 Ω. If an unknown emf E_1 is balanced at 600 cm of this wire, determine: i) the potential gradient of the potentiometer wire, ii) the value of unknown emf E_1.*

[CBSE 1997-98, 2005-06]

Solution: $l = 10$ m, $R = 20\ \Omega$, $E = 5$ V, $R_h = 480\ \Omega$, $l_1 = 600$ cm

The potential difference across the potentiometer wire may be given as:

$$V = \frac{20}{20 + 480} \times 5 = 0.2 \text{ V}$$

The potential gradient along the potentiometer wire may be given as:

$$k = \frac{V}{l} = \frac{0.2}{10} = 0.02 \text{ V/m}$$

The value of unknown emf may be given as:

$$E_1 = k\, l_1 = 0.02 \times 6 = 0.12 \text{ V}$$

Problem 3.313: *The wire AB is a uniform wire of resistance 15 Ω and length 1 m in the circuit diagram shown in the Fig. 3.204. It is connected to a series arrangement of cell E_1 of emf 2 V and negligible internal resistance and a resistor R. Terminal A of the wire is also connected to an electrochemical cell E_2 of emf 75 mV and a galvanometer G in this set up, a balancing point is obtained at 30 cm mark from A. Determine the resistance of resistor R. If E_2 were to have an emf of 300 mV, where will you expect the balancing point to be?* [CBSE 1998-99]

Solution: $R = 15\ \Omega$, $l = 1$ m, $E_1 = 2$ V, $E_2 = 75$ mV, $l_1 = 30$ cm, $E'_1 = 300$ mV

The potential difference across the potentiometer wire *AB* may be given as:

$$V_{AB} = \frac{15}{R + 15} \times 2 \text{ V}$$

The potential gradient along the wire *AB* may be given as:

$$k = \frac{V_{AB}}{l} = \frac{15}{R+15} \times 2 \text{ V/m}$$

The emf E_2 connected across point *A* and jockey (*J*) may be given as:

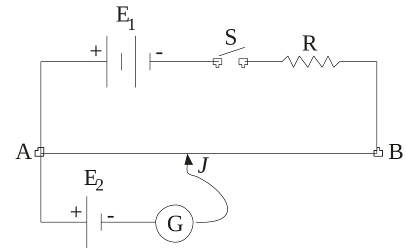

Fig. 3.204

$$E_2 = k\, l_1 = \frac{15}{R+15} \times 2 \times 0.3 = 75 \times 10^{-3}$$

or, $$R = \frac{15 \times 2 \times 0.3}{75 \times 10^{-3}} - 15 = 120 - 15 = 105\ \Omega$$

For E'_1 = 300 mV, the balance point must be given by the expression:

$$\frac{E'_1}{E_1} = \frac{l'_1}{l_1} \qquad \text{or,} \qquad l_1' = \frac{E'_1}{E_1} \times l_1 = \frac{300}{75} \times 30 = 120 \text{ cm}$$

As the length of the potentiometer wire is only 100 cm, so this point cannot be located on this potentiometer wire.

Problem 3.314: *The length of a potentiometer wire is 5 m. It is connected across a battery of constant emf, for a given Lechlanche cell, the position of zero galvanometer deflection is obtained at 100 cm. If the length of the potentiometer wire be made 8 m instead of 5 m, determine the length of wire for zero deflection of the galvanometer for the same cell.* [CBSE 1996-97]

Solution: l = 5 m, l_1 = 100 cm, l' = 8 m

The emf of the cell in first case, when the length of potentiometer wire is 5 m, may be given as:

$$E = k\, l_1 = \frac{I R}{l} \times l_1$$

The emf of the cell in second case, when the length of potentiometer wire is 8 m, may be given as:

$$E = k\, l_1' = \frac{I R}{l'} \times l_1'$$

Now, $$E = \frac{I R}{l} \times l_1 = \frac{I R}{l'} \times l_1' \qquad \text{or,} \qquad l_1' = \frac{l_1}{l} \times l' = \frac{1}{5} \times 8 = 1.6 \text{ m}$$

Problem 3.315: *A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance (R), a battery of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. Determine the value of external resistance.* [IIT]

Solution: l = 100 cm = 1 m, R = 10 Ω, E_1 = 2 V, E_2 = 10 mV, l_1 = 40 cm

The potential difference across the potentiometer wire *AB* may be given as:

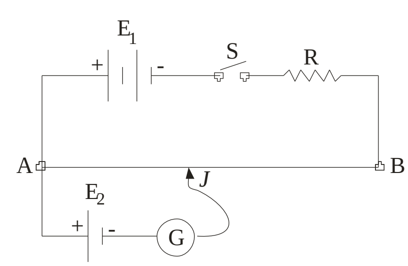

Fig. 3.205

$$V_{AB} = \frac{10}{R+10} \times 2 \text{ V}$$

The potential gradient along the wire *AB* may be given as:

$$k = \frac{V_{AB}}{l} = \frac{10}{R+10} \times 2 \text{ V/m}$$

The emf E_2 connected across point A and jockey (J) may be given as:

$$E_2 = k\,l_1 = \frac{10}{R+10} \times 2 \times 0.4 = 10 \times 10^{-3}$$

or, $$R = \frac{10 \times 2 \times 0.4}{10 \times 10^{-3}} - 10 = 800 - 10 = 790\ \Omega$$

Problem 3.316: *The uniform wire AB is 1 m long and has a resistance of 10 Ω. Other data are shown in the Fig. 3.206. Determine: i) potential gradient along the wire AB, ii) the length AO, when galvanometer shows null deflection.* [CBSE 1999-2000]

Solution: $l = 1$ m, $R = 10\ \Omega$

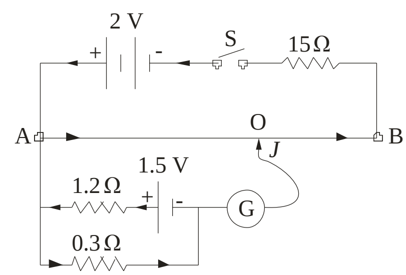

Fig. 3.206

The potential difference across the potentiometer wire AB may be given as:

$$V_{AB} = \frac{10}{10+15} \times 2 = 0.8 \text{ V}$$

So, the potential gradient along the wire AB may be given as:

$$k = \frac{V_{AB}}{l} = \frac{0.8}{1} = 0.8 \text{ V/m}$$

The current in the secondary circuit at the null deflection of the galvanometer may be given as:

$$I_2 = \frac{1.5}{1.2+0.3} = 1 \text{ A}$$

So, potential difference across the secondary circuit (AO) may be given as:

$$V_{AO} = 0.3 \times 1 = 0.3 \text{ V}$$
$$= 1.5 - 1.2 \times 1 = 0.3 \text{ V}$$

So, the length of the wire AO may be given as:

$$l_{AO} = \frac{V_{AO}}{k} = \frac{0.3}{0.8} = 0.375 \text{ m} = 37.5 \text{ cm}$$

Problem 3.317: *A cell gives balance with 85 cm of a potentiometer wire. When the terminals of the cell are shorted through a resistance of 7.5 Ω, the balance is obtained at 75 cm. Determine the internal resistance of the cell.* [ICSE 1994-95]

Solution: $l_1 = 85$ cm, $R = 7.5\ \Omega$, $l_2 = 75$ cm

The internal resistance of the cell may be given as:

$$r = R \times \left(\frac{l_1 - l_2}{l_2}\right) = 7.5 \times \left(\frac{85-75}{75}\right) = 1\ \Omega$$

Problem 3.318: *When a resistor of 5 Ω is connected across a cell, its terminal voltage is balanced by 150 cm of potentiometer wire, and when a resistor of 10 Ω resistance is connected across the cell, the* *terminal voltage is balanced by 175 cm of the same potentiometer wire. Determine the internal resistance of the cell.*

Solution: $R_1 = 5\ \Omega$, $l_2 = 150$ cm, $R_2 = 10\ \Omega$, $l_2' = 175$ cm

First Case: The internal resistance may be given as:

$$r = R_1 \times \left(\frac{l_1 - l_2}{l_2}\right) \quad \text{or,} \quad r \times \frac{l_2}{R_1} = l_1 - l_2 \qquad (3.240)$$

Second Case: The internal resistance may be given as:

$$r = R_2 \times \left(\frac{l_1 - l'_2}{l'_2}\right) \quad \text{or,} \quad r \times \frac{l'_2}{R_2} = l_1 - l_2' \qquad (3.241)$$

Equation (3.240) – (3.241):

$$r \times \left(\frac{l_2}{R_1} - \frac{l'_2}{R_2}\right) = l_2' - l_2 \quad \text{or,} \quad r = \frac{l'_2 - l_2}{\left(\frac{l_2}{R_1} - \frac{l'_2}{R_2}\right)} = \frac{175 - 150}{\left(\frac{150}{5} - \frac{175}{10}\right)} = 2\ \Omega$$

Problem 3.319: *A potentiometer wire is 10 m long and a potential difference of 6 V is maintained across its ends. Determine the emf of a cell which balances against a length of 180 cm of the potentiometer wire.*

Solution: $l = 10$ m, $E = 6$ V, $l_1 = 180$ cm $= 1.8$ m

The emf of the cell may be given as:

$$E = k\, l_1 = \frac{E}{l} \times l_1 = \frac{6}{10} \times 1.8 = 1.08\ \text{V}$$

Problem 3.320: *Two cells of emf E_1 and E_2 are connected together in two different possible ways to locate the null point against a potentiometer wire. The balance points are found corresponding to 351 cm and 70.2 cm of potentiometer wire respectively. Determine the ratio of emf's of two cells.*

Solution: $l_{1(A+B)} = 351$ cm, $l_{2(A-B)} = 70.2$ cm

The ratio of the emf's in two cases may be given as:

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{l_{1(A+B)}}{l_{1(A-B)}} = \frac{351}{70.2} = 5$$

or, $E_1 + E_2 = 5\,E_1 - 5\,E_2$

or, $\dfrac{E_1}{E_2} = \dfrac{6}{4} = \dfrac{3}{2}$

So, $E_1 : E_2 = 3 : 2$

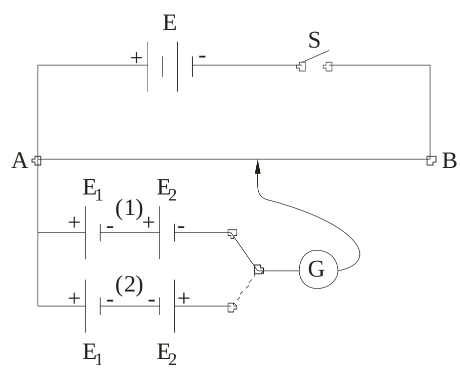

Fig. 3.207

Problem 3.321: *The resistance of a potentiometer wire of length 10 m is 20 Ω. A resistance box and a 2 V accumulator are connected in series with it. Determine the resistance to be inserted in the box which will result in a potential drop of 1 µV/mm of the potentiometer wire.* [Kerala 1993-94]

Solution: $l = 10$ m, $R = 20\ \Omega$, $E = 2$ V, $k = 1\ \mu\text{V/mm} = \dfrac{1 \times 10^{-6}}{10^{-3}} = 1 \times 10^{-3}$ V/m

Let the resistance introduced in the resistance box is R_{ext}.

The potential difference across the potentiometer wire may be given as:

$$V_{AB} = \frac{20}{R_{ext} + 20} \times 2 \text{ V}$$

The potential gradient along the wire may be given as:

$$k = \frac{V_{AB}}{l} = \frac{20}{10 \times (R + 20)} \times 2 = 1 \times 10^{-3}$$

or, $$R = \frac{20 \times 2}{10 \times 1 \times 10^{-3}} - 20 = 4000 - 20 = 3980\ \Omega$$

Problem 3.322: *A cell of 1.2 V gives a balance point at 30 cm length of a potentiometer wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of the two cells is 1.5, determine the difference in the balancing length of the potentiometer wire in two cases.* [CBSE 2005-06]

Solution: $E_1 = 1.2$ V, $l_1 = 30$ cm, $\frac{E_1}{E_2} = 1.5$

The ratio of two emf's in terms of the balancing lengths of the potentiometer wire may be given as:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \qquad \text{or,} \qquad l_2 = \frac{E_2}{E_1} \times l_1 = \frac{1}{1.5} \times 30 = 20 \text{ cm}$$

and, the difference between the balancing length may be given as:

$$l_1 - l_2 = 30 - 20 = 10 \text{ cm}$$

Problem 3.323: *A potentiometer has 400 cm long wire which is connected to an auxiliary steady voltage of 4 V. A Lechlanche cell gives null point at 140 cm and Daniel cell at 100 cm. i) Compare emf's of two cells, ii) If the length of the wire is increased by 100 cm, determine the position of the null point with the Lechlanche cell.*

Solution: $l = 400$ cm, $E = 4$ V, $l_1 = 140$ cm, $l_2 = 100$ cm

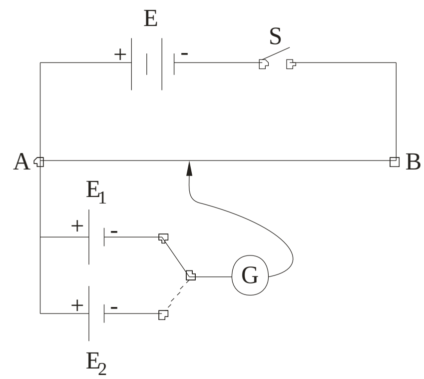

Fig. 3.208

The ratio of emf's of two cells may be given as:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} = \frac{140}{100} = \frac{7}{5}$$

$$E_1 : E_2 = 7 : 5$$

The emf of the Lechlanche cell, in two cases, may be given as:

$$E_1 = k\, l_1 = \frac{E}{l} \times l_1 = \frac{E}{(l + 100)} \times l_1'$$

So, $$l_1' = \frac{(l + 100)}{l} \times l_1 = \frac{(400 + 100)}{400} \times 140 = 175 \text{ cm}$$

Problem 3.324: *The balance point is obtained at 60 cm from the zero end of the potentiometer wire for a certain cell. The balance point with another cell, differing by 0.1 V than that of the first cell, is obtained at 55 cm. Determine the emf of two cells.*

Solution: $l_1 = 60$ cm, $E_1 - E_2 = 0.1$ V, $l_2 = 55$ cm

The ratio of two emf's may be given as:

$$\frac{E_1}{E_2} = \frac{E_1}{E_1 - 0.1} = \frac{l_1}{l_2} = \frac{60}{55} = \frac{12}{11}$$

or, $11\ E_1 = 12\ E_1 - 1.2$

or, $E_1 = 1.2$ V and, $E_2 = E_1 - 0.1 = 1.2 - 0.1 = 1.1$ V

Problem 3.325: *A potentiometer wire has a potential gradient of 2.5 mV/cm along its length. Determine the length of the wire at which null point is obtained for a 1.025 V standard cell. Also, determine the emf of another cell for which the null point is obtained at 860 cm length.*

Solution: $k = 2.5 \text{ mV/cm} = \dfrac{2.5\times10^{-3}}{10^{-2}} = 0.25$ V/m, $E_1 = 1.025$ V, $l_2 = 860$ cm

The length of the wire for null point of 1.025 V cell may be given as:

$$l_1 = \frac{E_1}{k} = \frac{1.025}{0.25} = 4.1 \text{ m} = 410 \text{ cm}$$

The ratio of emf's of two cells may be given as:

$$\frac{E_2}{E_1} = \frac{l_2}{l_1} \qquad \text{or,} \qquad E_2 = \frac{l_2}{l_1} \times E_1 = \frac{860}{410} \times 1.025 = 2.15 \text{ V}$$

Problem 3.326: *A potentiometer wire AB has a length of 100 cm, which is connected across a constant d.c. voltage source of E_1 = 2 V. If a cell E_2 is connected across AC (75 cm). No current flows through the cell E_2. Determine: i) the potential gradient along AB, ii) emf of the cell E_2.*

Solution: $l = 100$ cm $= 1$ m, $E_1 = 2$ V, $l_1 = 75$ cm

The potential gradient across the wire may be given as:

$$k = \frac{E_1}{l} = \frac{2}{1} = 2 \text{ V/m}$$

The emf of the cell E_2 may be given as:

$$E_2 = k\ l_1 = 2 \times 0.75 = 1.5 \text{ V}$$

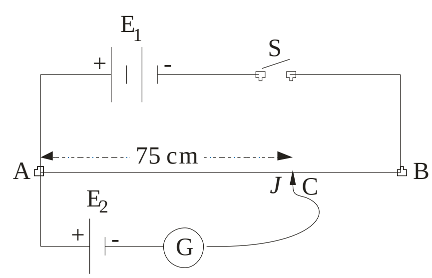

Fig. 3.209

Problem 3.327: *A cell can be balanced against 110 cm and 100 cm of potentiometer wire respectively when in open circuit and shorted through a resistance of 10 Ω. Determine the internal resistance of the cell.*

Solution: $l_1 = 110$ cm, $l_2 = 100$ cm, $R = 10\ \Omega$

The internal resistance of the cell may be given as:

$$r = R \times \left(\frac{l_1 - l_2}{l_2}\right) = 10 \times \left(\frac{110 - 100}{100}\right) = 1\ \Omega$$

Problem 3.328: *The null point is detected at 220 cm against a potentiometer wire, when a cell is shunted through a resistance of 5 Ω. On the other hand, the null point is detected at 300 cm, when the same cell is shunted through a resistance of 20 Ω. Determine the internal resistance of the cell.*

Solution: $R_1 = 5\ \Omega$, $l_2 = 220$ cm, $R_2 = 20\ \Omega$, $l_2' = 300$ cm

First Case: The internal resistance may be given as:

$$r = R_1 \times \left(\frac{l_1 - l_2}{l_2}\right) \qquad \text{or,} \qquad r \times \frac{l_2}{R_1} = l_1 - l_2 \qquad (3.242)$$

Second Case: The internal resistance may be given as:

$$r = R_2 \times \left(\frac{l_1 - l'_2}{l'_2}\right) \qquad \text{or,} \qquad r \times \frac{l'_2}{R_2} = l_1 - l_2' \qquad (3.243)$$

Equation (3.242) – (3.243):

$$r \times \left(\frac{l_2}{R_1} - \frac{l'_2}{R_2}\right) = l_2' - l_2 \qquad \text{or,} \qquad r = \frac{l'_2 - l_2}{\left(\frac{l_2}{R_1} - \frac{l'_2}{R_2}\right)} = \frac{300 - 220}{\left(\frac{220}{5} - \frac{300}{20}\right)} = 2.759\ \Omega$$

Problem 3.329: *The potentiometer wire AB of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected as shown in the circuit given in the Fig. 3.210, with a resistance 'r' and a switch 'S'. The null position is detected at 51 cm from the point A with switch S open. Determine: i) potential gradient along the potentiometer wire, ii) emf of the cell E_1, iii) if switch S is closed, the null point will move in which direction: towards point P or towards point Q?* [CBSE 2003-04]

Solution: $l = 1$ m, $E_2 = 1.02$ V, $l_1 = 51$ cm

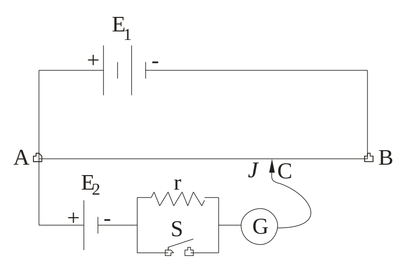

Fig. 3.210

The potential gradient along the potentiometer may be given as:

$$k = \frac{E_2}{l_1} = \frac{1.02}{0.51} = 2 \text{ V/m}$$

The null point will remain same, as the secondary circuit is neither drawing nor delivering any current from / to the potentiometer circuit in any case.

Problem 3.330: *A standard cell of emf 1.08 V is balanced by the potential difference across 91 cm of a meter long wire supplied by a cell of emf 2 V through a series resistor of resistance 2 Ω. The internal resistance of the cell is zero. Determine the resistance per unit length of the potentiometer wire.*

Solution: $E_2 = 1.08$ V, $l_1 = 91$ cm, $l = 1$ m, $E_1 = 2$ V, $R_h = 2\ \Omega$

Let the resistance per unit length of the potentiometer wire is R Ω/m.

So, the resistance of the potentiometer wire = $R \times l = R \times 1 = R$ Ω

The potential difference across the potentiometer wire may be given as:

$$V_{AB} = \frac{R}{R+2} \times 2 \text{ V}$$

The ratio of two voltages may be given as:

$$\frac{V_{AB}}{E_1} = \frac{l}{l_1}$$

$$\text{or,} \quad \frac{\left(\frac{2R}{R+2}\right)}{1.08} = \frac{100}{91} \qquad \text{or,} \qquad \frac{2R}{R+2} = 1.187$$

or, $2R = 1.187\,R + 2.374$ or, $R = \dfrac{2.374}{(2-1.187)} = 2.92\ \Omega/\text{m}$

Problem 3.331: *A battery E_1 of 4 V and a variable resistance R_h are connected in series with the wire AB (of length one meter) of the potentiometer. If a cell E_2, of emf 1.5 V is connected across points A and C, no current flows through E_2. Length AC = 60 cm. i) Determine the potential difference across the points A and B of the potentiometer. ii) Would the method work, if the battery E_1 is replaced by a cell of emf 1 V?* [CBSE 2002-03]

Solution: $E_1 = 4$ V, $E_2 = 1.5$ V, $l_1 = 60$ cm

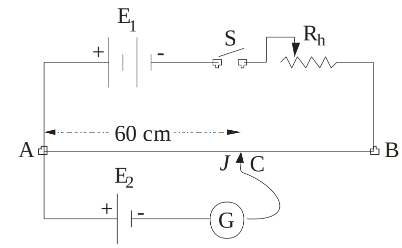

Fig. 3.211

The ratio of two voltages may be given as:

$$\frac{E_1}{E_2} = \frac{l}{l_1}$$

So, $E_1 = \dfrac{l}{l_1} \times E_2 = \dfrac{100}{60} \times 1.5 = 2.5$ V

The setup will not work, if the battery E_1 (4 V) is replaced by a cell of emf 1 V. The null point will not be detected in this case, also the mandatory condition of satisfactory working of a potentiometer is $E_1 > E_2$.

Problem 3.332: *The potentiometer wire of length 200 cm has a resistance of 20 Ω. It is connected in series with a resistance 10 Ω and an accumulator of emf 6 V having negligible internal resistance. A source of 2.4 V is balanced against a length 'L' of the potentiometer wire. Determine the length 'L'.* [CBSE 2002-03]

Solution: $l = 200$ cm, $R = 20\ \Omega$, $R_h = 10\ \Omega$, $E_1 = 6$ V, $E_2 = 2.4$ V

The potential difference across the potentiometer wire AB may be given as:

$$V_{AB} = \frac{20}{20+10} \times 6 = 4 \text{ V}$$

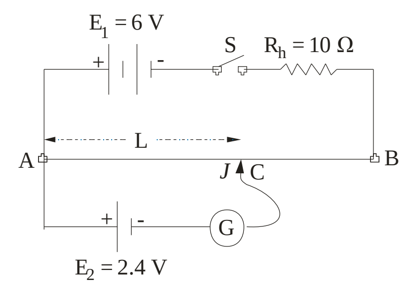

Fig. 3.212

The ratio of two voltages may be given as:

$$\frac{V_{AB}}{E_2} = \frac{l}{L}$$

or, $L = \dfrac{E_2}{V_{AB}} \times l = \dfrac{2.4}{4} \times 200 = 120$ cm

Problem 3.333: *A potentiometer wire carries a steady current. The potential difference across 70 cm length of it balances the potential difference across a 2 Ω coil supplied by a cell of emf 2 V and an unknown internal resistance r. When a 1 Ω coil is placed in parallel with the 2 Ω coil, a length equal to 50 cm of the potentiometer wire is required to balance the potential difference across the parallel combination. Determine the value of the internal resistance 'r' of the cell.*

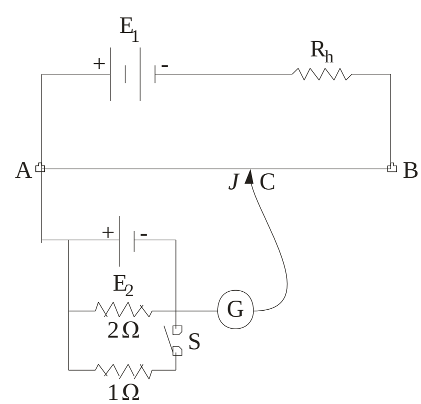

Fig. 3.213

Solution: $l_2 = 70$ cm, $R = 2\ \Omega$

$l_2' = 50$ cm, $R' = (2 \parallel 1) = \dfrac{2}{3}\ \Omega$

First Case: The internal resistance may be given as:

$$r = R_1 \times \left(\frac{l_1 - l_2}{l_2}\right)$$

or, $$r \times \frac{l_2}{R_1} = l_1 - l_2 \qquad (3.244)$$

Second Case: The internal resistance may be given as:

$$r = R_2 \times \left(\frac{l_1 - l'_2}{l'_2}\right) \qquad \text{or,} \qquad r \times \frac{l'_2}{R_2} = l_1 - l_2' \qquad (3.245)$$

Equation (3.245) – (3.244):

$$r \times \left(\frac{l'_2}{R_2} - \frac{l_2}{R_1}\right) = l_2 - l_2' \qquad \text{or,} \qquad r = \frac{l_2 - l'_2}{\left(\frac{l'_2}{R_2} - \frac{l_2}{R_1}\right)} = \frac{70 - 50}{\left(\frac{50}{(2/3)} - \frac{70}{2}\right)} = 0.5\ \Omega$$

3.34 Wheatstone Bridge: *Wheatstone Bridge is a setup of four resistances* (*the value of three resistances are known while one is unknown resistance*) *in the form of a bridge, to measure / determine the value of one unknown resistance out of the four resistances*.

A Wheatstone bridge consists of four resistances *P*, *Q*, *R* and *S*; out of which three resistances are known while one is unknown. These four resistances are connected to form the arms of a quadrilateral *ABCD*. A battery of emf *E* is connected across the points *A* and *C*, and a very sensitive galvanometer (*G*) is connected across the points *B* and *D*. as shown in the Fig. 3.214.

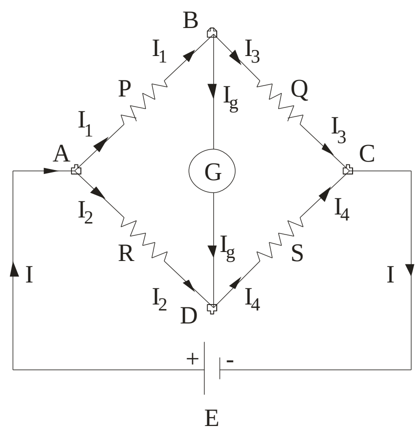

Fig. 3.214

Let *S* is the unknown resistance to be measured. The resistance *R* is so adjusted that there is no deflection in the galvanometer. The bridge is said to be balanced when the potential difference across the galvanometer is zero, *i.e.* it is not carrying any current and hence zero deflection is shown by the galvanometer. For the balanced condition of the bridge, we have:

$$\frac{P}{Q} = \frac{R}{S} \qquad (3.246)$$

So, the unknown resistor, $S = \frac{Q}{P} \times R$ (3.247)

We can determine the unknown resistance *S* by the knowledge of the ratio of resistances *P* and *Q*, and the resistance *R*. So, the arms containing resistances *P* and *Q* are known as *ratio arms*, the arm containing resistance *R* is known as *standard arm* and the arm containing resistance *S* is known as *unknown arm*.

Derivation of Balanced Condition of Wheatstone Bridge:

If the Wheat stone bridge is in balanced condition, it implies that the current flowing through the galvanometer (*G*) is zero.

i.e. $I_g = 0$

Assuming node *C* at zero Volts, the node *A* will be at *E* Volts due to the battery connected there. The voltage at node *B* is assumed as V_B while at node *D* is assumed as V_D. Since the current flowing through the galvanometer (I_g) is zero, so the voltages at node *B* and *D* are equal.

i.e. $V_B = V_D = V$

Now, applying *KCL* at node *B* and node *D* respectively:

$$\frac{E-V}{P} = \frac{V-0}{Q} = \frac{V}{Q} \qquad (3.248)$$

and, $$\frac{E-V}{R} = \frac{V-0}{S} = \frac{V}{S} \qquad (3.249)$$

Equation (3.249) / (3.248):

$$\frac{(E-V)/R}{(E-V)/P} = \frac{V/S}{V/Q}$$

or, $\frac{P}{R} = \frac{Q}{S}$ or, $\frac{P}{Q} = \frac{R}{S}$ (3.250)

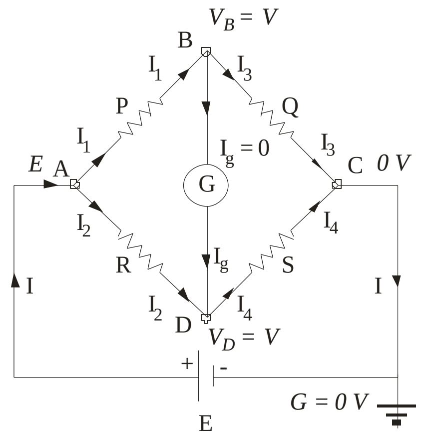

Fig. 3.215

Sensitivity of a Wheatstone Bridge: *A Wheatstone bridge is said to be more sensitive if it shows a large deflection in the reading of galvanometer for a small change in the resistance of the resistance arm* (*R*).

So, the sensitivity of a Wheatstone bridge depends on two factors:

i) Relative magnitudes of the resistances in the four arms of the bridges. The bridge is most sensitive when all the four resistances are of same order.

ii) Relative positions of battery and the galvanometer.

Advantages of Wheatstone Bridge Method: The bridge method of resistance measurement has following advantages over other traditional methods of resistance measurement:

i) Since it is a null method, so the internal resistance of the battery and the resistance of the galvanometer do not introduces any error in the measurement.

ii) As the method do not involve any measurement of current or potential difference, so the resistances of ammeters and voltmeters (which are absent now) do not introduce any errors in the measurement.

iii) The unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of the resistances in the arms *P* and *Q*.

3.35 Meter Bridge or Slide Wire Bridge: *It is the simplest practical application of the Wheatstone bridge, used to measure an unknown resistance*.

Principle of Operation: It's principle of operation is based on the working principle of *Wheatstone Bridge*.

i.e. when the bridge is balanced:

$$\frac{P}{Q} = \frac{R}{S}$$

Construction: It consists of one meter long manganin wire of uniform cross section, stretched along a meter scale fixed over a wooden board and with its two ends soldered to two *L*-shaped thick copper strips *A* and *C*. Another copper strip is fixed between these two copper strips, so as to provide two gaps *ab* and a_1b_1. A resistance box *R.B.* is connected across the gap *ab* and the unknown resistance *S* is connected across the gap a_1b_1. A source of emf *E* is connected across the ends of wire *AC*. A movable jockey and a galvanometer are connected across *BD*, as shown in the Fig. 3.216.

Working: After inserting a suitable resistance R in the resistance box (*R.B.*), the jockey (J) is moved along the wire AC till the null deflection is shown by the galvanometer. This is the balanced condition of *Meter Bridge*. If P and Q are the resistances of the parts AB and BC of the wire, we have for balanced condition of the bridge:

$$\frac{P}{Q} = \frac{R}{S}$$

As, the total length of the wire AC = 100 cm, and if $AB = l$, then $BC = (100 - l)$. Since, the bridge wire is having a uniform cross sectional area, so the resistance of the wire is proportional to its length (*i.e.* $r \propto l$).

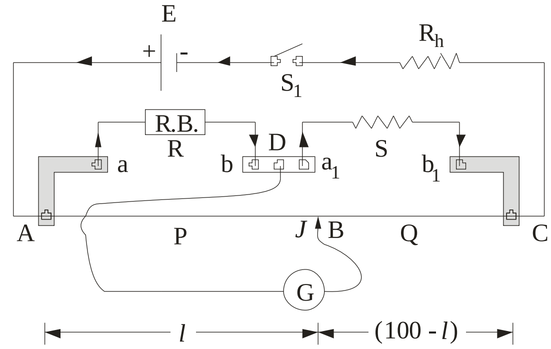

Fig. 3.216

Now, $$\frac{P}{Q} = \frac{\text{Resistance of AB}}{\text{Resistance of BC}} = \frac{l}{(100-l)} = \frac{R}{S}$$

So, $$S = \frac{R \times (100-l)}{l} \tag{3.251}$$

So, the unknown resistance S may be determined by the knowledge of R and l.

Problem 3.334: *Determine the magnitude of resistance X in the circuit shown in the Fig. 3.217, when no current flows through the 5 Ω resistor.*

Solution: $P = X$, $Q = 18\ \Omega$, $R = 2\ \Omega$, $S = 6\ \Omega$

Since, no current is flowing through the 5 Ω resistor, the given bridge is under balanced condition.

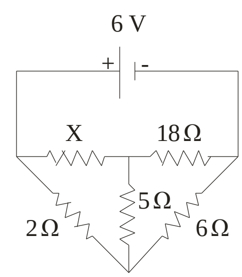

Fig. 3.217

So, $$\frac{P}{Q} = \frac{R}{S}$$

or, $$P = \frac{R}{S} \times Q$$

So, $$X = \frac{2}{6} \times 18 = 6\ \Omega$$

Problem 3.335: *In the arrangement of a Wheatstone bridge, the ratio arms P and Q are approximately equal. The bridge is balanced when R = 500 Ω. On interchanging the resistors P and Q the value of R for balancing is 505 Ω. Determine the value of S and the ratio P/Q.*

Solution: $R = 500\ \Omega$, $R' = 505\ \Omega$ (with P and Q interchanged)

When P and Q are at their respective places; R = 500 Ω:

$$\frac{P}{Q} = \frac{R}{S} = \frac{500}{S} \tag{3.252}$$

When P and Q are interchanged from their respective places; R = 505 Ω:

$$\frac{P'}{Q'} = \frac{Q}{P} = \frac{R'}{S} = \frac{505}{S} \tag{3.253}$$

Equation (3.252) × (3.253):

$$\frac{P}{Q} \times \frac{Q}{P} = \frac{500}{S} \times \frac{505}{S} = \frac{500 \times 505}{S^2}$$

or, $S = \sqrt{500 \times 505} = 502.49\ \Omega$

Putting this value in equation (3.252):

$$\frac{P}{Q} = \frac{500}{S} = \frac{500}{502.49} = \frac{1}{1.005} \qquad \text{or,} \qquad P : Q = 1 : 1.005$$

Problem 3.336: *P, Q, R and S are four resistance wires of resistances 2 Ω, 2 Ω, 2 Ω and 3 Ω respectively, connected in a Wheatstone bridge. Determine the resistance with which S must be shunted in order to balance the bridge.*

Solution: $P = 2\ \Omega$, $Q = 2\ \Omega$, $R = 2\ \Omega$, $S = 3\ \Omega$

Let the resistor S be shunted with R_{sh}, in order to balance the bridge as shown in the Fig. 3.218. The equivalent resistance of the resistor S shunted with R_{sh} may now be given as:

$$S' = \frac{R_{sh} \times S}{R_{sh} + S} = \frac{3R_{sh}}{R_{sh} + 3}$$

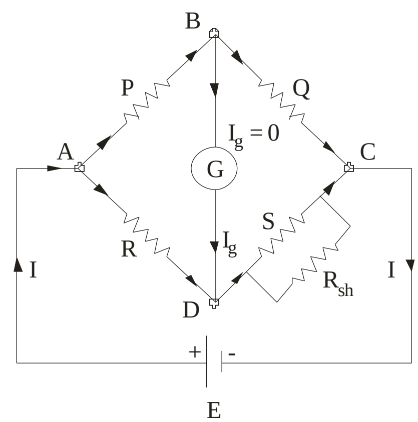

Fig. 3.218

For balanced condition of the bridge, we have:

$$\frac{P}{Q} = \frac{R}{S'}$$

or, $P \times S' = R \times Q$

or, $2 \times \dfrac{3R_{sh}}{R_{sh} + 3} = 2 \times 2$

or, $3\ R_{sh} = 2\ R_{sh} + 6$

or, $R_{sh} = 6\ \Omega$

Problem 3.337: *P, Q, R and S are the resistances taken in cyclic order in a Wheatstone Bridge network. P and Q are the ratio arms, S is unknown resistance and R is a 10 Ω resistor. A balance is obtained when R is shunted with a resistance of 190 Ω. If P and Q are interchanged, the balance is restored by altering the shunt across R to 265 Ω. Determine the value of resistance of S and the ratio P : Q.*

Solution: $R = 10\ \Omega$, $R_{sh} = 190\ \Omega$, $R_{sh}' = 265\ \Omega$ (with P and Q interchanged)

When *P* and *Q* are at their respective places; $R_{sh} = 190\ \Omega$:

The equivalent resistance of arm R may be given as:

$$R_{eq} = \frac{R \times R_{sh}}{R + R_{sh}} = \frac{10 \times 190}{10 + 190} = 9.5\ \Omega$$

$$\frac{P}{Q} = \frac{R_{eq}}{S} = \frac{9.5}{S} \qquad (3.254)$$

When *P* and *Q* are interchanged from their respective places; $R_{sh}' = 265\ \Omega$:

The new equivalent resistance of arm R may be given as:

$$R_{eq}' = \frac{R \times R'_{sh}}{R + R'_{sh}} = \frac{10 \times 265}{10 + 265} = 9.64\ \Omega$$

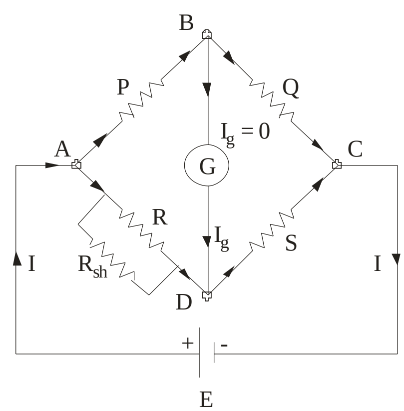

Fig. 3.129

$$\frac{P'}{Q'} = \frac{Q}{P} = \frac{R'_{eq}}{S} = \frac{9.64}{S} \qquad (3.255)$$

Equation (3.254) × (3.255):

$$\frac{P}{Q} \times \frac{Q}{P} = \frac{9.5}{S} \times \frac{9.64}{S} = \frac{9.5 \times 9.64}{S^2}$$

or, $S = \sqrt{9.5 \times 9.64} = 9.57\ \Omega$

Putting this value in equation (3.254):

$$\frac{P}{Q} = \frac{9.5}{S} = \frac{9.5}{9.57} = \frac{1}{1.007}$$

or, $P : Q = 1 : 1.007$

Problem 3.338: *Determine the current drawn from the battery by the network of resistors shown in the Fig. 3.220 (a).* [CBSE 1996-97, 2008-09]

Solution: The given network may be redrawn as shown in the Fig. 3.220 (b).

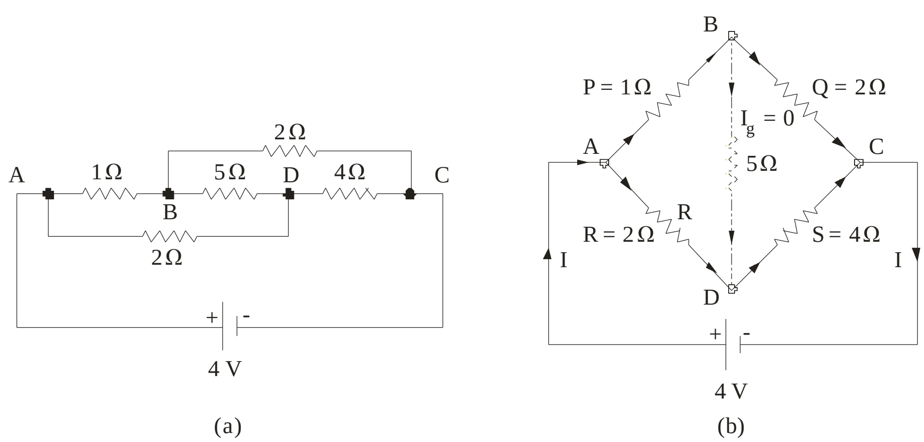

Fig. 3.220

Now, the reader may clearly observe that this is a balanced Wheatstone bridge.

As, $\frac{P}{Q} = \frac{1}{2} = \frac{R}{S} = \frac{2}{4}$

So, the central branch (5 Ω) will carry no current and may be assumed as open circuited. So, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (1 + 2) \parallel (2 + 4) = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

So, the current drawn by the network may be given as:

$$I = \frac{E}{R_{eq}} = \frac{4}{2} = 2\text{ A}$$

Problem 3.339: *Each of the resistances in the network shown in the Fig. 3.221 equals R. Determine the resistance between two terminals A and C.* [Himachal 1994-95, CBSE 2007-08]

Solution: The given network may be redrawn as shown in the Fig. 3.221 (b). Now, the reader may observe that this is a Wheatstone bridge, which is in its balanced position. So, the central branch across the points B and D may be assumed as open circuited, as it is carrying no current at the balanced position of the Wheatstone bridge. So, the equivalent resistance across the points A and C may be given as:

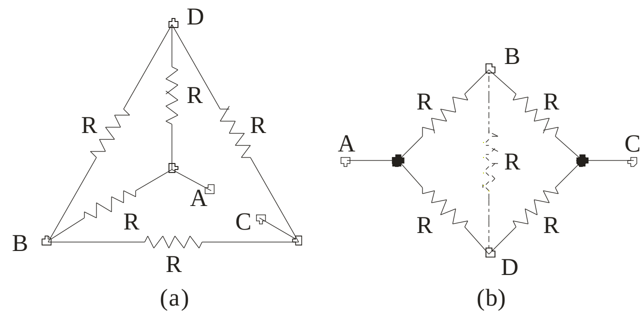

Fig. 3.221

$$R_{AC} = (R + R) \parallel (R + R) = \frac{2R \times 2R}{2R + 2R} = R$$

Problem 3.340: *A potential difference of 2 V is applied across the points A and B in the network shown in the Fig. 3.222 (a). Determine: i) the equivalent resistance of the network across the points A and B, ii) the magnitude of currents flowing through the arms ACB and ADB.*

Solution: $E = 2$ V, $R = 2\ \Omega$

The given network may be redrawn as shown in the Fig. 3.222 (b). Now, the reader may observe that this is a Wheatstone bridge, which is in its balanced position. So, the central branch across the points C and D may be assumed as open circuited, as it is carrying no current at the balanced position of the Wheatstone bridge. So, the equivalent resistance across the points A and B may be given as:

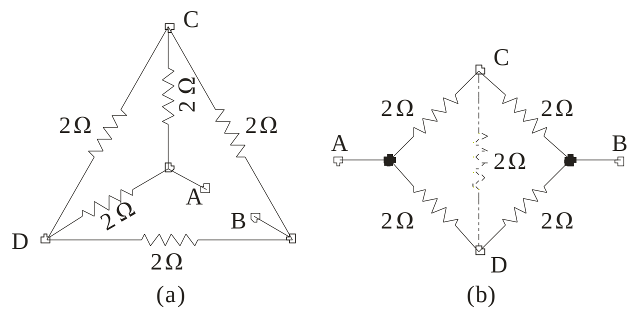

Fig. 3.222

$$R_{AC} = (2 + 2) \parallel (2 + 2) = \frac{4 \times 4}{4 + 4} = 2\ \Omega$$

The current flowing through each branch may be given as:

$$I = \frac{E}{R_{ACB}} = \frac{E}{R_{ADB}} = \frac{2}{2 + 2} = 0.5 \text{ A}$$

Problem 3.341: *Determine the value of unknown resistance X, in the circuit shown in the Fig. 3.223 (a), if no current flows through the branch AO. Also determine the current drawn by the circuit from the battery.* [CBSE 2001-02]

Solution: The given network may be redrawn as shown in the Fig. 3.223 (b). Now, the reader may observe that this is a Wheatstone bridge, which is in its balanced position, if there is no current flowing through the branch AO. So, the central branch across the points A and O may be assumed as open circuited, as it is carrying no current at the balanced position of the Wheatstone bridge. We have the relationship between the values of resistances in four arms of a Wheatstone bridge at balanced position as:

$$\frac{P}{Q} = \frac{R}{S} \qquad \text{or,} \qquad \frac{2}{3} = \frac{4}{X}$$

So, $X = \frac{3}{2} \times 4 = 6\ \Omega$

The equivalent resistance of the network as seen by the battery may be given as:

$$R_{\text{eq}} = [(2 + 4) \parallel (3 + 6)] + 2.4$$

$$= \frac{6\times 9}{6+9} + 2.4 = 6\ \Omega$$

So, the current drawn by the network from the source may be given as:

$$I = \frac{E}{R_{eq}} = \frac{6}{6} = 1\ \text{A}$$

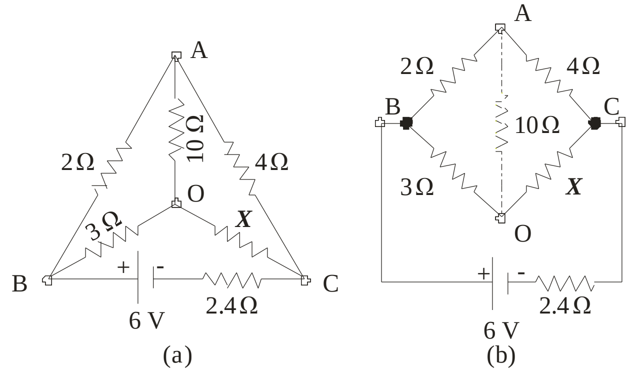

Fig. 3.223

Problem 3.342: *Six equal resistors, each of value R, are connected together as shown in the Fig. 3.224 (a). Determine the equivalent resistance across the points A and B. If a supply of emf E is connected across AB, compute the current through the points AB and arm CD.*

Solution: The given network may be redrawn as shown in the Fig. 3.224 (b). Now, the reader may observe that this is a Wheatstone bridge, which is in its balanced position. So, the central branch across the points *C* and *D* may be assumed as open circuited, as it is carrying no current at the balanced position of the Wheatstone bridge. The equivalent resistance of the network across the points *A* and *B* (as seen by the battery) may be given as:

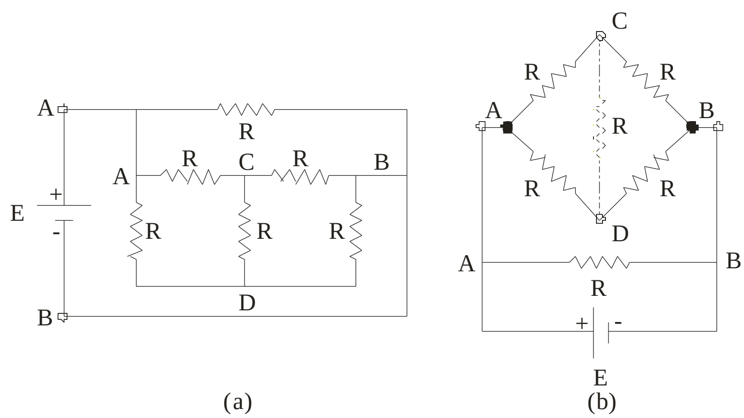

Fig. 3.224

$$R_{\text{eq}} = R \parallel (R + R) \parallel (R + R)$$

or, $$R_{\text{eq}} = \frac{1}{\left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}\right)} = \frac{2R}{(2+1+1)} = \frac{R}{2}$$

The current flowing through the points *A* and *B*, is the current drawn by the network from the battery and may be given as:

$$I_{\text{AB}} = \frac{E}{R_{eq}} = \frac{E}{(R/2)} = \frac{2E}{R}$$

The current through the arm *CD* (the central branch of the balanced Wheatstone bridge) may always be given as:

$$I_{\text{CD}} = 0$$

Problem 3.343: *Determine the ratio of the heat produced in the four arms of the Wheatstone bridge shown in the Fig. 3.225.*

Solution: The reader may easily observe that:

$$\frac{40}{10} = \frac{60}{15} = 4$$

So, this Wheatstone bridge is at its balanced position. So, the galvanometer is not carrying any current and shows the null deflection and hence the galvanometer may be assumed as open circuited.

Now, the voltage across various branches may be given as:

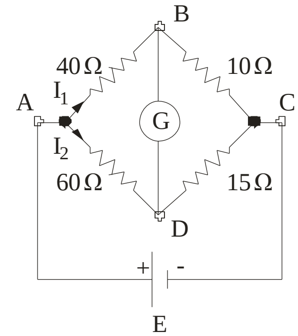

Fig. 3.225

$$V_{AB} = \frac{40}{40+10} \times E = 0.8\,E$$

and, $$V_{BC} = \frac{10}{40+10} \times E = 0.2\,E$$

and, $$V_{AD} = \frac{60}{60+15} \times E = 0.8\,E$$

and, $$V_{DC} = \frac{15}{60+15} \times E = 0.2\,E$$

So, the ratio of heat produced in four branches may be given as:

$$H_{AB} : H_{BC} : H_{AD} : H_{DC} = \frac{V_{AB}^2}{R_{AB}} \times t : \frac{V_{BC}^2}{R_{BC}} \times t : \frac{V_{AD}^2}{R_{AD}} \times t : \frac{V_{CD}^2}{R_{CD}} \times t$$

$$= \left[\frac{(0.8E)^2}{40} : \frac{(0.2E)^2}{10} : \frac{(0.8E)^2}{60} : \frac{(0.2E)^2}{15}\right] \times 750$$

$$= 12 : 3 : 8 : 2$$

Problem 3.344: *When two known resistances, R and S, are connected across the left and right gaps of a meter bridge, the balance point is located at a distance l_1 from the zero end of the meter bridge wire. An unknown resistance X is now connected in parallel to the resistance S and the balance point is now located at a distance l_2 from the zero end of the meter bridge wire. Obtain a formula for X in terms of l_1, l_2 and S.*

Solution: **First Case:**

$$\frac{R}{S} = \frac{l_1}{(100-l_1)} \qquad (3.256)$$

Second Case (When *X* is in parallel with the arm *S*):

$$\frac{R}{X\,S/(X+S)} = \frac{l_2}{(100-l_2)} \qquad (3.257)$$

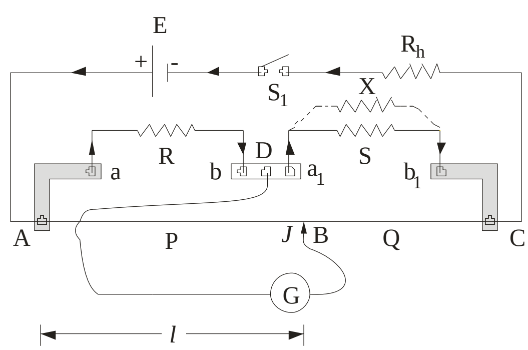

Fig. 3.226

Equation (3.257) / (3.256):

$$\frac{\left(\dfrac{R}{X\,S/(X+S)}\right)}{\left(\dfrac{R}{S}\right)} = \frac{\left(\dfrac{l_2}{(100-l_2)}\right)}{\left(\dfrac{l_1}{(100-l_1)}\right)}$$

or, $$\frac{X+S}{X} = 1 + \frac{S}{X} = \frac{l_2 \times (100 - l_1)}{l_1 \times (100 - l_2)}$$ or, $$X = \frac{S}{\left(\frac{l_2 \times (100 - l_1)}{l_1 \times (100 - l_2)}\right) - 1}$$

Problem 3.345: *The Wheatstone bridge shown in the Fig. 3.228 is showing no deflection in the galvanometer connected across the points B and D. Determine the value of R_{sh}.*

Solution: The equivalent resistance of shunted resistance 100 Ω may be given as:

$$Q_{\text{eq}} = \frac{100\,R_{sh}}{100 + R_{sh}}$$

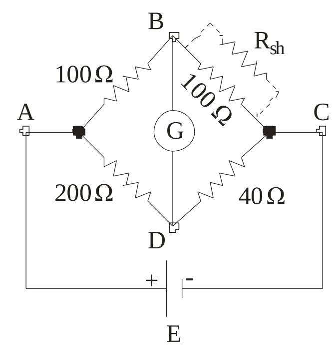

Fig. 3.227

For balanced condition of the bridge:

$$\frac{P}{Q_{eq}} = \frac{100}{100\,R_{sh}/(100 + R_{sh})} = \frac{R}{S} = \frac{200}{40}$$

or, $$\frac{100\,R_{sh}}{100 + R_{sh}} = 20$$

So, $$R_{\text{sh}} = \frac{2000}{(100 - 20)} = 25\ \Omega$$

Problem 3.346: *Four resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively are connected in cyclic order to form a Wheatstone bridge. Is the network balanced? If not, determine the resistance to be connected in parallel with the resistance of 10 Ω to balance the network.*

Solution: The ratio, $\frac{P}{Q} = \frac{15}{12} = \frac{5}{4}$

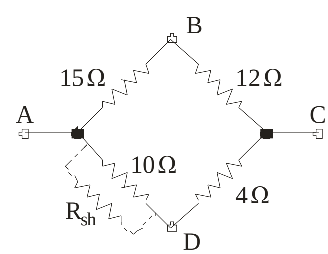

Fig. 3.228

The ratio, $\frac{R}{S} = \frac{10}{4} = \frac{5}{2}$

Since two ratios are not equal, so ***the bridge is not balanced.***

The equivalent resistance of shunted resistance 10 Ω may be given as:

$$R_{\text{eq}} = \frac{10\,R_{sh}}{10 + R_{sh}}$$

For balanced condition of the bridge:

$$\frac{P}{Q} = \frac{R_{eq}}{S}$$ or, $$\frac{5}{4} = \frac{10\,R_{sh}/(10 + R_{sh})}{4}$$

or, $$\frac{10\,R_{sh}}{10 + R_{sh}} = 5$$ or, $$R_{\text{sh}} = \frac{50}{(10 - 5)} = 10\ \Omega$$

Problem 3.347: *Determine the equivalent resistance across the points A and B in the network shown in the Fig. 3.229 (a).*

Solution: The given network may be redrawn as shown in the Fig. 3.229 (b).

Now, the reader may clearly observe that this is a balanced Wheatstone bridge.

As, $\dfrac{P}{Q} = \dfrac{1}{2} = \dfrac{R}{S} = \dfrac{2}{4}$

So, the central branch (10 Ω) will carry no current and may be assumed as open circuited. So, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (1 + 2) \parallel (2 + 4) = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

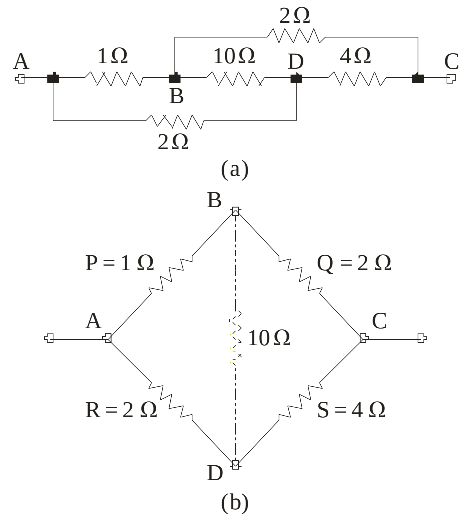

Fig. 3.229

Problem 3.348: *Determine the equivalent resistance across the points A and B in the network shown in the Fig. 3.230 (a).*

Solution: The given network may be redrawn as shown in the Fig. 3.230 (b).

Now, the reader may clearly observe that this is a balanced Wheatstone bridge.

As, $\dfrac{P}{Q} = \dfrac{R}{S} = 1$

So, the central branch (R) will carry no current and may be assumed as open circuited. So, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (R + R) \parallel (R + R) = \frac{2R \times 2R}{2R + 2R} = R$$

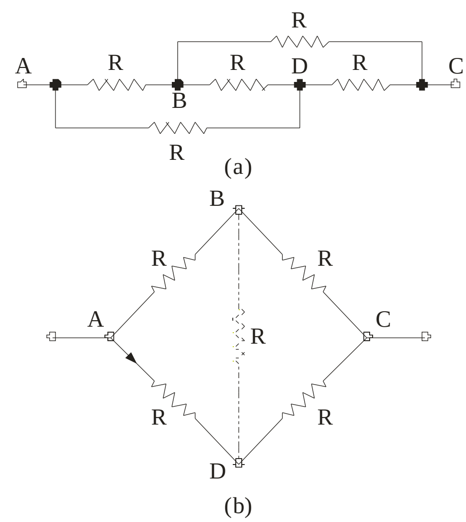

Fig. 3.230

Problem 3.349: *Determine the value of R and current through it, in the network shown in the Fig. 3.231 (a), if the current flowing through the resistance X is zero.*

Solution: The given network may be redrawn as shown in the Fig. 3.231 (b). The reader may observe that this is a Wheatstone bridge. We have for the balanced condition of Wheatstone bridge:

$$\frac{1}{4} = \frac{1.5}{R}$$

So, $R = 4 \times 1.5 = 6\ \Omega$

The equivalent resistance of the network as seen by the battery may be given as:

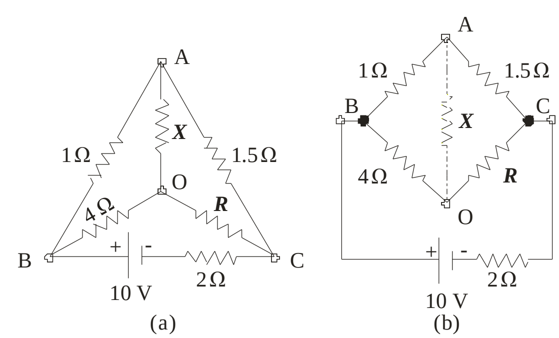

Fig. 3.231

$$R_{eq} = [(1 + 1.5) \parallel (4 + 6)] + 2 = \frac{2.5 \times 10}{2.5 + 10} + 2 = 2 + 2 = 4\ \Omega$$

The current drawn by the network from the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{10}{4} = 2.5\text{ A}$$

The current flowing through the resistance R (= 6 Ω) may be given as:

$$I_R = \frac{(1+1.5)}{(4+6)+(1+1.5)} \times 2.5 = 0.5\text{ A}$$

Problem 3.350: *Determine the equivalent resistance across the points A and B in the network shown in the Fig. 3.232 (a).*

Solution: The given network may be redrawn as shown in the Fig. 3.232 (b). The reader may observe that this is a balanced Wheatstone bridge. So, the central branch across the points C and D carries no current and hence may be assumed as open circuited. Now, the equivalent resistance across the points A and B may be given as:

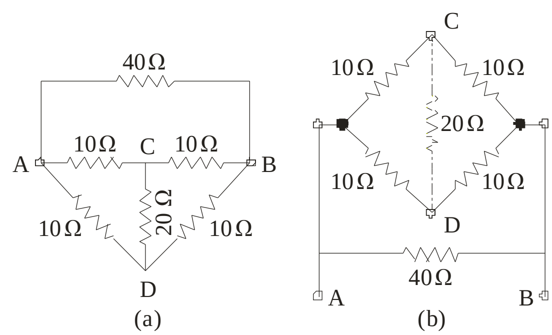

Fig. 3.232

$$R_{eq} = (10 + 10) \parallel (10 + 10) \parallel 40$$

$$= \frac{1}{\left(\frac{1}{20} + \frac{1}{20} + \frac{1}{40}\right)} = \frac{40}{(2+2+1)} = 8\ \Omega$$

Problem 3.351: *The potentiometer wire AB shown in the Fig. 3.233 is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection?*

Solution: Let the null point is detected at a distance of l from the point A.

So, according to the principle of potentiometer and the Wheatstone bridge:

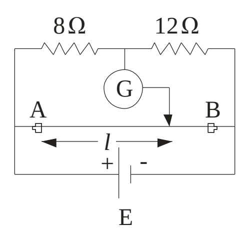

Fig. 3.233

$$\frac{8}{12} = \frac{l}{40 - l}$$

or, $80 - 2\,l = 3\,l$

So, $l = \frac{80}{5} = 16$ cm

Problem 3.352: *The potentiometer wire AB shown in the Fig. 3.234 is 50 cm long. When AD = l = 30 cm, a null deflection occurs in the galvanometer. Determine the value of R.*

Solution: According to the principle of potentiometer and the Wheatstone bridge:

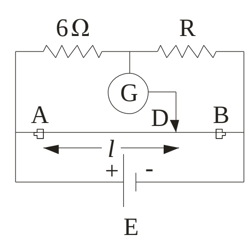

Fig. 3.234

$$\frac{6}{R} = \frac{l}{50 - l} = \frac{30}{50 - 30} = \frac{3}{2}$$

So, $R = \frac{2}{3} \times 6 = 4\ \Omega$

Problem 3.353: *Determine the value of unknown resistance X and the current drawn by the circuit, if no current flows through the galvanometer in the Fig. 3.235. Assume the resistance per unit length of the wire AB to be 0.01 Ω/cm.* [CBSE 2000-2001]

Solution: According to the principle of potentiometer and the Wheatstone bridge:

$$\frac{X}{4} = \frac{l}{100-l} = \frac{60}{100-60} = \frac{3}{2}$$

So, $X = \frac{3}{2} \times 4 = 6\ \Omega$

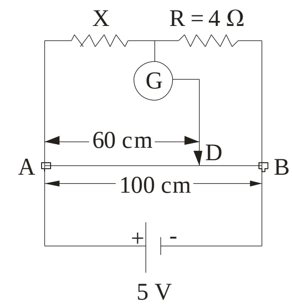

Fig. 3.235

The equivalent resistance of the circuit may be given as:

$$R_{eq} = R_{AB} \parallel (X+4) = 0.01 \times 100 \parallel (6+4)$$

$$= \frac{1 \times 10}{1+10} = \frac{10}{11}\ \Omega$$

So, the current flowing through the circuit may be given as:

$$I = \frac{E}{R_{eq}} = \frac{5}{(10/11)} = 5.5\ \text{A}$$

Problem 3.354: *In the circuit shown in the Fig. 3.236, P = 3 Ω, Q = 2 Ω, R = 6 Ω, S = 4 Ω and X = 5 Ω. Determine the value of current I drawn from the battery of emf 2 V.* [CBSE 1991-92]

Solution: The reader may observe that this is a Wheatstone bridge and is at its balanced position,

as, $$\frac{P}{Q} = \frac{3}{6} = \frac{R}{S} = \frac{2}{4}$$

So, the central branch connected across the points B and D will carry no current and may be assumed as open circuited.

Now, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (3+2) \parallel (6+4)$$

or, $$R_{eq} = \frac{5 \times 10}{5+10} = \frac{10}{3}\ \Omega$$

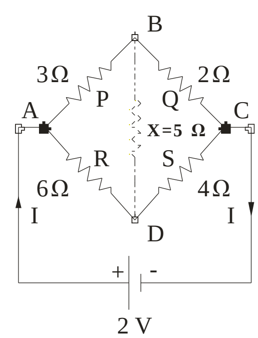

Fig. 3.236

So, the current drawn from the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{2}{(10/3)} = \frac{3}{5}\ \text{A} = 0.6\ \text{A}$$

Problem 3.355: *The ratio arms of a post office box are 1000 : 10. Determine the value of unknown resistance, if third resistance is 948 Ω.*

Solution: For a Wheatstone bridge under balanced condition:

$$\frac{P}{Q} = \frac{R}{S} \qquad \text{or,} \qquad \frac{1000}{10} = \frac{948}{S}$$

So, $S = \frac{10}{1000} \times 948 = 9.48\ \Omega$

Problem 3.356: *Each resistor r in the network shown in the Fig. 3.237 is of 10 Ω and the battery has an emf of 6 V. Determine the current supplied by the battery.*

Solution: The reader may observe that this is a Wheatstone bridge and is at its balanced position,

as, $$\frac{P}{Q} = \frac{r}{r} = \frac{R}{S} = \frac{r}{r}$$

So, the central branch connected across the points *B* and *D* will carry no current and may be assumed as open circuited.

Now, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (r + r) \parallel (r + r)$$

or, $$R_{eq} = \frac{2r \times 2r}{2r + 2r} = r = 10\ \Omega$$

So, the current drawn from the battery may be given as:

$$I = \frac{E}{R_{eq}} = \frac{6}{10} = 0.6\text{ A}$$

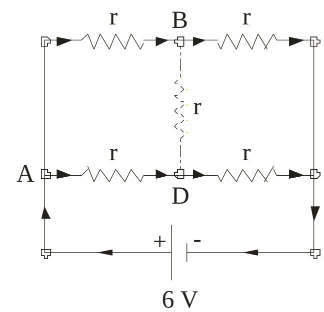

Fig. 3.237

Problem 3.357: *Determine the equivalent resistance across the points A and C in the network shown in the Fig. 3.238.*

Solution: The reader may observe that this is a Wheatstone bridge and is at its balanced position,

as, $$\frac{P}{Q} = \frac{10}{10} = \frac{R}{S} = \frac{10}{10}$$

So, the central branch connected across the points *B* and *D* will carry no current and may be assumed as open circuited.

Now, the equivalent resistance of the network as seen by the battery may be given as:

$$R_{eq} = (10 + 10) \parallel (10 + 10)$$

or, $$R_{eq} = \frac{20 \times 20}{20 + 20} = 10\ \Omega$$

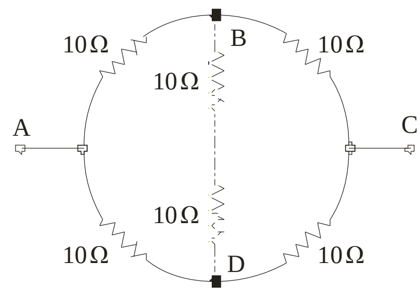

Fig. 3.238

Problem 3.358: *An unknown resistance X is placed in the left gap and a known resistance of 60 Ω is placed in the right gap of a meter bridge. The null point is obtained at 40 cm from the left end of the bridge. Determine the unknown resistance.*

Solution: $R = X$, $S = 60\ \Omega$, $l = 40$ cm

For a meter bridge:

$$\frac{R}{S} = \frac{l}{100 - l} \quad \text{or,} \quad \frac{X}{60} = \frac{40}{100 - 40}$$

So, $$X = \frac{40}{60} \times 60 = 40\ \Omega$$

Problem 3.359: *In a meter bridge setup, two resistances P and Q are connected in series in the left gap. When the resistance in the right gap is 50 Ω, the balance point is at the center of the slide wire. If P* *and Q are connected in parallel in the left gap, the resistance in the right gap has to be changed to 12 Ω so as to obtain the balance point at the same position. Determine P and Q.*

Solution: $S = 50\ \Omega$ (P and Q in series), $l = 50$ cm, $S' = 12\ \Omega$ (P and Q in parallel), $l' = 50\ \Omega$

First Case (P and Q in series):

$$\frac{R}{S} = \frac{P+Q}{50} = \frac{l}{100-l} = \frac{50}{100-50} = 1$$

or, $P + Q = 50$ (3.258)

Second Case (P and Q in parallel):

$$\frac{R'}{S'} = \frac{l'}{100-l'} \qquad \text{or,} \qquad \frac{(P\times Q/P+Q)}{12} = \frac{50}{100-50} = 1$$

or, $P \times Q = 1 \times 12 \times (P + Q) = 1 \times 12 \times 50 = 600$ (3.259)

Now, $(P - Q)^2 = (P + Q)^2 - 4\ P \times Q = (50)^2 - 4 \times 600 = 100$

or, $P - Q = 10$ (3.260)

Equation (3.258) + (3.260):

$$2\ P = 60 \qquad \text{or,} \qquad P = \frac{60}{2} = 30\ \Omega$$

Equation (3.258) – (3.260):

$$2\ Q = 40 \qquad \text{or,} \qquad Q = \frac{40}{2} = 20\ \Omega$$

Problem 3.360: *In a meter bridge, when the resistance in the left gap is 2 Ω and an unknown resistance in the right gap, the balance point is obtained at 40 cm from the zero end. On shunting the unknown resistance with 2 Ω, determine the shift of the balance point on the bridge wire.*

Solution: $R = 2\ \Omega$, $l = 40$ cm, $R_{sh} = 2\ \Omega$

For a meter bridge:

$$\frac{R}{S} = \frac{2}{S} = \frac{l}{100-l} = \frac{40}{100-40} = \frac{2}{3}$$

So, $S = 3\ \Omega$

The equivalent resistance of the resistance S on shunting with 2 Ω resistance may be given as:

$$S_{eq} = \frac{3\times 2}{3+2} = \frac{6}{5}\ \Omega$$

The new balance point may be given as:

$$\frac{R}{S_{eq}} = \frac{2}{(6/5)} = \frac{l'}{100-l'}$$

or, $1000 - 10\ l' = 6\ l'$ or, $l' = \dfrac{1000}{16} = 62.5$ cm

So, the shift in balance point may be given as:

$$\Delta\ l = l' - l = 62.5 - 40 = 22.5 \text{ cm}$$

Problem 3.361: *An experimental setup of a meter bridge is shown in the Fig. 3.239. When the two unknown resistances X and Y are inserted, the null point B is obtained 40 cm from the end A. When a resistance of 10 Ω is connected in series with X, the null point shifts by 10 cm. Determine the position of the null point when the 10 Ω resistance is connected in series with resistance Y. Also determine the values of the resistances X and Y.*

Solution: $l = 40$ cm, $l' = 40 + 10 = 50$ cm [for $(X + 10)$ Ω]

First Case (for X and Y in gaps):

$$\frac{X}{Y} = \frac{l}{l-100} = \frac{40}{100-40} = \frac{2}{3}$$

or, $X = \frac{2}{3}\,Y$

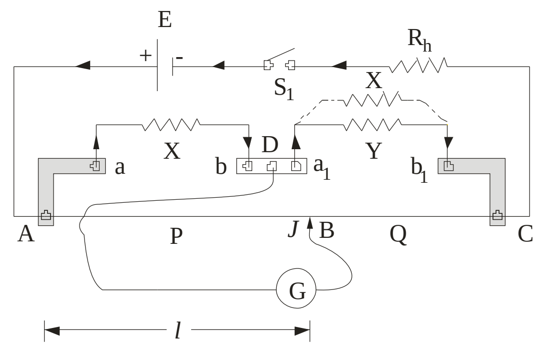

Fig. 3.239

Second Case [for $(X + 10)$ and Y in gaps]:

$$\frac{X+10}{Y} = \frac{l'}{100-l'} = \frac{50}{100-50} = 1$$

or, $X + 10 = Y$

or, $Y - X = 10$

or, $Y - \frac{2}{3}\,Y = 10$ or, $Y = 10 \times 3 = 30$ Ω

So, $X = \frac{2}{3}\,Y = \frac{2}{3} \times 30 = 20$ Ω

Third Case [for X and $(Y + 10)$ in gaps]:

$$\frac{X}{Y+10} = \frac{20}{30+10} = \frac{l''}{100-l''}$$

or, $100 - l'' = 2\,l''$ or, $l'' = \frac{100}{3} = 33.333$ cm